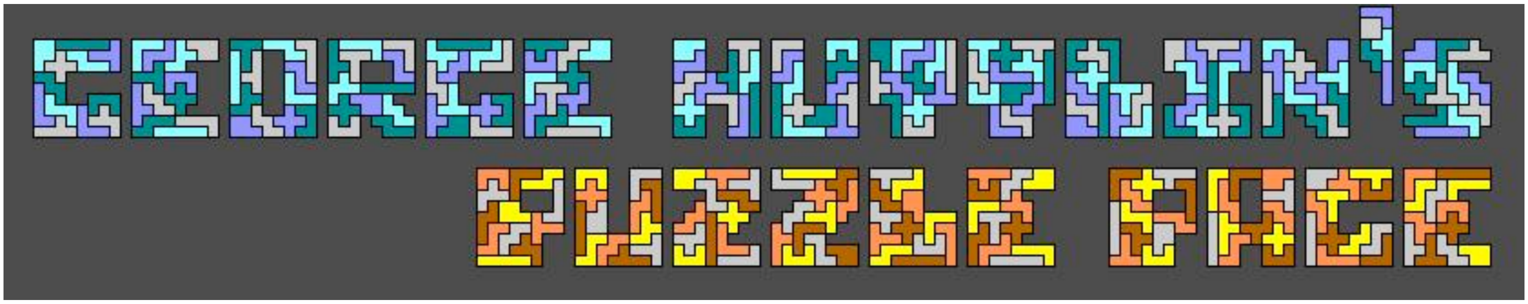


...some ramblings in the world of *polyominoes*.

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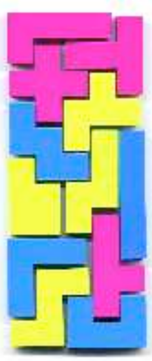
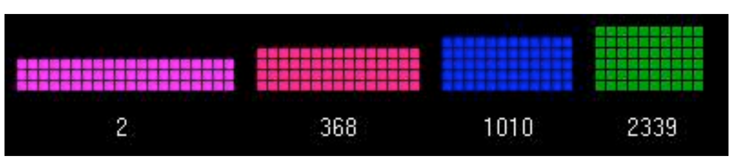
## Pentominoes

Each of the letters above is formed from the puzzle pieces known as *pentominoes*. A pentomino is the shape of five connected checkerboard squares. There are only twelve different pentomino shapes. Since each of these shapes covers an area of five squares, the total set of twelve covers an area of 60 squares. The object of the puzzle is to choose a shape made with 60 squares and then cover it using all 12 pentominoes.



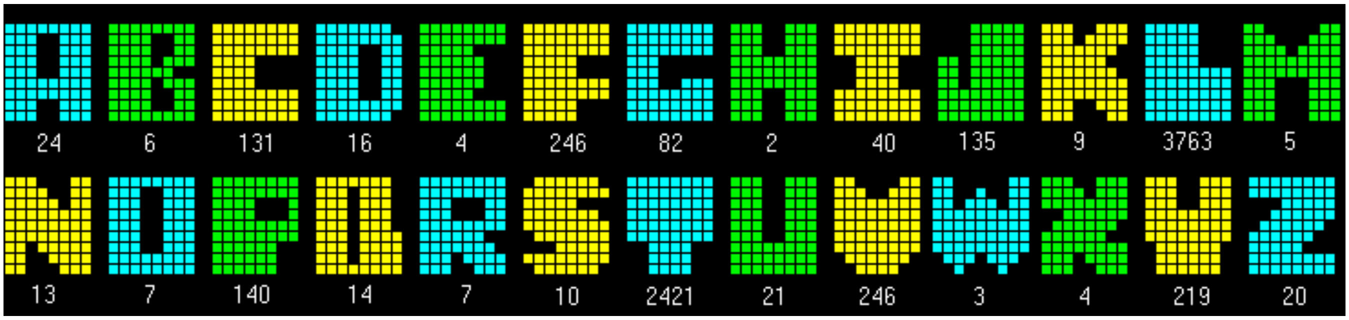
Puzzles based on pentominoes have been around for nearly a century and have been marketed under a variety of trade names. In one edition, the puzzle is to return the pieces to their 6 by 10 box as shown to the right. Of course you might consider covering rectangles of other proportions: 5 by 12 (as on the left), 4 by 15, and 3 by 20. The set of pentominoes pictured on the right is from the 1950's. The pieces on the left are formed from unit cubes, so that solid figures (such as a 3 x 4 x 5 box) can be made.

Computer searches for all the solutions to all of these rectangles have been done by many, myself included. Despite the fact that computer programs like this are recursive, I used FORTRAN, my native tongue. (I guess that means I speak BASIC and C with a FORTRAN accent.) If you'd like to see a really fast Java approach to solving problems of this sort, check out what Gerard has to offer. By the way, the numbers of distinct solutions for the rectangles are



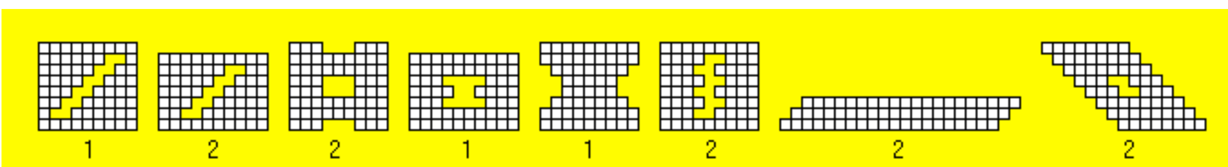

Many claim that the 3 by 20 rectangle is a tough problem because it has only 2 solutions. I disagree. The narrowness of the 3 by 20 rectangle so severely limits what's reasonable that getting to its solutions is no more difficult than any of the other rectangles. The speed with which the program completes its search supports this observation.

As can be seen from the letters above, pentominoes can form other shapes as well. The letters are based on rectangles 8 squares wide by 9 squares high, leaving 12 squares blank to give each letter its character, with the exception of "M" and "W" for which I used 9 by 9 squares. The outlines for all the letters are below along with the numbers of distinct solutions found by my program.




Letters with a more traditional appearance can be made by going to the 9 by 9 rectangle for the whole alphabet. However, as the territory spanned by the letters increases, the width of the lines forming them decreases and the number of solutions falls drastically. Shapes and solutions for "A," "B," and "C" have been given in the booklet accompanying the edition by Discovery Toys. I also found shapes for "D," "F," and "G," but abandoned the 9 by 9 project for the time being because a very beautiful shape for "E" has no solution and that seems to be the only reasonable 9 by 9 shape for the letter.

I have spent a great amount time and used lots of ink, paper, and plastic sheet protectors finding, printing, and archiving all the solutions to other interesting shapes, including those listed as problems in the book *Polyominoes* by Solomon Golomb [Princeton University Press, (Princeton NJ, 1994)]. What intrigues me more, however, is coming up with shapes that have very few solutions, such as the letter "H" shown above. Below are my other favorite one-and-two solution problems, the last two of which are proposed in Golomb's book.




I've also worked with Golomb's "Problem 81" for which my program determined there is no solution. When I get a chance, I'll give a more complete list of problems and numbers of solutions. Thanks to W. D. M. Laan for pointing out my error in the number of solutions for the first shape immediately above. Originally, I listed "2" as the number of solutions, but I failed to read my notes correctly when I prepared the original graphic.

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## Tetrominoes

You might have noticed that the "apostrophe" in the header is not formed with pentominoes. These shapes are the 5 tetrominoes, each of which is made with four checkboard squares. The tetrominoes are somewhat vexing because they do not cover the 4 by 5 rectangle. You see, a 4 by 5 rectangle on a checkerboard includes equal numbers of black and red squares. Four of the five tetrominoes cover equal numbers of black and red squares. Therefore, no matter how you do it, when you place those four "balanced" tetrominoes on the rectangle, two red and two black squares remain. Unfortunately, the remaining piece (the "unbalanced" piece) wants to cover three of one color and one of the other.

Puzzles based on tetrominoes are manufactured, however, using a double set covering 40 squares. These will cover rectangles since there are two unbalanced pieces. The solution counts I've found are:

5 by 8 ..... 783
4 by 10 ... 449

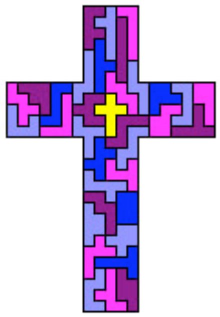
An example of the 5 by 8 puzzle is shown in the picture. Note the model number "783" used by the manufacturer.


## Hexominoes, etc.


Beyond tetrominoes and pentominoes a puzzle based on the 35 hexominoes has been avilable. The hexominoes will not cover a rectangle for reasons akin to those regarding tetrominoes. [See Golomb, page 10.] However, they will cover a rectangle with a pimple in the center of the long edge. This is expected to have billions of solutions, each of which is much more difficult to find than a pentomino solution.

My old computer algorithm, which gave the ordering of the pieces the highest priority, is too slow for hexominoes. I have completed an algorithm that prioritizes things by position. (This is probably the way a human would approach the puzzle.) This is orders of magnitude faster. Nevertheless a billion solutions would take a lot of ink, paper, sheet protectors and binders, not to mention computer time, so I don't intend to list all the solutions.

Since there are 35 of them, hexominoes are more convenient than pentominoes for forming pictures. I have used the trees and cross below for seasonal greeting cards, and have plans for the church. The particular solution for the cross contains a few sub-groupings of hexominoes which are symmetric, effortlessly giving quite a few solutions through rotations and reflections that are technically different. (If you're interested in a novel design for a stained-glass window, please contact me!) The same applies to the windows and door on the church.

Solutions to hexomino problems of particular interest to railroad buffs are shown on my home page.

"-Ominoes" beyond hex have been considered and some sets are even available commercially. See Golomb's book for more details.

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## Solid Pentominoes, etc.

The pieces in one of the sets of pentominoes shown at the top of this page are made with five cubes rather than plane squares. This adds another dimension for possible shapes. In addition to the 2-dimensional figures, these pieces can be used to form 3-dimensional things, such as a 3 x 4 x 5 box. I have not resolved some issues with my count of solutions for the box, for which I have attempted to suppress all solutions duplicated through rotation or reflection. However, the situation is much simpler with the 3 x 3 x 3 Soma cube, for which I've obtained, what I believe to be, the accepted number of 240 solutions. (The Soma people claim 1,105,920 solutions, but that includes all reflections and rotations of the entire cube as well as the individual pieces.) By the way, the Soma cube is made with all the irregular 3-D shapes that can be formed with 3 and 4 unit cubes.

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## Hexiamonds

If you are willing to leave the comfortable world of squares for the world of equilateral triangles, you'll find such things as hexiamonds. Hexiamonds constitute the set of 12 shapes made by joining 6 equilateral triangles on an isometric grid.

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Let me offer a few words on the meaning of the phrase "distinct solutions." Imagine that you've found a solution to the 6 by 10 pentomino problem. Imagine gluing the pieces together. Now you can lift your solution, rotate it, and perhaps flip it over putting it back in the 6 by 10 box so what you have looks different. Well, this isn't a different solution. However, if precautions weren't taken, a computer might find such solutions as "different." To prevent such duplications, I limit the permissible orientations for certain pieces. For instance, consider any solution to the 6 by 10 pentomino puzzle. You can glue together all the pieces for that solution and then rotate it and flip it as needed so the piece shaped like "V" always winds up oriented in one particular way. Therefore, in the computer's search through the different configurations of the pieces, I restrict the "V" to one particular orientation. For overall shapes with maximum symmetry such as an 8 by 8 square with a 4 by 4 hole in the center (as shown below), I take a piece with no symmetry at all (such as the "L," upside down, next to the straight piece in the upper left corner) and restrict it to one orientation.

CLICK HERE for George Huttlin's home page and to send comments.

CLICK HERE for more information on hexiamonds.

Last updated 22 August 2003