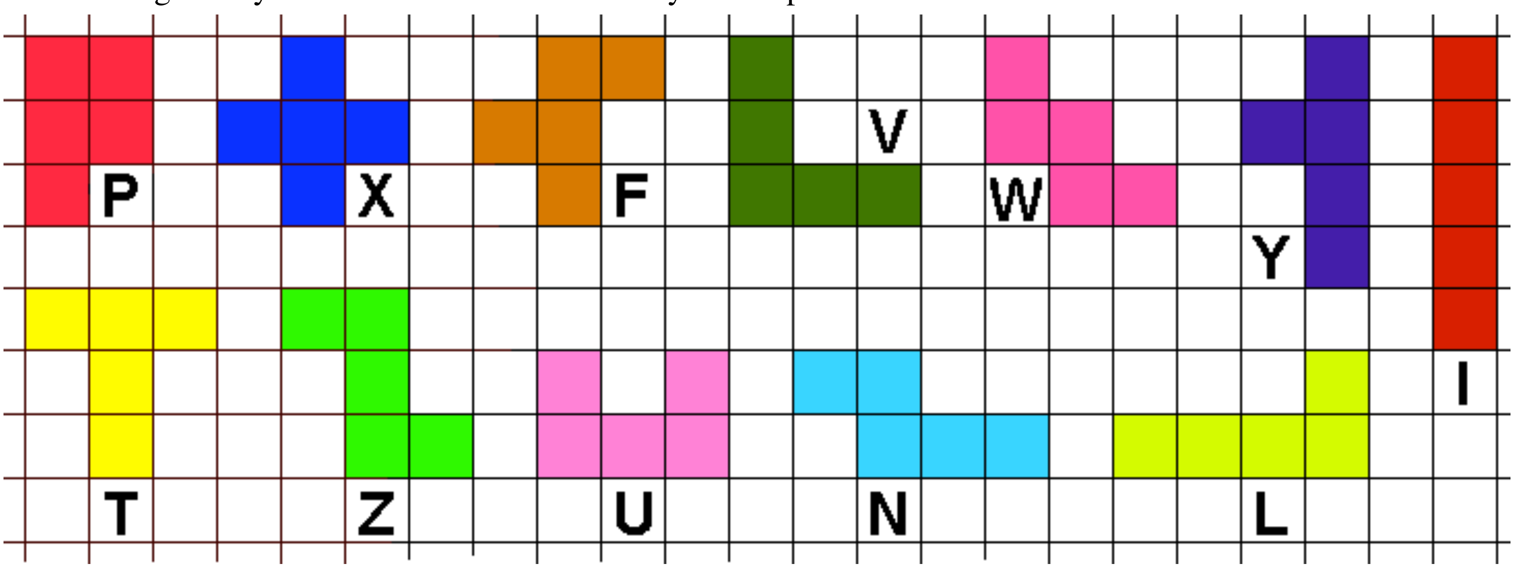
[CENTRE for INNOVATION in MATHEMATICS TEACHING](http://www.cimt.plymouth.ac.uk)

PENTOMINOES - An Introduction

Polyominoes is the general name given to plane shapes made by joining squares together. Note that the squares must be 'properly' joined edge to edge so that they meet at the corners.
Each type of polyomino is named according to how many squares are used to make it. So there are monominoes (1 square only), dominoes (2 squares), triominoes (3 squares), tetrominoes (4 squares), pentominoes (5 squares), hexominoes (6 squares) and so on.
Though the idea of such shapes has been around in Recreational Mathematics since the beginning of the 1900's, it was not until the latter half of the century that they became as popular as they are today.
In 1953 Solomon W Golomb (an American professor) first introduced their names and outlined their possibilities to mathematicians, who seized on them with considerable interest. They were not brought to the notice of the world in general until 1957 when Martin Gardner (in his famous column in the Scientific American) wrote about them, and they have remained a rich source of spatial recreation ever since.
Pentominoes (made from 5 squares) are the type of polyomino most worked with. There are only 12 in the set (because shapes which are identical by roatation or reflection are not counted). This means that they are few enough to be handleable, yet quite enough to provide diversity.

To make a set of Pentomino pieces
Mark out (lightly) a grid of squares on a piece of card.
Squares having an edge length of 1 cm (or half-an-inch or one-and-a-half cm) are a good size to use.
Draw and cut out the shapes show below.
The letters given by each are those used to identify the shapes.



There are many problems and investigations associated with Pentominoes.

Some which are available from this site are -

Answers are not given to problems, but hints are offered in appropriate places.

Other sources of information -

Polyominoes, puzzles, patterns, problems and packings	Simon W Golomb
Princeton University Books 1994	ISBN 0-691-08573-0
This the only complete book on the subject but it contains a very large list of other published sources about Polyominoes and similar sorts of ideas.	

Go to the top | Puzzles Index Page | CIMT Home Page

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PENTOMINOES
Space-filling Problems 1

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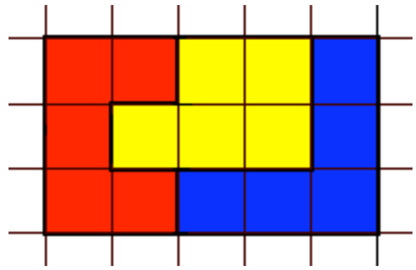
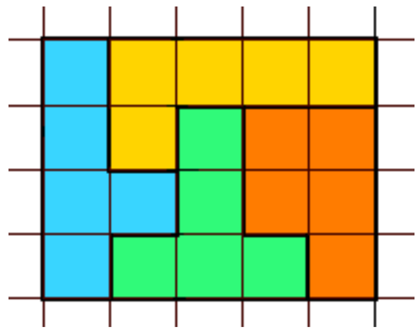
1. On the right is shown how 3 pentominoes (P, U and V) can be put together to fill a rectangle which covers an area of 15 squares.

Find **at least** two other ways of filling a rectangle of the same size with three pentominoes.



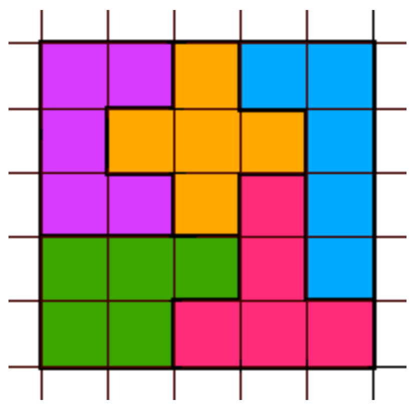
2. On the right is shown how 4 pentominoes (L, P, T and Y) can be put together to fill a rectangle which covers an area of 20 squares.

Find **at least** two other ways of filling a rectangle of the same size with four pentominoes.



3. On the right is shown how 5 pentominoes (L, P, T, U and X) can be put together to fill a rectangle which covers an area of 25 squares.

Find **at least** two other ways of filling a rectangle of the same size with five pentominoes.



Go to the top | Pentominoes Introduction | Next Problem page | Puzzles Index Page | CIMT Home Page

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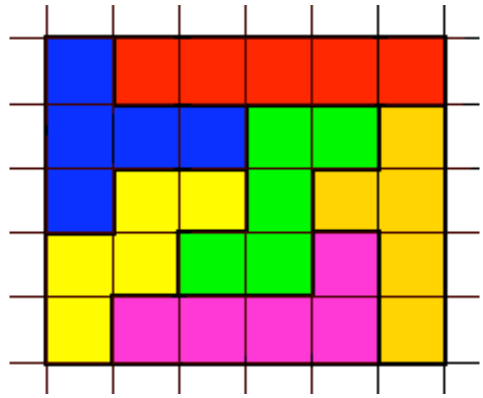
PENTOMINOES
Space-filling Problems 2

4. On the right is shown how 6 pentominoes can be put together to make a 5 by 6 rectangle.

Use the **other** 6 pentominoes from the full set of 12, to make another 5 by 6 rectangle.



5. Use the I, N, T, V, W, Y and Z pentominoes to make a 5 by 7 rectangle.

Then use the **other** 5 pentominoes from the full set of 12, to make a 5 by 5 square.

Go to	the top	Previous Problem page	Next Problem page	Puzzles Index Page	CIMT Home Page

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PENTOMINOES Tessellations

Any one of the 12 pentominoes can be used as the basis of a tessellation. With most of them (I, L, N, P, V, W, Z) it is easy to see how it can be done. But the F, T, U and X are a little more difficult and, if you are not careful, you will soon find 'holes' in your tessellation.

6. Make a drawing (1cm squared paper is good for this) to show how one of the F, T, U or X pentominoes will tessellate.

You will need to make the drawing big enough so it is easy to see how the pattern repeats - that probably means drawing 20 or more copies of the shape you are using. The proper use of colour also helps to show how the pattern repeats itself.

Go to the top | Previous Problem page | Next Problem page | Puzzles Index Page | CIMT Home Page

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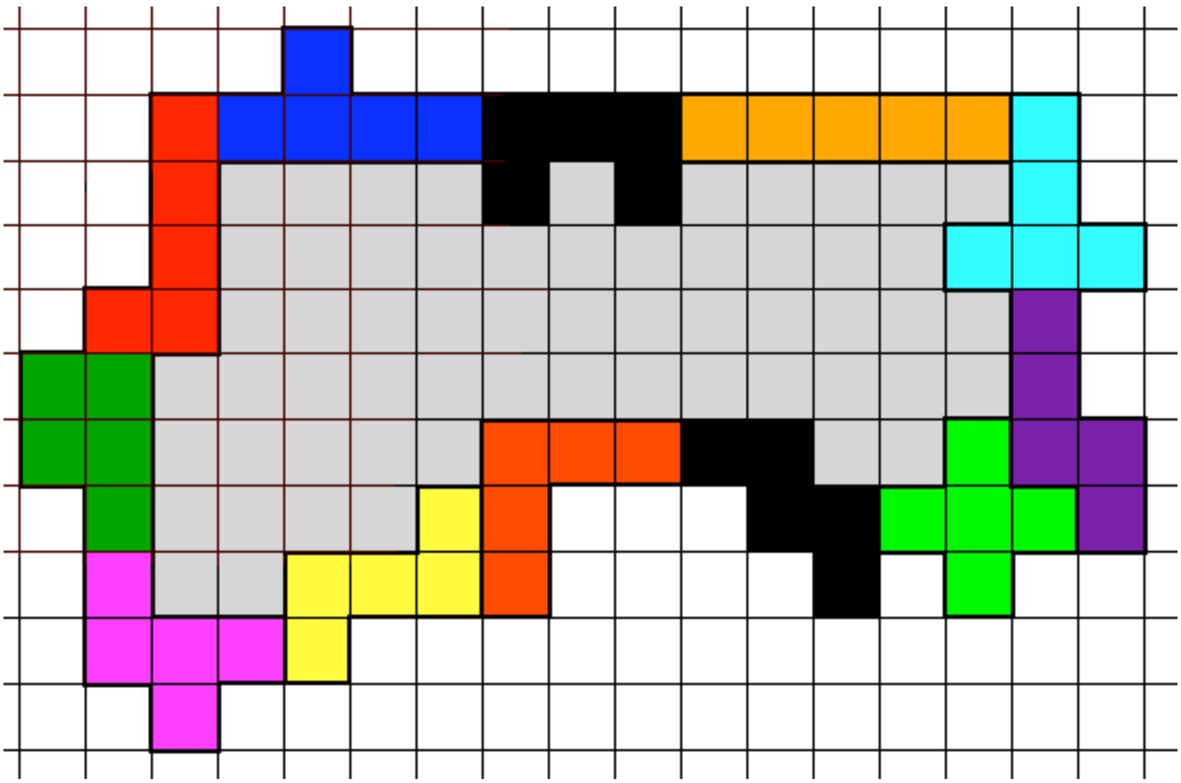
PENTOMINOES
The Enclosure Problem

The drawing below shows the full set of 12 pentominoes arranged to enclose a 'field'.
Notice the rule used to join them is that they must touch along the full edge of a square and not just at the corners.



It can be seen that an area of 59 (grey) squares is enclosed within the field.
It can also be noticed that the pentominoes have not been used very efficiently.

7. The problem is, to find a pentomino field enclosing the **greatest** possible area (= number of squares).

You can grade your attempts by reference to this table

AREA	GRADE
under 70	not trying!
71 - 80	E
81 - 90	D

91 - 100	C
101 - 110	B
111 - 120	A
over 120	Super!

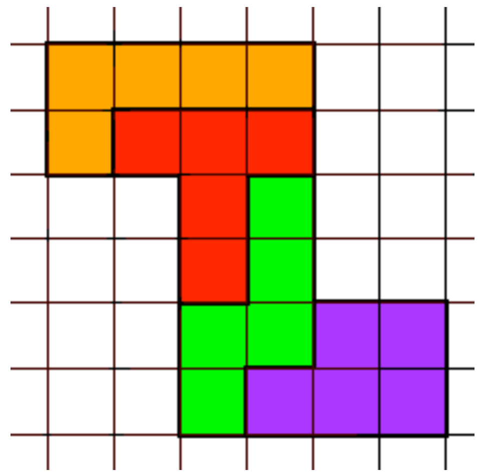
There are two variations on this puzzle -
1. That the **outside** edge of the enclosure must be a rectangle (having straight edges).
2. That the **inside** edge of the enclosure must be a rectangle (having straight edges).
The grading table given above does NOT hold for these variations.

Go to	the top	Pentominoes Introduction	Previous Problem page	Next Problem page	Puzzles Index Page	CIMT Home Page

CENTRE for INNOVATION in MATHEMATICS TEACHING

PENTOMINOES
The Duplication Problem

The drawing on the right shows how 4 pentominoes have been put together to make a copy of the Z-pentomino which is 2 times bigger in its dimensions than the original. (Note that it is 4 times bigger in area.)



8. Choose 4 pentominoes to make a twice-size copy of either the F, I, L, N, P, T, U, W, or Y pentomino.
Note the V and X cannot be done.

You can try the lot if you like!

Go to	the top	Pentominoes Introduction	Previous Problem page	Next Problem page	Puzzles Index Page	CIMT Home Page

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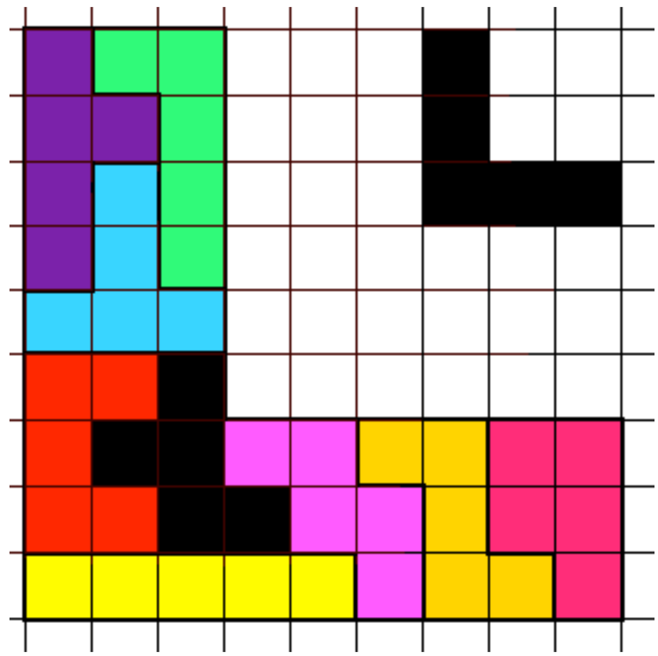
PENTOMINOES
The Triplication Problem

To work on this problem, select any ONE of the pentominoes and set it apart from the rest.
Then, using 9 from the 11 pentominoes remaining, make a shape which is the same as the originally selected pentomino only 3 times bigger.

In the example on the right, in the top right-hand corner is
the V-pentomino which was the one selected.
Also shown is another V made from 9 pentominoes and
which is 3 times bigger in its dimensions than the original.
(Note that it is 9 times bigger in area.)
The X and N pentominoes were not used.



9. Try triplicating any one of the other pentominoes.

Try the lot!

Go to	the top	Pentominoes Introduction	Previous Problem page	Next Problem page	Puzzles Index Page	CIMT Home Page

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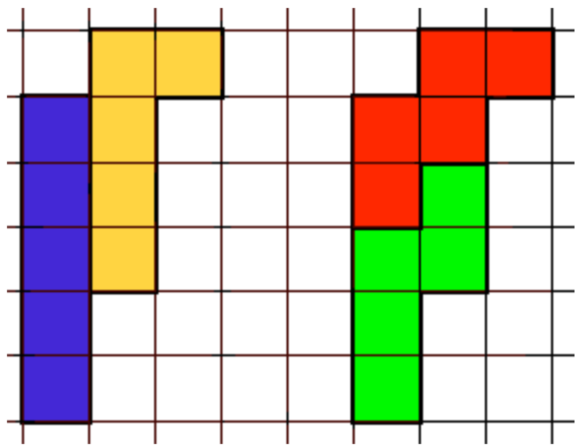
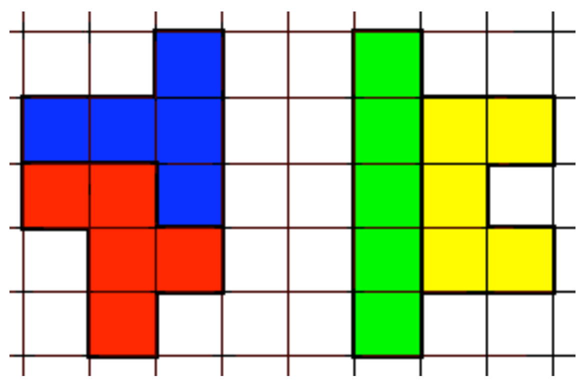
PENTOMINOES
The Matching Problem

This problem requires 2 (or more) pentominoes to be put together to make one total shape, and then 2 other pentominoes found which can be put together to make the SAME total shape
In the example below, the I and L pentominoes have been fitted together on the left, and then the W and N pentominoes have been fitted together (on the right) to make the same shape.



Diagrams for these two problems are given on the right.



10. Find 2 pentominoes which will make the same total shape as the one given for the F and T.

11. Find 2 pentominoes which will make the same total shape as the one given for the I and U.

12. Put V and Z together to make the same total shape as L and N.

13. Put W and X together to make the same total shape as P and Y.

14. Put V and X together to make the same total shape as U and Y.

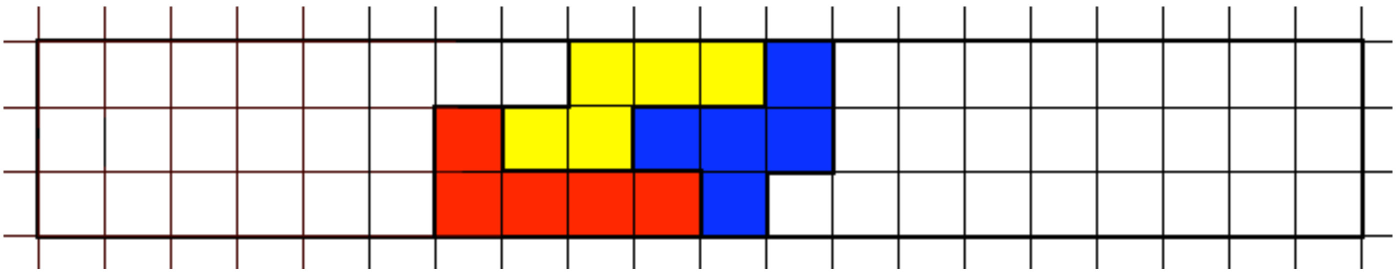
15. Divide the full set into 3 groups. Y N L P, F W T I, and X U V Z. Then assemble each group into a shape so that all 3 groups show the same total shape.

Go to	the top	Pentominoes Introduction	Previous Problem page	Next Problem page	Puzzles Index Page	CIMT Home Page

CENTRE for INNOVATION in MATHEMATICS TEACHING

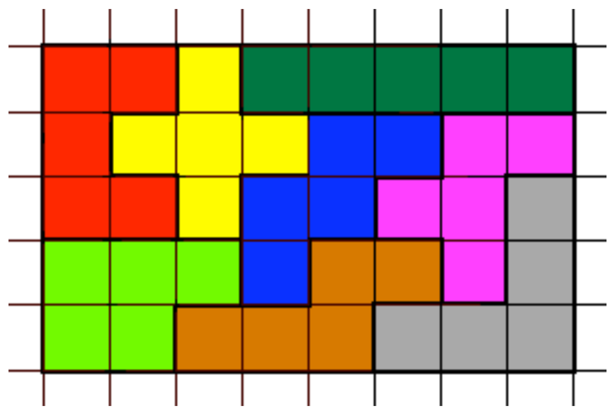
PENTOMINOES
Space-filling Problems 3

12 pentominoes (each made of 5 squares) must cover between them, a total of 12 x 5 = 60 squares.
One of the ways this can be done is by filling a rectangle measuring 3 by 20.
Such a rectangle, with just 3 pentominoes placed in it is shown below. It is the start of a solution.



16. Find the complete solution to filling the 3 by 20 rectangle. If you need it you can get a hint.

17. A rectangle of area 40 squares can be filled with pentominoes.
One way of doing this is in a 5 by 8 rectangle and one is shown on the right.
Find another way of doing it in a 4 by 10 rectangle.



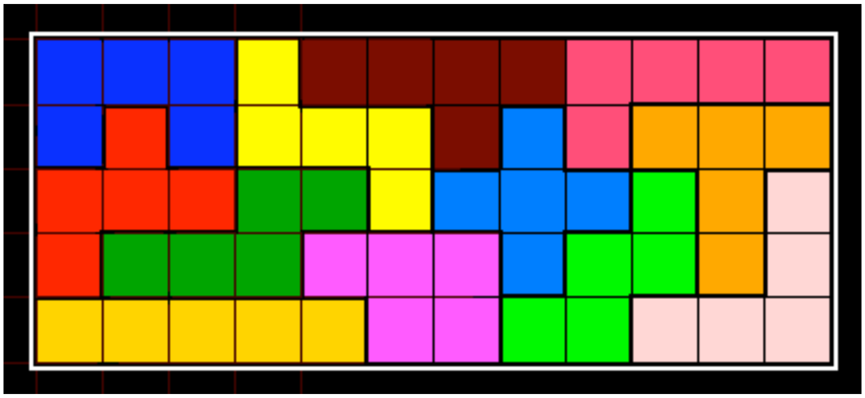
18. If you solved the triplication problem [#8] for the I-pentomino then that created a 3 by 15 rectangle having an area 45 squares.
Now find a way of filling a 5 by 9 rectangle which also has an area of 45 squares.

19. Find a way of filling a rectangle of area 50 squares.

20. Find a way of filling a rectangle of area 55 squares.

Using all 12 pentominoes to fill a rectangle can be done in 3 different ways.
As a 3 by 20 rectangle; as a 5 by 12 rectangle; as a 6 by 10 rectangle.

The 3 by 20 was done in problem #15 (above).
One solution to the 5 by 12 (from among several thousand) is shown below.



21. Use the full set of 12 pentominoes to fill a 6 by 10 rectangle.

Go to	the top	Pentominoes Introduction	Previous Problem page	Next Problem page	Puzzles Index Page	CIMT Home Page

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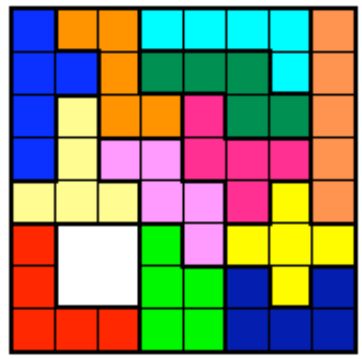
PENTOMINOES
Space-filling Problems 4

A normal sized chess-board is made of 64 squares (8 x 8).
Since there are only 60 squares in a full set of pentominoes it would not be possible to use them to cover a complete chess-board.
However it is possible to cover a chess-board provided a 'hole' of 4 squares (2 x 2) is left in it. One example of that is shown on the right.
It always possible to find a solution no matter where the hole is placed on the board.



22. Find a solution to covering the chess-board when the hole is placed exactly in the middle.

Go to	the top	Pentominoes Introduction	Previous Problem page	Next Problem page	Puzzles Index Page	CIMT Home Page

CENTRE for INNOVATION in
MATHEMATICS TEACHING

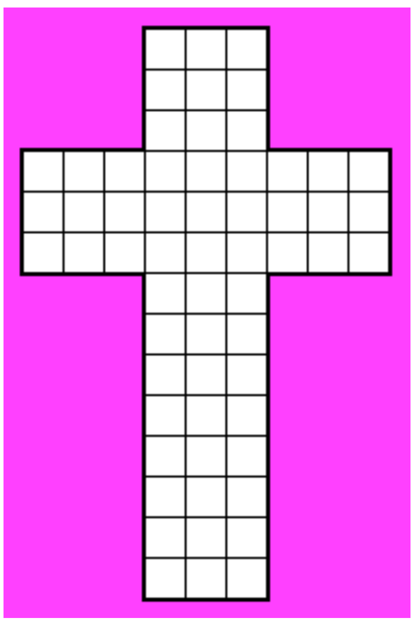
PENTOMINOES
A Miscellany

23. Of the 12 pentominoes, which ones could be used as the net for making an open-topped box?

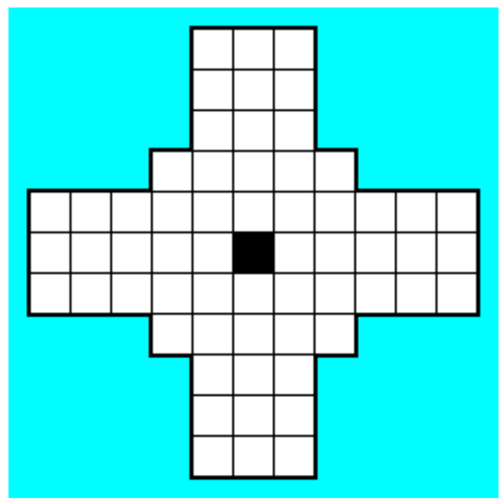
24. Find the smallest number of Y-pentominoes needed to fill a rectangle completely. (It is less than 20.)

Make each of these shapes with a full set of 12 pentominoes.
Note that 26, 27 28 and 29 have (black) holes in them.

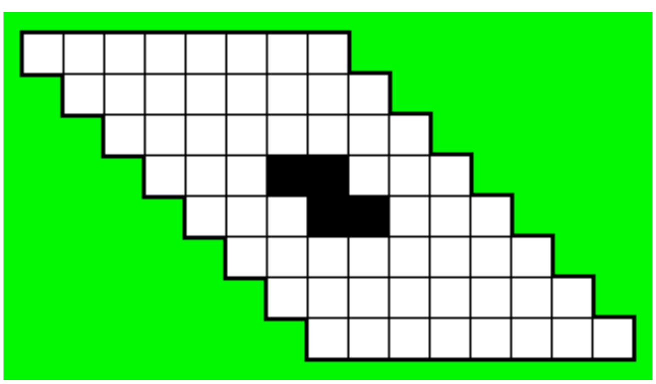
25.



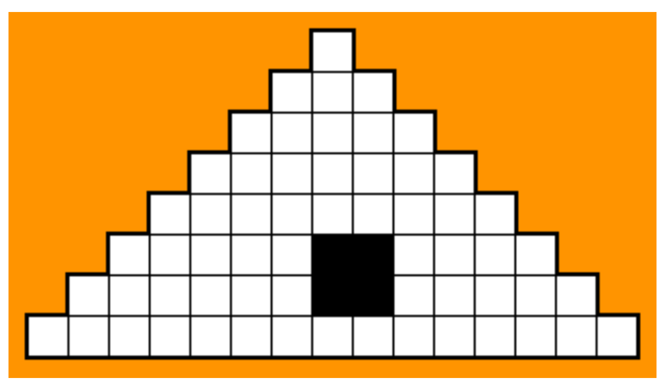
26.



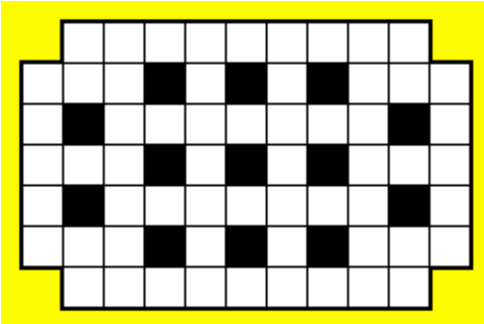
27.



28.



29.



Solid pentominoes

If the pentominoes are made using cubes instead of squares then it becomes possible to work with 3-dimensional shapes.
The simplest problem then is to put all 12 together to make a cuboid. Clearly it will have a volume of 60 cubes.
It can be done as
a 3 by 4 by 5; or a 2 by 5 by 6; or a 2 by 3 by 10.
All of these are possible.

8 of the solid pentominoes can be assembled to make a twice-size representation of almost any one of them. This is NOT possible in the case the I, T, W and X.

Go to the top | Pentominoes Introduction | Previous Problem page | Puzzles Index Page | CIMT Home Page