## Towers Of Hanoi



## a.k.a. Pyramid Piling Puzzle, Rainbow Puzzle, Brahma Puzzle

Very old design, this puzzle purchased from Bits And Pieces 2007. (wood stand and seven wood discs, $2.3^{\prime \prime}$ by $5.3^{\prime \prime}$ base by $3.5^{\prime \prime}$ high)
On Post $A$ there are $n$ rings of different sizes, in the order of the largest ring on the bottom to the smallest one on top. Posts $B$ and $C$ are empty. The object is to move the $n$ rings from Post $A$ to Post $B$ by successively moving a ring from one post to another post that is empty or has a larger diameter ring on top.


A


B


C

Solution: Since any of the rings 1 through $n-1$ can be placed on top of ring $n$, all $n$ rings can be moved by invoking the recursive procedure TOWER:

```
procedure TOWER(n,x,y,z)
    if }n>0\mathrm{ then begin
            TOWER( }n-1,x,z,y
            write "Move ring n from }x\mathrm{ to }y\mathrm{ ."
            TOWER( }n-1,z,y,x
            end
    end
```

$\operatorname{TOWER}(n, x, y, z)$ makes $2^{n}-1$ moves; for example, $\operatorname{TOWER}(3, \mathrm{~A}, \mathrm{~B}, \mathrm{C})$ takes 7 steps:

"Unwinding" the recursion of TOWER, yields the following simple iterative algorithm that moves the discs on post in the clockwise direction:
if $n$ is odd then $d:=$ clockwise else $d:=$ counterclockwise
repeat
Move the smallest ring one post in direction $d$.
Make the only legal move that does not involve the smallest ring.
until all rings are on the same post

## The Pyramid Piling Puzzle Version Of Towers Of Hanoi




A challenge to Adults and Children
AFTER DINNER ENTERTAINMENT
Approved therapy for all people
a pleasing GIFT at any time
Interesting LEGEND and Directions inside
made by
Wel-Maid Wood Products
Ray DeP. Haas
P. O. Box 44D Suffield, Conn. 06078

## PYRAMID PILING PUZZLE <br> Directions

Assemble parts as in picture
Move top dise to right dowel; Move second dise to left dowel, then transfer top dise from right dowel to left dowel; put third dise on right dowel, then move both dises from left dowel to right dowel, REMEMBERING to move ONLY ONE at a time and NEVER placing a larger dise on top of a smaller one. Continue moving in same manner until all dises are back in same form as at beginning, but on another dowel.

IT CAN BE DONE.
GOOD LUCK to you.
WEL-MAID WOOD PRODUCTS Ray DeP. Haas
P. O. Box 44

Suffield, Conn.


PYRAMID PILING PUZZLE
This puzzle is unique in that it is not only a challenge but gives interesting enjoyment while doing it. It takes time and concentration, and brings a happy feeling of satisfaction when completed. It grips one's attention so they are reluctant to leave it until done. A real challenge to children.

The object: All eight discs are to be moved from center dowel to another dowel, moving ONLY one at a time, and NEVER place a larger disc above a smaller one. All three dowels may be used, and when completed, the discs are to be in pyramid form as at the beginning, but on another dowel.

It is a fascinating PASTIME for retired folk, providing relaxation and rest from work and worries; for travelers on plane, bus or train.

Endorsed by doctors and nurses as excellent THERAPY for children and adults, convalescents, and finger dexterity, as well as useful to people subject to mental depressions,

It has proven to be an Ideal GIFT item. Keep one or two handy at home for entertaining visitors, using as a conversation plece. Have one at both ends of dining table for a contest as it makes a wonderful "AFTER DINNER ENTERTAINMENT," Say nothing, it will get attention, you watch for comments.

READ the following LEGEND of this puzzle.
LEGGEND: This is a mathematical puzzle, sald to be about 5,000 years old, known in very early days as the "Tower of Hanol," first sold as a toy by \& French mathematiclan. Edouard Lucas, in 1883. Also, bore the name of "Prof. Ctaus" of the college of "LiSou-Stian." The origina description of the toy called it a simplified version of a mythical "Towes of Brahma" in a temple in the Indian city of Benares. This pyramid puzzle has 8 discs and requires 255 moves to complete. The origina tower, the description read, consisted of 64 dises, all made of gold, now In the process of being transferred by the temple priests. Before they complete their task, it was sald, the temple will crumble into dust and the world will vanish in a clap of thunder. The disappearance of the world mary be questioned, but there is little doubt about the crumbling of the ancient temple. The formula $2 / 64-1$ yields the twenty-digit numbe $18,446,744,073,709,551,615$. Assuming that the priests worked night and day, moving one disc every second, it would take them many thousands of millions of years to finish the job.

Do not let these figures scare you. This puzzle with 8 dises can be done, and is about what the majority of people can do.

Wel-Maid Wood Products<br>Ray DeP. Haas<br>P. 0. BOX 44<br>SUFFIELD, CONN. 06078

Pyramid Piling Puzzle, Well-Maid Wood Products, Suffield, CT, unknown age. (cardboard box 3.7" x 5" x 3/4", wood base and pieces, directions, info. sheet; discs were lost at some point and replaced)

## The Rainbow Puzzle Version Of Towers Of Hanoi



Rainbow Puzzle, unknown age.
(2.5" diameter by $1.6^{\prime \prime}$ wood box with three $3 / 16^{\prime \prime}$ diameter $2^{\prime \prime}$ long wood pegs, and 8 wood discs, from $11 / 16^{\prime \prime}$ diameter to $1+15 / 16^{\prime \prime}$ diameter; to play, turn box over and put pegs into the holes in the bottom)

## Further Reading

Excerpt from J. A. Storer's book.
Wikipedia Page from: http://en.wikipedia.org/wiki/Towers_of_hanoi
Wolfram Mathworld Page from: http://mathworld.wolfram.com/TowerofHanoi.html
Claus Page from: http://www.cs.wm.edu/~pkstoc/toh.html
Ajtai Patent, from: www.uspto.gov - patent no. 5,992,851

