## Rubik's Clock

Rubik's Clock is one of the easiest to solve with pen and paper (My girlfriend calls it cheating...(-:). Just write down what happens in matrix form and you soon find what you need.

## First some notations:

I will write $\mathbf{U}$ (resp.D) for a pin which is Up (so not pushed down) (resp. Down) and i will write the four pins in a matrix as you see them. For example:
U D
D U
means that the topleft pin is up, the bottomright is up and the others are (pushed) down.
And I will use a T which wheel I turn. So in our previous example I put the T in like:
UT D
D U
it means that i first make sure that the pins are as expected and then turn (such that the clocks turn clockwise) one tick (="hour"). Further more, if I don't put a T anywhere it means you can turn anywhere.

We will now look at the basic-"moves". The first matrix is obtained by looking at the front and the second is obtained by turning Rubik's clock upside down SUCH THAT THE LEFT SIDE GOES TO THE RIGHT SIDE. This is important.
Of course if you want to mess things up, then don't (-; .
In the matrices

```
+
means that that clock moves one tick clockwise,
0
means that that clock stays still,
means that that clock moves one tick counter clockwise.
```

| $\begin{aligned} & \mathrm{U} \mathrm{U} \\ & \mathrm{U} \mathrm{U} \end{aligned}$ | $\begin{aligned} & +++ \\ & +++ \\ & +++ \end{aligned}$ |  | $\begin{aligned} & -0- \\ & 000 \\ & -0- \end{aligned}$ |  | $\begin{aligned} & \text { DT U } \\ & \text { U U } \end{aligned}$ | $\begin{array}{lll} + & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}$ | $\begin{aligned} & 0-- \\ & 0-- \\ & 000 \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & \text { D UT } \\ & \text { U U } \end{aligned}$ | $\begin{aligned} & 0++ \\ & +++ \\ & +++ \end{aligned}$ | $\begin{aligned} & -00 \\ & 000 \\ & -0- \end{aligned}$ | $\begin{aligned} & \text { DT D } \\ & \text { U U } \end{aligned}$ | $\begin{array}{ll} + & 0 \\ 0 & + \\ 0 & 0 \\ 0 & 0 \end{array}$ | $000$ |  |  |
| $\begin{aligned} & \text { DT U } \\ & \text { U D } \end{aligned}$ | $\begin{array}{lll} + & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 \end{array}$ | $\begin{aligned} & 0-- \\ & --- \\ & --0 \end{aligned}$ |  |  |  |  |  |

Note that if you admit turning over (flip) and turning (rotating over Pi/2 etc.) of Rubik's Clock these basic moves are the only ones you have to look at for obtaining invariants. The group of moves are generated by the moves above and 'flip' and 'rotate'.

With this in mind it is easily to be seen that the following moves are all you need to get the whole group (and hence is a solution we where looking for). Since one basic move after (the inverse of) another one is just adding (substracting) the corresponding matrices the following should be self-exemplary

| $+{ }_{+}^{\text {U U DT }}+$ | $\begin{aligned} & \text { DT U } \\ & \text { U D } \end{aligned}$ | 000 | 000 |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  | 000 | 0-0 |  |
|  |  | 000 | 000 |  |
| $\begin{aligned} & \text { DT U } \\ & \text { D U } \end{aligned}$ | $\mathrm{U} \mathrm{U}_{\mathrm{U}}+$ | D UT | 000 | 0-0 |
|  |  | U U | 000 | 000 |
|  |  |  | 000 | 000 |
| U U D UT | + 00 | $00-$ |  |  |
| $\mathrm{UC}^{-} \mathrm{U}$ U | 000 | 000 |  |  |
| UU UU | 000 | 000 |  |  |

Using these moves the method to solve the Clock will be clear, although not optimal.
Easy wasn't it...


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