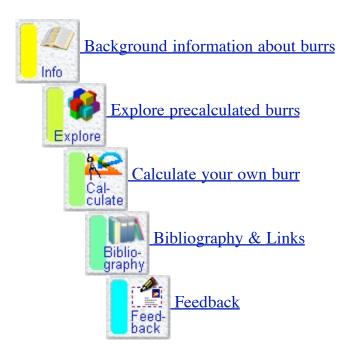
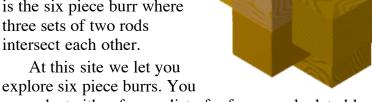
The burr puzzles site 03/21/2007 07:53 PM



The burr puzzles site



A Burr Puzzle consists of at least three rods intersecting each other at right angles. The most famous and well known type is the six piece burr where three sets of two rods intersect each other.



can select either from a list of a few precalculated burrs, or you can select six pieces of your own making and then have us calculate the possible solutions for you.

Note, that you will need a Java enabled browser to visit some pages of this site. The image hafter a hyper-link indicates which pages require a Java enabled browser. Virtually all pages require JavaScript to properly display.

Optimized for Netscape Navigator 3.0 and Microsoft Internet Explorer 3.0

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Burr puzzles background information

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A <u>Historical overview</u> gives a short introduction to the relatively few known facts about the roots of burr puzzles.

The <u>Definitions</u> section contains a rigorous collection of definitions used in the following sections. A few simple Lemmas are also introduced to show some trivial conclusions following directly from the definitions.

The <u>Three-piece burrs</u> are the simplest possible burrs. That is - they have the fewest number of rods possible. However, there are no regular three-piece burrs possible. This section explains why, and shows some non-regular designs to build a three-piece burr after all.

Six-piece burrs are the most common burrs. This section introduces previously published material as well as new insights gathered from our own work.

The <u>Bibliography</u> lists books, articles, and other web-sites which were used to start our research into the area of burrs. There are not many such publications, and few of them deal exclusively with burrs.

[IBM Research | Burr Puzzles Site]



Historical overview

Little is known about the early history of interlocking puzzles, but they were certainly produced both in Asia and Europe as early as the 18th century. Six-piece burrs were shown as early as 1803 in the Bestelmeier Toy catalogs. But it wasn't until Edwin Wyatt published Puzzles in Wood in 1928 that a book devoted to many interlocking puzzles was available. Puzzles of this type used to be known as "Chinese" puzzles, probably because they were mass-produced in the Orient since the early 1900s, but there does not appear to be any evidence that the idea originated there. Nowadays they are commonly referred to as "burr" puzzles. Wyatt introduced the term because the puzzles looked like a seed burr.

By far the most familiar burr is the six-piece burr. The six-piece burr is actually a large family of designs, since the designer has a wide choice of how to notch each piece. Several versions have been patented and manufactured. The earliest US patent is No. 1,225,760 of Brown, dated 1917, with several others following shortly thereafter.

Most toy and novelty stores have a few burr puzzles on their shelves. Unfortunately these are often the uninspired, time-worn versions with sliding keys and internal symmetries. The puzzle has suffered from this tarnished image, and to make matters worse, inventors have tinkered with bizarre embellishments like cords through holes to give the basic puzzle their twist.

In 1978, Bill Cutler has returned to the basic design and published a paper describing the solid six-piece burr completely [Cutler78]. He has shown that there are 25 possible notchable pieces to make solid six-piece burrs and that they can be put together in 314 ways. He also showed that there are 369 general pieces usable to make solid burrs and that they can be put together in 119,979 ways.

More recently, Stewart Coffin, Bill Cutler, Philippe Dubois, and Peter Marineau have come up with higher-level six-piece burr designs that are not solid and in which several pieces must be moved before one can be taken out. Stewart Coffin has enhanced the art of burr designs into abstract geometrical forms. His book The Puzzling World of Polyhedral Dissections contains close to 100 exquisite designs of burr and other wooden puzzles.

From the late 1980s to the mid 1990 Bill Cutler and others undertook a complete analysis of all six-piece burrs [Cutler94]. Original estimates by Cutler assumed it would take about 62.5 years to run the analysis on a PC AT. With some help from others who run Cutlers programs on faster and bigger machines, total calculation time was cut down to 2.5 years. From this analysis we now know that there are roughly 35.65 billion ways to assemble burr puzzles pieces (71.3 billion if mirror images are counted also). Of these 35.65 billion logical assemblies 5.95 billion can be taken

apart. The highest level found was a level 12 puzzle. Unfortunately it is not unique, i.e. it has more than one assembly, most of which are of a much lower level. The highest level unique sixpiece burr is of level 10 if the pieces are 8 units long and level 9 if the pieces are 6 units in length. If all pieces are notchable, the highest level is 5 for a unique burr.

[IBM Research | Burr Puzzles Site]



Definitions

The definitions on this page introduce some basic terms used to describe and define burr-puzzles. The definitions are all numbered in a decimal classification scheme to simplify their referencing. The definitions are mostly trivial and kept brief for that matter.

A few basic lemmas have been interspersed where they follow immediately from the given definitions.

Rods

Rods are the basic building blocks from which burrs are made. There are quite a number of terms used to qualify the peculiarities of rods. Note, that we often use the term *piece* interchangeably with the term rod.

Def 1.0: Rod

A rod is a piece [of wood] with one dimension usually longer than the other two.

Def 1.1: Notched Rod

A rod with notches cut out.

Def **Disjoint Rod**

1.1.1: A notched rod where the notches leave the rod in more than one piece.

Def **Connected Rod**

1.1.2: A notched rod where the notches leave the rod in one piece.

Def 1.2: Rectangular Rod

A rod of dimensions w^*h^*l , with l > w and l > h, is said to be rectangular.

Def Square Rod

1.2.1: A rectangular rod of dimensions 2*2*n, with n > 2, is said to be square.

Def Regular Rod

1.2.1.1: A regular rod is a notched, connected, square rod with notches in multiples of 1*1*1 cubes.

Def Solid Rod

1.2.1.1.1: A solid rod is a regular rod with no notches.

Def **Notchable Rod**

1.2.1.1.2: A rod is said to be notchable when each cross-section perpendicular to its length axis is convex. This restriction is often made on pieces to allow them to be easily produced by saw cuts perpendicular to the axes.

Burrs

Burrs are what this is all about. Using the definitions of the Rod from above we can now define what a Burr is.

Def 2.0: Burr

A burr can be roughly described as an interlocking geometrical puzzle with a high degree of external symmetry which is composed of notched, connected, rods (often made out of wood) [Cutler78].

Def 2.1: Regular Burr

A regular burr consists of three sets of regular rods intersecting each other at right angles. The length of the regular rods is depending on the particular burr. The burr must not show any external holes, but it can have internal voids.

Def Three-piece Burr

2.1.1: A three-piece burr consists of three square rods intersecting each other at right angles. The rods have the dimensions 2*2*4. Note: The length used in this definition is 4, the minimum required. However, the length used most commonly is 6.



Lemma The three-piece burr is the simplest regular burr

2.1.1.1: *Proof:* Each of the three sets consists of exactly one rod. This cannot be further reduced without making one of the sets empty.

Def Six-piece Burr

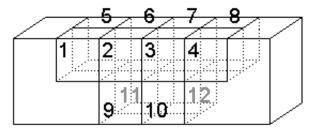
A six-piece burr consists of three sets of two rods intersecting each other at right angles. The rods have the dimensions 2*2*6. Note: The length used in this definition is 6, but there are many six-piece burrs where the rods can be made longer if desired; many use 8 as a length to make real wood burrs for easier handling. Few use longer pieces of 10 or 12 units.

Lemma Not all six-piece burrs with a rod-length greater than 6 can be taken apart.

2.1.2.1: *Proof*: Philipe Dubois' burr cannot be taken apart if the rods have a length greater than 6. [Slocum86].

Lemma For regular six-piece burrs, the pieces can **2.1.2.2:** only have the cubes 1 through 12 notched

Proof: This is necessary to assure that there are no exterior holes in the burr.



Related terms and definitions

The following terms will be used in conjunction with regular Burrs to describe their properties.

Def 3.0: <u>Core</u>

The core of a regular burr is the volume taken up by the intersection of the rods.

Def 4.0: Weight

The weight of a regular burr is the number of 1*1 cubes of the core which are occupied by rods.

Def 4.1: Solid

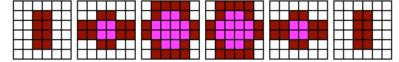
A regular burr is said to be solid if it is of maximum weight. That is, if the core of the burr is completely occupied by the rods, and there are no interior holes.

Lemma A solid three-piece burr has a weight of 8.

4.1.1: *Proof:* Looking at the pictures in the section about <u>Three-piece Burrs</u> will make it immediately evident, that the core consists of a 2 by 2 by 2 cube and its maximum weight is therefore 8.

Lemma A solid six-piece burr has a weight of 32.

4.1.2: *Proof:* Slicing a six-piece burr into six layers reveals the 32 internal cubes.



Def 5.0: Level

The level of a burr is the number of moves needed to remove a piece. For example a level 2.3 burr means there are two moves needed to remove the first piece (or pieces) and three more for the second piece (or pieces).

Note, that this definition as stated in several books is not very accurate, and has lead our calculation routines to label several puzzles with the wrong level. For a discussion and more accurate definition see the section about <u>Six-piece Burrs</u>.

Lemma Solid burrs are all of level 1.

5.0.1: *Proof:* If there are no interior holes, then it must be possible to slide one or more key pieces out in the first move.

Def 6.0: <u>Type</u>

The type of a burr is the number of pieces which get removed at each consecutive move. For example a type 2.2.1.1 for a six-piece burr means that first 2 pieces are removed, then another 2, and the remaining 2 pieces are removed 1 by 1.

Note: This definition is different from Bill Cutler's definition! He defines it as the way the first move can be made to take the puzzle apart [Cutler78]. In his definition there are the following types:

- 1.5 One piece can be taken out (this denotes all burrs with a solid key)
- 2.4 The burn can be separated into two sets of 2 and 4 pieces.
- 2.1.3 The burr can be separated by either pulling out 2 or 3 pieces
- 2.1.1.2 The burr comes apart by pulling out either set of 2 pieces.
- 3.3 The burr comes apart into two halves.
- 2.4-3.3, 2.1.3-3.3, 2.1.1.2-3.3, 3.3-3.3 The initial move can be made in one of two ways.

Def 7.0: <u>Fitable</u>

A set of rods is said to be fitable if they can be logically assembled into the shape of the burr. That is if there is a way to fit all rods into the burr.

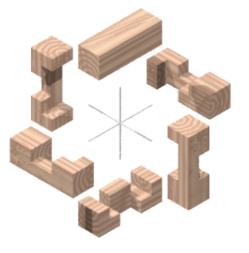
Def 8.0: Solvable

A set of fitable rods is said to be solvable if the given burr can be taken apart by only moving the rods along the three perpendicular axes. No twisting or turning of the pieces is allowed.

Lemma There are fitable burrs which are not solvable.

8.0.1: *Proof:* The following set of pieces fits in exactly one way but cannot be taken apart.

See a java or a larger static version of this burr.



[IBM Research | Burr Puzzles Site]

Three-piece burrs

For the simplest of all burrs, there exists no regular solution. The first part will show why. The second part will show how people have overcome the problem by relaxing some of the constraints to construct non-regular three-piece burrs.

Regular three-piece burrs

For regular three piece burrs we are looking at rods of the size 2*2*6. We use the more common length of 6 in preference over the minimal size 4. Since the core of a three piece burr is 8 (<u>Lemma 4.1.1</u>), there are potentially 2⁸=256 possible rods. By excluding disjoint and duplicate rods there are exactly 28 unique ones, of which 9 are notchable. A list of all the pieces can be found on the <u>Three-piece burr pieces</u> page.

Not all of those rods could actually be used in the construction of a regular three-piece burr.

Lemma The solid rod may not be used in the construction of a regular three-piece burr.

1.1.1.2: Proof: Using a solid rod would leave the other two rods disjoint.

Theorem There is no solvable regular three-piece burr.

1.1.1.3: Proof: After the first two pieces have been assembled, there is no way left to

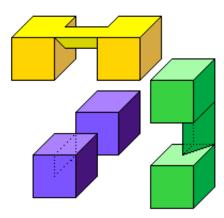
slide the third one in.

Non-regular three-piece burrs

While there are no regular three piece burrs, there are, however, a few solutions to a non-regular three-piece burr. Some of these designs are described in the following sections.

Segerblom's three-piece burr

| History | Definitions | 3-piece | 6-piece | Bibliograph



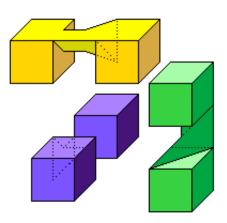
In April 1899, Scientific American published a puzzle, designed by Wilhelm Segerblom of Wakefield, Massachusetts. This design is also described in "Puzzles Old & New", p.66 [Slocum86], and in the "Book of Ingenious & Diabolical Puzzles", p. 73 [Slocum94].

All three pieces are identical. There exists only one solution since the pieces and their mirror images are identical.

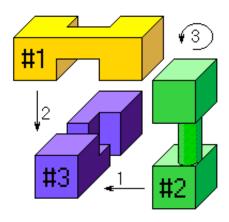
To assemble, all three pieces have to be slid together in a diagonal movement. It is not possible to put two pieces together and then slide the third one in.

Improved Segerblom three-piece burr

The "Book of Ingenious & Diabolical Puzzles" mentions an improved version of the Segerblom design [Slocum94]. The picture in the book is too unclear to give an exact design. However, it looks as if the improvement consist in the addition of a triangular section to improve stability, but without filling the center completely. My best guess is a design as shown in the figure to the right.



Wyatt's three-piece burr



Edwin Wyatt, describes in his booklet "Puzzles in Wood" a version which requires a piece which can be rotated to make room to slide the other two pieces together [Wyatt28].

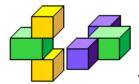
This design relaxes the constraint that rods can only have notches in multiples of 1*1*1 cubes. One of the rods has a notch leaving only a cylindrical part around which the piece can be rotated.

Since piece #3 has a mirror image of itself, there are two possible solutions to this design. They are mirror images of each other.

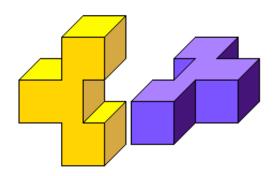
Nob Yashigahara's fake three-piece burr

Nob Yashigahara invented this three-piece burr which is made up of only two pieces! [Slocum94, p.72].

The design relaxes the constraint of square rods by taking two rods of size 2*4*6 and cutting them as shown in the following figure.



If you make the two rods from three kinds of woods, it will be hard to tell that this is not a three-piece burr, but just looks like one.



[IBM Research | Burr Puzzles Site]





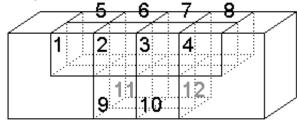
Six-piece burrs are the simplest regular burrs. To analyze the six-piece burrs, we first discuss the pieces used to build them. Then we introduce a few interesting facts about six-piece burrs. Finally we present some statistics from a (mostly) complete analysis made by Bill Cutler.

Six-piece burr pieces

Before an analysis of six-piece burrs can be conducted, the usable pieces need to be computed. William Cutler was the first to completely analyze six-piece burr pieces used to make solid sixpiece burrs [Cutler78].

To get a complete analysis we need to find a way to enumerate the pieces. To be able to refer to particular pieces, we introduce the following numbering scheme:

Def 9.0: Binary Piece Number



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pieces. To be able to refer to

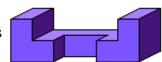
re seen in Lemma 2.1.2.2 a six-piece may have only the cubes 12 notched out. If we write or 0 for the presence or absence. As we have seen in Lemma 2.1.2.2 a sixpiece burr piece may have only the cubes 1 through 12 notched out. If we write down a 1 or 0 for the presence or absence of any of the twelve cubes we get the binary number of a piece.

Lemma The solid six-piece burr piece has a binary number of 111111111111.

Proof: The solid piece does not have any cubes removed, hence all of the twelve cubes 9.0.1: are present.

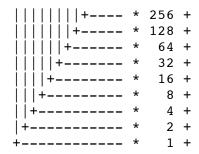
Lemma The lightest usable piece has the binary number 00000000011.

9.0.2: *Proof:* The following image shows the cubes 1 through 10 are absent and 10 through 12 are present. Removing any more cubes would leave the piece disjoint.



Def 9.1: Decimal Piece Number

The decimal piece number is defined as 4096 minus the binary piece number, expressed as a decimal number as shown below:



Note: the reason for this seemingly reverse definition from normal mathematical notations of binary numbers was chosen for a graphical reason. If a program lists all the pieces sorted by decimal numbers, then this definition will yield generally better visible 3D representations of the pieces.

Lemma The solid six-piece burr piece has the decimal number 1.

9.1.1: $Proof: 4096 - 111111111111_b = 4096 - 4095 = 1.$

Note: The fact that the solid piece comes out as number 1 is the main reasons for choosing to subtract the binary number from 4096.

Lemma The lightest usable piece has the decimal number 1024.

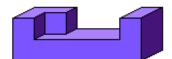
9.1.2: $Proof: 4096 - 000000000011_b = 4096 - 2048 - 1024 = 1024.$

Note: If you rotate the piece by 180° you get the piece 00000001100 which has the decimal number $4096 - 000000001100_{b} = 4096 - 512 - 128 = 3456$.

We have already introduced notchable pieces in <u>Def 1.2.1.1.2</u>. now we are going to define the type of a six-piece burr piece with a finer grain of detail.

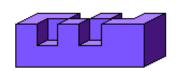
Def 9.2: internal corner

A piece is said to have an internal corner, if the sides of three cubes meet inside the piece in a concave fashion. The piece shown to the right has one internal corner.



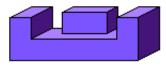
Lemma The maximum number of internal corners a piece can have is **9.2.1:** 8.

Proof: The piece on the right shows 8 internal corners, trying to remove any more cubes e.g from the bottom either reduces the number of internal corners or makes the piece disjoint.



Def 9.3: millable

A piece is called millable if it can be produced by a milling machine. This means the piece may not have any internal corners. For example the piece shown in the picture is millable.



Lemma Notchable pieces are millable.

9.3.1: *Proof:* Since notchable pieces have no concave crosscuts perpendicular to the axis, no internal corners may be formed. Hence they are millable.

Lemma There are millable pieces which are not notchable.

9.3.2: *Proof:* The piece shown in the definition 9.3 above, is millable but not notchable.

Def 9.4: Piece Type

Piece types are introduced as to whether a piece is notchable, millable, or neither. And within those three categories they are divided as to whether they can be used to build solvable, solid six-piece burrs.

Type Description

notchable32 notchable piece that can be used for solid burrs notchable notchable piece that can be used for general burrs millable32 millable piece that can be used for solid burrs millable millable piece that can be used for general burrs general32 general piece that can be used for solid burrs general piece that can be used for general burrs disjoint unusable piece

Note: the numeral 32 signifies the fact that a solid burr has a weight of 32 (see <u>Lemma 4.1.2</u>).

Theorem A notched piece is usable for solid burrs (of type notched32) if it meets the following **9.4.1:** criteria:

- 1. The piece is solid
- 2. The following cubes may only be removed as a pair: 1 and 5, 4 and 8, 9 and 10, 11 and 12. Cubes 2, 3, 6, and 7 may be removed individually.

Proof: see [Cutler78].

Theorem A general piece is usable for solid burrs (of type general32) if it meets the following **9.4.2:** criteria:

- 1. The piece is solid
- 2. Cubes 1, 2, 5 and 6 removed; others may vary
- 3. Cubes 3, 4, 7 and 8 removed; others may vary
- 4. Cubes 2, 3, 9 and 10 removed
- 5. Cubes 6, 7, 11 and 12 removed
- 6. Cubes 1 and 5 removed and either (a) 3 and 7 removed, (b) 4 and 8 removed, or (c) 4 and 8 not removed
- 7. Cubes 2 and 6 removed and either (a) 4 and 8 removed, or (b) 4 and 8 not removed
- 8. Cubes 1 and 5 not removed and (a) 3 and 7 removed, or (b) 4 and 8 removed

Proof: see [Cutler78].

Theorem For each type there exist the following number of usable, unique pieces. That is duplicates (from rotations) have been removed.

Type	Nbr of Pieces List
notchable32	25 🗎 (20KB)
notchable	59 🗎 (44KB)
millable32	78 🗎 (57KB)

general 32 369 (250KB) takes long to load general 837 (554KB) takes forever to load

Proof: enumeration by computer programs, using Theorems 9.4.1 and 9.4.2.

Now we have enumerated all the pieces and we could move on to analyze what six-piece burrs can be built using these pieces. But before we do so, we want to mention a few more facts about six-piece burr pieces:

Def 9.5: Piece Weight

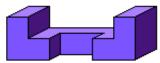
The weight of a six-piece burr piece is 12 minus the number of 1*1*1 cubes which have been removed.

Lemma The solid six-piece burr piece has a weight of 12.

9.5.1: *Proof:* The solid piece does not have any cubes removed, hence the weight is 12-0 = 12.

Lemma The lightest usable piece has a weight of 2.

9.5.2: *Proof:* The following image shows the piece with all but 2 cubes removed. Removing any more cubes would render the piece disjoint and thus unusable.



Lemma The weight of a piece is equal to the number of the 12 center cubes which have not **9.5.3:** been removed.

Proof: This follows immediately from the definition.

Burr weights

In <u>Def 4.0</u> we have defined the weight of a regular burr as the number of 1*1 cubes of the core which are occupied by pieces. Given the above definitions on piece weights, we can now formulate the relationship between piece weights and burr weights:

Theorem The weight of a regular six-piece burr is equal to the sum of the weights of the pieces.

10.0: $W_B = W_{P_1} + W_{P_2} + W_{P_3} + W_{P_4} + W_{P_5} + W_{P_6}$

Proof: If we would take six pieces of weight 0 (which are disjoint), the core of the burr would be completely empty. The weight of the six-piece burr would then be 0. Now we add one cube to one of the pieces, this will increase the sum of the piece weights by one as well as the burr weight by one. We can continue to do so until we have inserted 32 cubes and the six-piece burr is solid.

Lemma If the sum of the weight of six given pieces is greater than 32, they cannot be assembled into a six-piece burr.

Proof: According to the above Theorem 10.0 the weight of the six-piece burr would be greater than 32, which cannot be according to Lemma 4.1.2.

Burr levels

In <u>Def 5.0</u> we have defined a level as the number of moves required to release a piece. This definition, however, leaves a couple of open questions. First, what constitutes a move, and second, is the move taking the piece out counted or not. If we assume that a move is considered as moving a piece in a particular direction, and we count the removing of the piece as a move as well, then we arrive at the conclusion that the levels published for the <u>Fearsome Four</u> in Puzzles Old & New [Slocum86] are correct. However the level 9 for Peter Marineau's is not. It would be level 10. We have to further add the constraint that, if two independent pieces get moved in the same direction then we consider this only one move. Note that there is a difference between being able to move two pieces independently in the same direction versus two pieces which can only be moved as a unit. Since our computation algorithms favored the behavior to move pieces independently if possible, special code had to be added to the calculation routines to compact the moves if possible. With this further constraint we arrive at the following more exact definition of a level.

Def 11.0: Level

The level of a regular burr is the number of moves needed to remove the first piece or pieces. A move is counted as one irregardless of how far a piece is moved in one direction. Moving a piece in one direction and then immediately in another is considered two moves. The move to remove the piece is counted for the level also. Moving two or three pieces independently in the same direction immediately after each other is counted as one move. This process is continued for the second, third, etc. piece. The level is then expressed as a.b.c.d.e.f where a is the number of moves to remove piece one, b the number of moves to release piece two and so on.

This definition will however reclassify Philippe Dubois' burr from a level 7.4 to a level 6.4. But we decided to stay with this definition since it gets more of the published burr levels right.

Def 11.1: Level Type

The level of a regular burr generally changes with the length of the pieces used. Cutler introduced the level type to denote the variation in levels with the length of the pieces. It is a four digit number in hexadecimal notation to denote the four levels at the piece length 6, 8, 10, and 12. For example a type of 9900 indicates that the burr puzzle is of level 9 with pieces of length 6 and 8. It cannot be solved with pieces of length 10 or 12.

Note: the java program used to calculate burrs at this site only take into consideration a given piece length at a time. There is no code to calculate all four length in succession. Hence it does not calculate the level type.

Cutler's analysis of solid six-piece burrs

William Cutler was the first to completely analyze solid six-piece burrs in 1978 [Cutler78]. He showed that there are exactly 25 of the possible 59 notchable pieces which can be used to build solid six-piece burrs. With these 25 pieces, a total of 314 solvable burrs can be built. 158 of them use a solid piece as the key.

Cutler further published that there are 369 general pieces which can be used for solid burrs. With these, a total of 119'979 solvable solid burrs are possible. The following table summarizes his

findings.

Type	Notchable	All (including Notchable)
1.5	158	6'402
2.4	49	22'840
2.1.3	62	17'574
2.1.1.2	20	525
3.3	16	72'485
2.4-3.3	2	26
2.1.3-3.3	4	28
2.1.1.2-3.3	1	1
3.3-3.3	2	98
Totals	314	119'979

Cutler's analysis of holey six-piece burrs

From the late 1980s to the mid 1990 Bill Cutler and others undertook a complete analysis of all six-piece burrs [Cutler94]. Original estimates by Cutler assumed it would take about 62.5 years to run the analysis on a PC AT. With some help from others who run Cutlers programs on faster and bigger machines, total calculation time was cut down to 2.5 years. From this analysis we now know that there are roughly 35.65 billion ways to assemble burr puzzles pieces (71.3 billion if mirror images are counted also). Of these 35.65 billion logical assemblies 5.95 billion can be taken apart. The highest level found was a level 12 puzzle. Unfortunately it is not unique, i.e. it has more than one assembly, most of which are of a much lower level. The highest level unique six-piece burr is of level 10 if the pieces are 8 units long and level 9 if the pieces are 6 units in length. If all pieces are notchable, the highest level is 5 for a unique burr.

This analysis has completely explored all assemblies for the first piece to be taken out. From there only high level burrs have been completely analyzed. For lower level burrs the programs where not run to complete the disassembly. So the number of total solvable burrs is a statistical estimate.

The following table summarizes some of the findings for the high level burrs:

	Note	hable	General					
	Lev	vel 5	Level 8		Level 9		Level 10	
Holes	Count	Unique	Count	Unique	Count	Unique	Count	Unique
0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0
2	1	0	0	0	0	0	0	0
3	10	4	0	0	0	0	0	0

4	94	11	15	13	0	0	0	0
5	536	21	1031	828	23	21	0	0
6	1949	40	3730	2128	277	136	0	0
7	4990	31	6754	1681	924	154	13	8
8	9230	22	10353	485	1526	32	36	10
9	12917	10	14415	95	1714	11	58	0
10	14220	0	14264	21	1397	1	96	0
11	12846	0	7816	3	723	0	110	0
12	9431	0	2121	0	192	0	51	0
13	5196	0	268	0	16	0	8	0
14	2014	0	16	0	0	0	1	0
15	547	0	1	0	0	0	0	0
16	95	0	0	0	0	0	0	0
17	9	0	0	0	0	0	0	0
18	0	0	0	0	0	0	0	0
19	0	0	0	0	0	0	0	0
20	0	0	0	0	0	0	0	0
Total	74085	139	60784	5254	6792	355	373	18

[IBM Research | Burr Puzzles Site]

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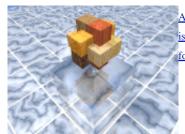
Explore precalculated six-piece burr puzzles

This site was inspired by a page in the book <u>Puzzles Old & New</u>, with the title **Super Burrs - The** Fearsome Four. It contained a black and white drawing of four six-piece burrs layed out as shown in the picture below. They were claimed to be higher level burrs and it was the first time I had seen any higher level burrs. Needless to say, I made the pieces out of wood to have a set of my own. The challenge - however - there were no solutions in the book. This brought on the creation of a six-piece burr puzzle solving computer program. One thing led to another and voila, the burr puzzles site was born. The reason the program was written was because I could not solve Bill's baffling burr, the computer program confirmed this. Further studies revealed that it was published wrong. Later I found it published correctly in Coffin's <u>Puzzling World of Polyhedral Dissections</u>.

The Fearsome Four



Other Burrs



An animated version of this image is also available (830KB, takes forever to load)

- Solid Burrs
- Notchable Higher Level Burrs
- General Higher Level Burrs

Definition: The **Level** indicates the number of shifts required to remove a piece. For example a level 7.4 requires seven moves for the first piece to be removed and 4 more for the second piece.



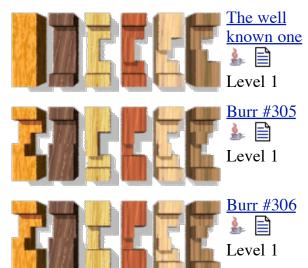


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Solid sample burr puzzles

Cutler has shown in his article "The Six-Piece Burr" that there are a total of 314 solutions to solid six piece burrs if only notchable pieces are used. One of them is particularly well known and has been often used to produce commercial six-piece burrs. Stewart Coffin has described two of the 314 puzzles as the most interesting (#305) and the most difficult (#306) ones in his book "The Puzzling World of Polyhedral Dissections".

Burr



Description

There are two solutions, one being the mirror image of the other.

Note that only the third piece is different from burr #305.

[IBM Research | Burr Puzzles Site]





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Notchable higher level sample burr puzzles

Notchable burr puzzles have the big advantage that they are easy to manufacture. They are thus given special consideration in the published literature about burr puzzles. Bill Cutlers analysis has shown that there are 13'354'991 possible assemblies using the 59 notchable pieces. The highest level notchable burr is a level 10, however it is not a unique solution and thus not that interesting. The highest level for unique solutions is 5 and there are 139 of them [Cutler94].

From the following table select the burr you want to explore. Select the icon for a static image representation of the burr. These static pages are good to print a solution on paper which you can take it into your work shop for making a physical model.

Burr Designs from Cutlers analysis [Cutler94]

Burr

Notchable 1-Hole Level-2

Description

1987: There are only 4 notchable burrs with 1 hole which have a level 2 (or higher) solution. One was chosen at random.

1987: the highest level for unique solutions is 5. There are 139 such burrs. This is one of them.



Level 2



Level 5

Notchable Level-5

Most







Level-10 <u>.</u>

1987: unique level 5 burr with the most (480) false assemblies. The piece length is 10. If the length is 6 there are 144 solutions with levels ranging from 2 to <u>Assemblies</u> 3. With length of 8 there are still two solutions.

Notchable 1987: the highest level for non-unique solutions is 10. non-unique There are only 2 such burrs. This is the one with fewer holes. Its level type is 46AA meaning that at piece length 6 the highest level is 4, at piece length 8 Level 1-10 the highest level is 6, and at length 10 and 12 the highest level is 10.

Burr Designs by David Winkler [David Winkler's Burr Page]



1996: One of the simplest higher level burrs, this level 3 burr has really only one solution, neglecting symmetries, and no false solutions.



David
Winkler's
favorite
level 5
burr

1996: This is a level-5.4 burr with only one solution, but with 23 apparent solutions.



David
Winkler's
complex
level 5
burr

Level 5.4

Level 5.4

1996: This is a level-5.4 burr with only one solution, but with 143 apparent solutions, the most for notchable burrs of piece-length 6.

Burr Designs by Jürg von Känel



Programs written to conduct my own research for creating this site.

[IBM Research | Burr Puzzles Site]





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General higher level sample burr puzzles

While general burr puzzles are harder to manufacture, some of them have much higher levels. The highest level is 12, however the only such burr - Love's Dozen - is not unique. At a piece length of 8 the highest level unique burr puzzles are 10. At a length of 6 the highest level is 9. Most fascinating is the fact, that Peter Marineau designed his level 9 Piston puzzle without the help of a computer and before Bill Cutler had completely analyzed all six-piece burrs. Most of the designs listed here are from Bill Cutler's two publications documenting his computer analysis of six-piece burrs [Cutler88 and Cutler94]. Through e-mail I got a few designs from Peter Rösler who, in 1989 together with Gerhard Dotzler and Thomas Kiss, two students at the Technical University of Munich, ran C programs to design their own burrs.

From the following table select the burr you want to explore. Select the icon for a static image representation of the burr. These static pages are good to print a solution on paper which you can take into your work shop for making a physical model.

Burr

Designs by Bruce Love



Level 1-12.3

Description

1987: this non-unique level 12 six-piece burr is the highest level possible. It is the only one at level 12, and there are none at level 11 at all. It has a total of 154 assemblies of which 89 can be taken apart!

Designs by Peter Marineau



Peter
Marineau's
Piston
Puzzle Burr

Level 9.3

1986: Peter Marineau designed this puzzle by hand. It was the highest level burr known before Bill Cutler did his exhaustive computer analysis.

Designs by Bill Cutler



Computer's
Choice
Unique
Level 10

1988: 10 is the highest level for which unique sixpiece burrs exist. There are 18 such unique level 10 solutions, all of which disassemble in a similar fashion. Note that this particular example is also unique at piece length 6, but the level is only 5 in that Level 9

Level 10

Computer's Choice 5-Hole

case.

1988: There are 6.5 billion 5-hole assemblies. For the highest level 9 there are 23 solutions of which 21 are unique.



Computer's Choice 3-Hole

1988: There are 2.5 billion 3-hole assemblies. For the highest level 7 there are 198 solutions of which 157 are unique.



Level 3

Bill Cutler's 1986: This is the least un-notchable level-3, 1-hole BB31-10-40 design, of a large number of such designs discovered.





#C "teuflische Verführung"

This puzzle has 5 false solutions and 1 which can be taken apart. Even in all the false solutions some pieces can be moved. The overall sequences of moves leads to a more satisfying pattern than the next puzzle.



#D "super" Level 9

Level 9

The piece which comes out first moves on a zig-zag pattern. This leaves a subjective feeling that the burr is of a lower level than it actually is.



A very satisfying puzzle due to the fact that the internal structure of the puzzle can be readily deduced by logical analysis (rather than brute force trial and error).

This is a very fascinating burr puzzle. Before it a

piece can be taken out, every move moves exactly 3

Designs by Stewart Coffin



Stewart Coffin's Triple Slide Level 3

pieces (that is half of all pieces). Two such moves of three pieces have to be executed before it comes apart

into 2 and 4 pieces.

Burr Designs by Jürg von Känel



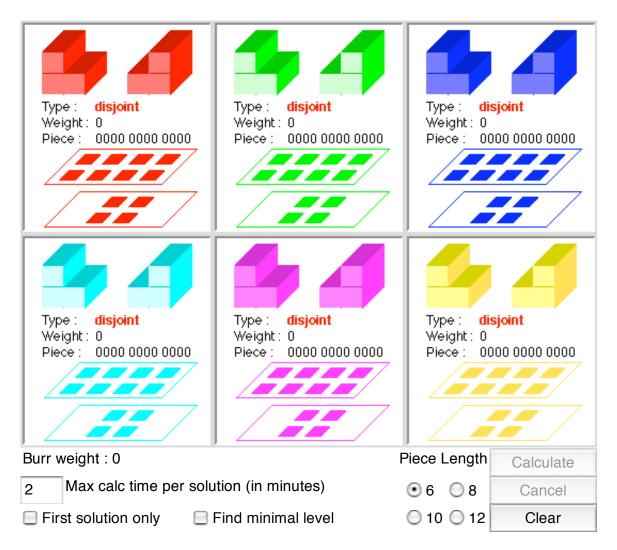
By going from notchable to general pieces, this puzzle has the advantage of having a weight of 29 (only three internal voids) as opposed to 27. The more interesting fact however is that the first three moves always move three pieces simultaneously until

the puzzle comes apart in two halves.

[IBM Research | Burr Puzzles Site]



Calculate your own burr puzzle



Explanations how to operate the applet.

[IBM Research | Burr Puzzles Site]

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Burr puzzles bibliography

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History | Definitions | 3-piece | 6-piece | Bibliography

Burr puzzles – Bibliography 03/21/2007 07:58 PM

		Jack Botermans	ISBN 0-295-96579-7
[Slocum92]	New Book of Puzzles, 101 Classic and Modern Puzzles to Make and Solve	Jerry Slocum, Jack Botermans	W. H. Freeman, New York, 1992, 128p., hb., ISBN 0-7167-2356-5
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[Wyatt28]	Puzzles in Wood	Edwin Mather Wyatt	Bruce Publishing Co, Milwaukee, Wisc, 1928; reprinted by Woodcraft Supply Corp, Parkersburg, WV, 1992, 64p. paperback, ISBN 0-918036-09-7
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Information on the web

On the web the amount of information is even scarcer. There are only a few other web sites on the topic of burr puzzles. There are some about puzzles in general, of which I have only listed the ones which have some relevance to interlocking puzzles (if you search for puzzles on the web you will find a whole slew of pages, most of which deal with crossword puzzles).

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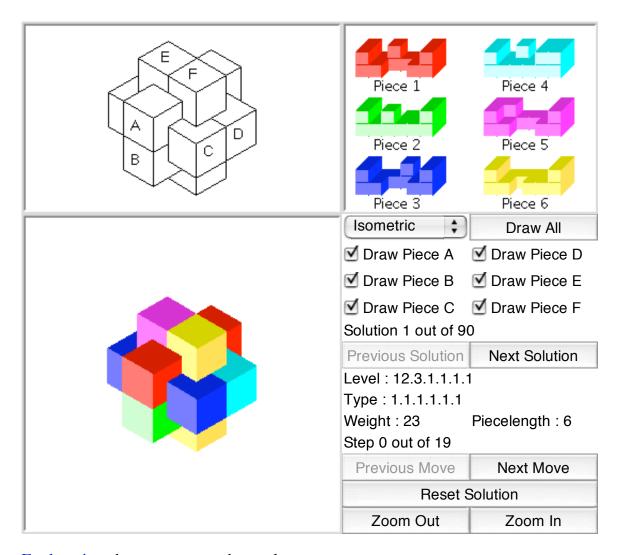
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[IBM Research | Burr Puzzles Site]



Bruce Love's Dozen



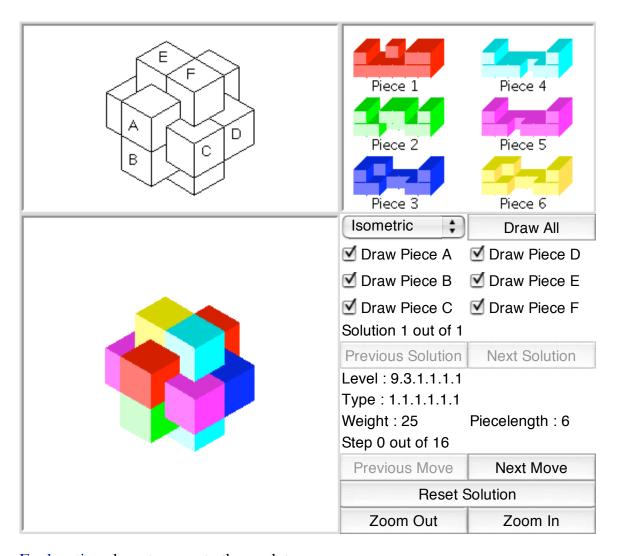
Explanations how to operate the applet.

generated by Burr6 Version: 1.10.000 elapsed CPU time: 206.19 second(s) 00:03:26.19

[IBM Research | Burr Puzzles Site]



Peter Marineau's level 9 burr



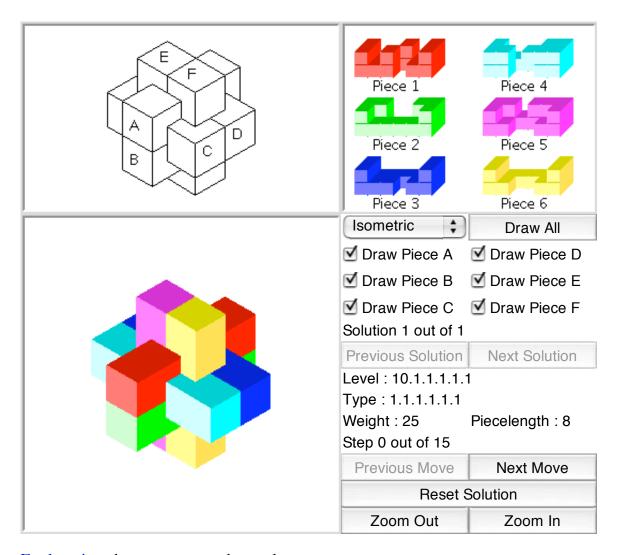
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[IBM Research | Burr Puzzles Site]



Computer's Choice Unique 10



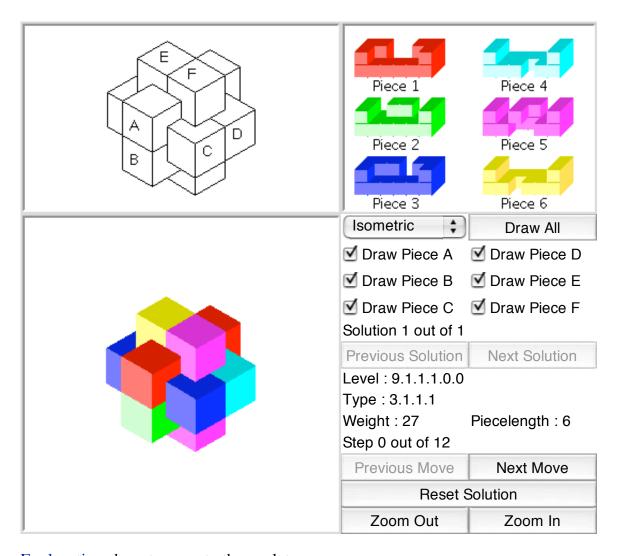
Explanations how to operate the applet.

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[IBM Research | Burr Puzzles Site]



Computer's Choice 5-Hole



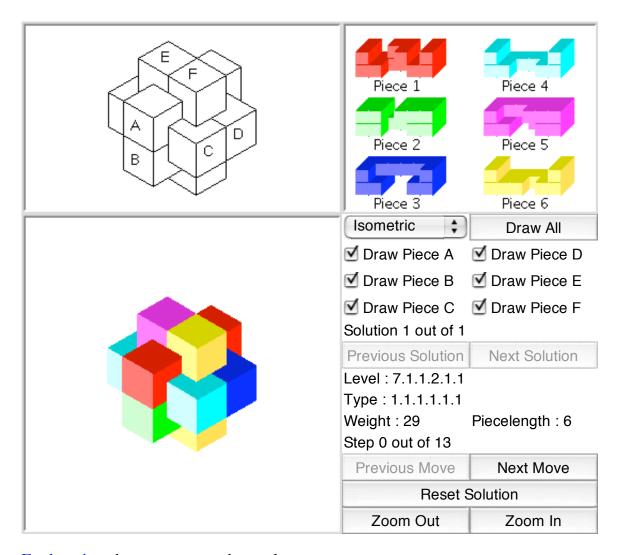
Explanations how to operate the applet.

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[IBM Research | Burr Puzzles Site]



Computer's Choice 3-Hole



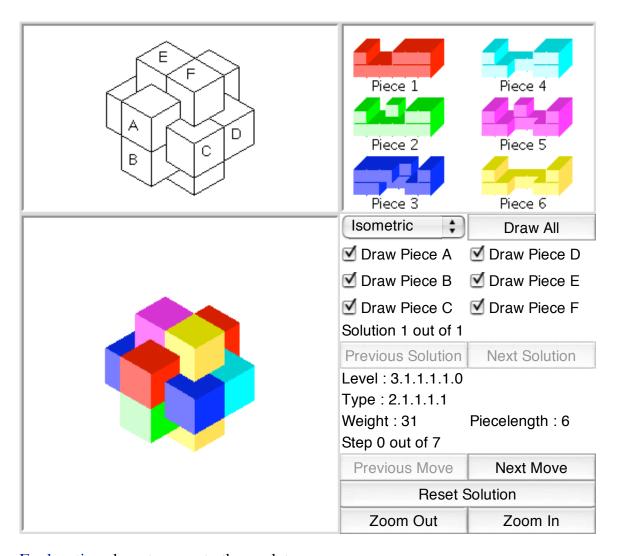
Explanations how to operate the applet.

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[IBM Research | Burr Puzzles Site]



Bill Cutler's BB31-10-40



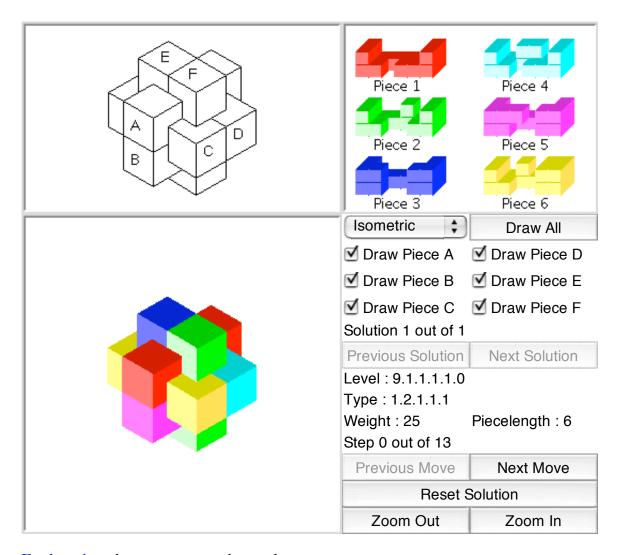
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generated by Burr6 Version: 1.10.000 elapsed CPU time: 1.15 second(s) 00:00:01.15

[IBM Research | Burr Puzzles Site]



Peter Rösler's Burr #C "teuflische Verführung"



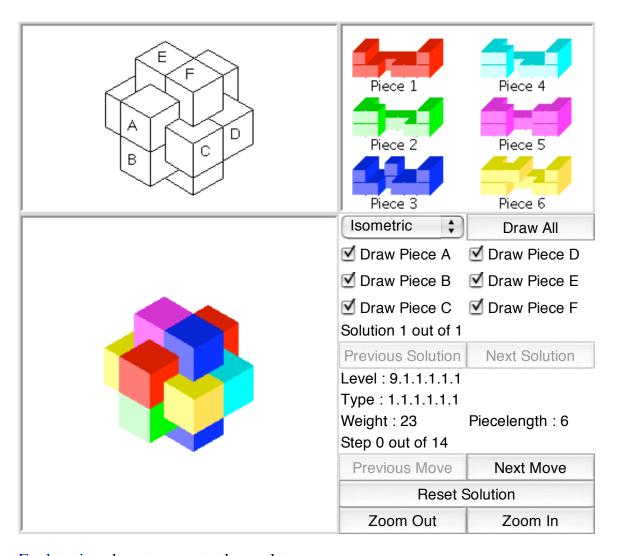
Explanations how to operate the applet.

generated by Burr6 Version: 1.10.000 elapsed CPU time: 7.86 second(s) 00:00:07.86

[IBM Research | Burr Puzzles Site]



Peter Rösler's Burr #D "super"



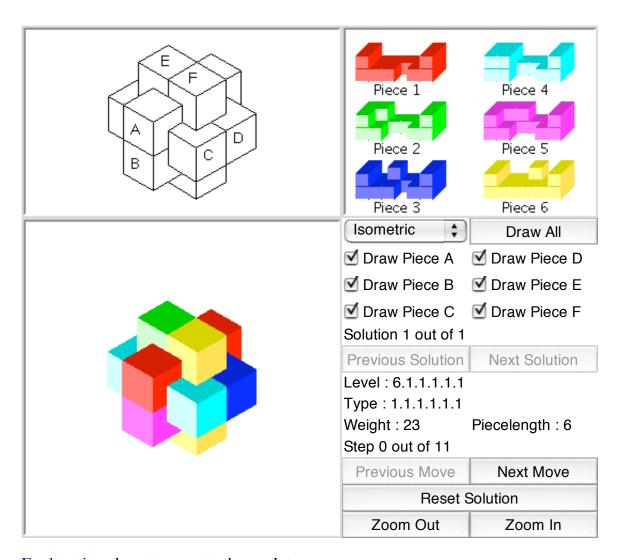
Explanations how to operate the applet.

generated by Burr6 Version: 1.10.000 elapsed CPU time: 30.65 second(s) 00:00:30.65

[IBM Research | Burr Puzzles Site]



Peter Rösler's Burr #G



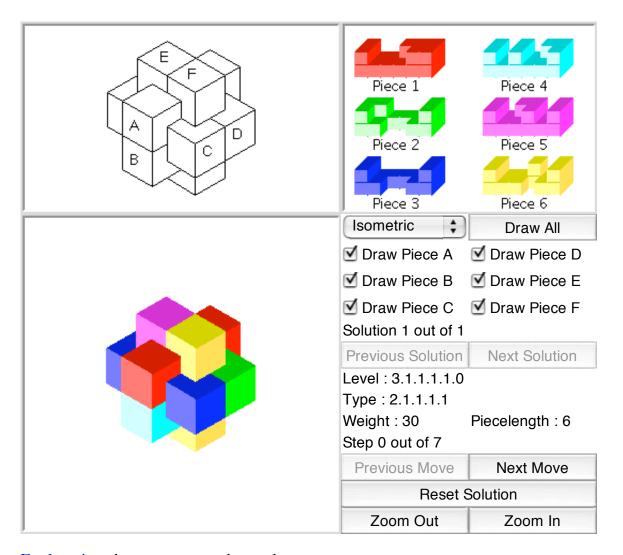
Explanations how to operate the applet.

generated by Burr6 Version: 1.10.000 elapsed CPU time: 44.11 second(s) 00:00:44.11

[IBM Research | Burr Puzzles Site]



Stewart Coffin's tripple slide



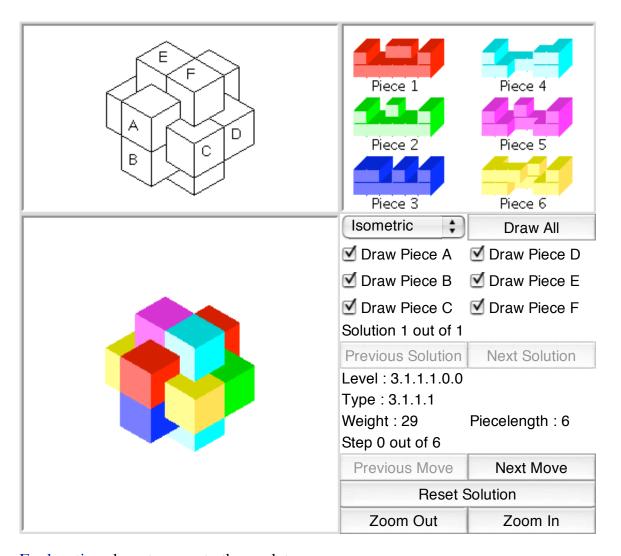
Explanations how to operate the applet.

generated by Burr6 Version: 1.10.000 elapsed CPU time: 0.99 second(s) 00:00:00.99

[IBM Research | Burr Puzzles Site]



jvk's 25.2 derivation



Explanations how to operate the applet.

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