Impossiball

Solution

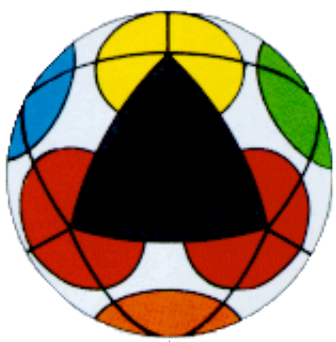
1. Description of the Impossiball

The surface of the Impossiball consists of twenty **Spherical Triangles**, five triangles forming a **spherical cap.** These caps can be turned, transporting five triangles at each turn.

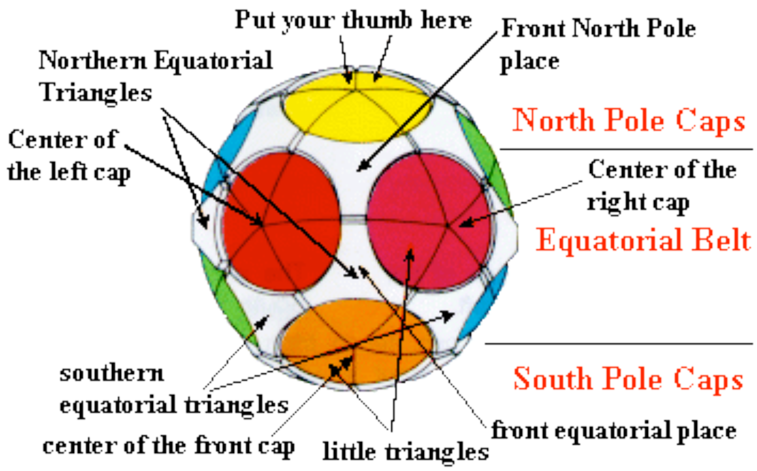
Each triangle has three **Little Triangles** in different colors, and five of these always form a **disc.** The aim of the game is to restore each of these twelve discs to one color or make beautiful patterns.

Removing one segment, turns the impossiball into the worlds first 19 piece 3-D slide puzzle, the segments can be rotated on the spot in just three moves.

When ordered, two opposite discs will have the same color. We will define the two yellow caps as the **North Pole** and **South Pole caps**. The belt between the north pole and South Pole cap we shall call the **equatorial belt**. You will notice that the equatorial belt consists of ten triangles of two kinds: five **Northern equatorial triangles,** which have an edge in common with the North Pole cap, and the five **Southern equatorial triangles**, which have an edge in common with the South Pole cap.

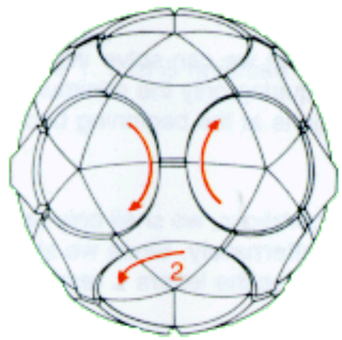


In order to have a definite orientation, put your left thumb on two of the North Pole triangles (of course, if you are left handed, use the right thumb). Then, of the remaining North Pole triangles, there is obviously one in front. We call this the **Front North Pole triangles**. Holding the ball in this manner, you can easily turn three edge caps, the **left, right** and **front cap.**



Turns of caps will be described by arrows in the corresponding discs, where an arrow always means a turn by on **click** (72°). When we turn by more than one click, the number of clicks will be specified.

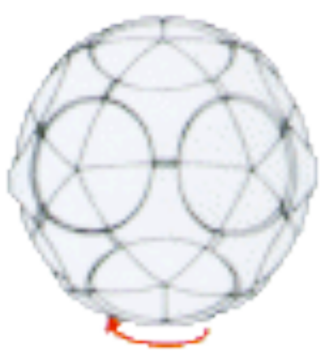
The arrow shows a turn of the left cap **Downwards**



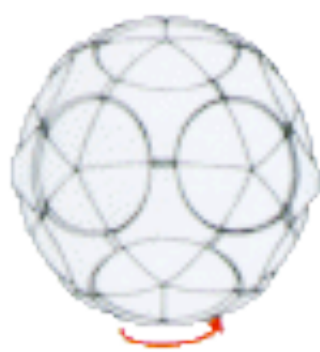
The arrow shows a turn of the right cap **Upwards**

The arrow shows a turn of the front cap **to the left** by two clicks

In two instances only, we shall use turns of the South Pole cap. Since the disc of the South Pole cap is not visible in our illustration of the Impossiball, we shall represent a turn of the South Pole cap by an arrow underneath.



The arrow shows a turn of the South Pole cap to the left.



The arrow shows a turn of the South Pole cap to the right.

2. The elementary operations

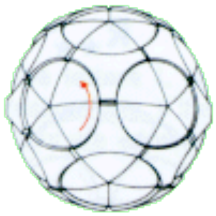
We want of course an optimal strategy which consists of a minimal number of short operations.

It is most astonishing that we can solve Impossiball with only two types of operations, requiring only the minimum number of four turns. I have put these operations at the begining because we shall use them right from the start.

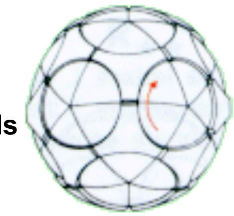
In these elementary operations, we shall only use the left and right caps which shall be turned **alternately.** Since we start either with the **Left** or **Right** cap, we shall use the letters **L** resp. **R** for these operations.

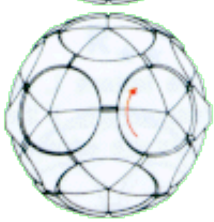
In the first two operations, the first two turns are **upwards.** This we indicate by putting two stars "upwards" beside the letters **L** and **R:** $\mathrm{L}^{\star\star}$, $\mathrm{R}^{\star\star}$. You will notice that these two operations are symmetric, only "left" and "right" are exchanged. They are of the same "type".

$\mathrm{L}^{\star\star}$ $\mathrm{R}^{\star\star}$

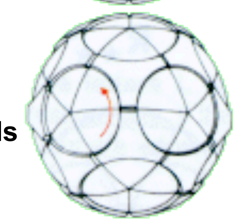


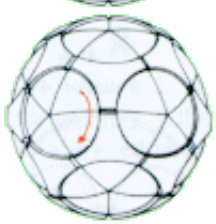
Step 1:
The **Left** or **Right Cap** respectively is turned **upwards**



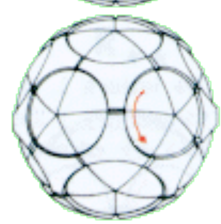


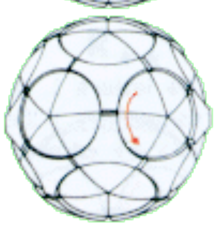
Step 2:
The **Right** or **Left Cap** respectively is turned **upwards**



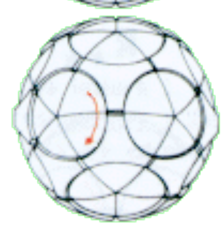


Step 3:
The first turn is reversed



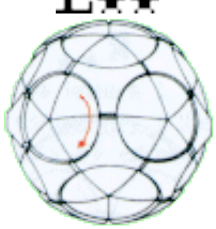


Step 4:
The second turn is reversed

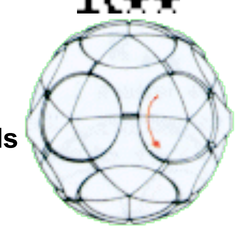


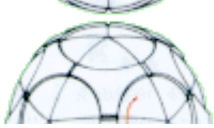
In the next two operations - again symmetric - the first two turns are **downwards.** This we indicate by putting two stars "downwards" beside the letters **L** and **R:** $\mathrm{L}_{\star\star}$, $\mathrm{R}_{\star\star}$.

$\mathrm{L}_{\star\star}$ $\mathrm{R}_{\star\star}$

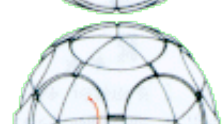


Step 1:
The **Left** or **Right Cap** respectively is turned **downwards**

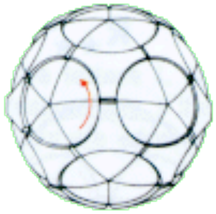




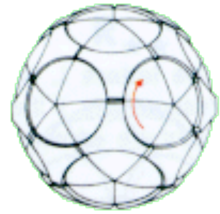
Step 2:

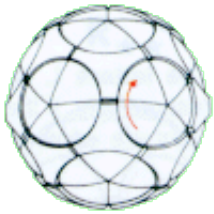


The **Right** or **Left Cap** respectively is turned **downwards**

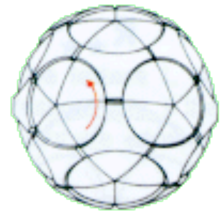


Step 3:
The first turn is reversed





Step 4:
The second turn is reversed



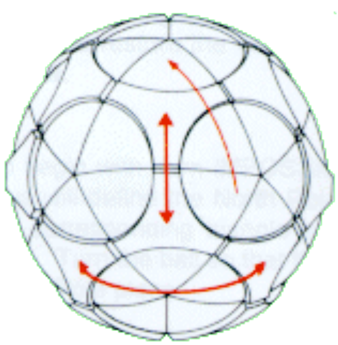
These operations are very fast. Please do me a favor and practice each of these operations for a few minutes. After a very short time you will need less than three seconds for one operation.

Let me summerize once more, and you will never forget these operations:

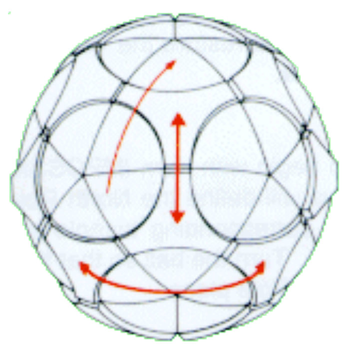
The Letters L or **R** indicate that you start with the **Left** or **Right** cap respectively. If the stars are **"up"** the first two turns are **upwards**, if the stars are **"down"**, the first two turns are **downwards.**

Let me point out very briefly the main properties of these operations:

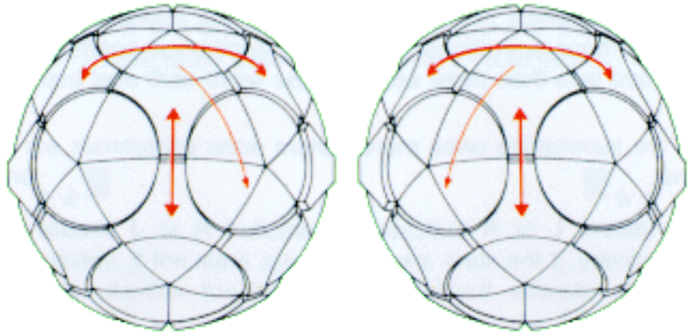
1. All these operations exchange one pair of triangles **in vertical direction**, namely the triangles at the frontNorth Pole place and the front northern equatorial place.
2. In addition, they exchange only one pair of triangles **in horizontal direction**, either in the North Pole cap or in the equatorial belt.



L★★



R★★



In applying our elementary operations, **it is most important to know where the horizontal exchange takes place.** You can easily remember this by the following "contradictory" observation.

If the stars are **"up"** then the exchange is **"down"** in the equatorial belt.

If the stars are **"down"** then the exchange is **"up"** in the North Pole cap.

3. Construction of the North Pole cap

Now the game starts. You will have no difficulties in messing up IMPOSSIBALL by a few turns. If you look at it again, the North Pole disc as well as the South Pole disc are gone, because all discs carry more than one color.

The situation is pretty funny! You want to arrange the discs, but you can turn only the caps. It turns out that the things you really have to deal with are the triangles.

As in almost all combinatorial games, restoring order means carrying out two activities:

Placing a triangle: This means that we transport a triangle to its proper place, not worrying about the relative position of its colors.

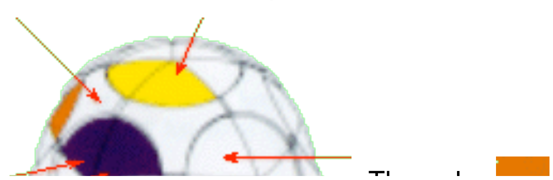
Orientating a triangle: This means that we turn a triangle "on the spot" such that its colors fit with its neighboring triangles.

If we succeed in placing and orientating at the same time, we shall say we have **set** the triangle.

Our strategy will be as follows:

We first construct the North Pole cap, then we set the five northern equatorial triangles, next we set the southern equatorial triangles, and finally we restore the South Pole Cap. **The Pole caps will always be yellow.**

To begin with, turn the Impossiball to bring "up" a little yellow triangle. This will define the North Pole. I advise you at the beginning to mark the corresponding triangle somehow, perhaps with a piece of colored tape. Turn the ball so that this first triangle will be left of the front North Pole place.

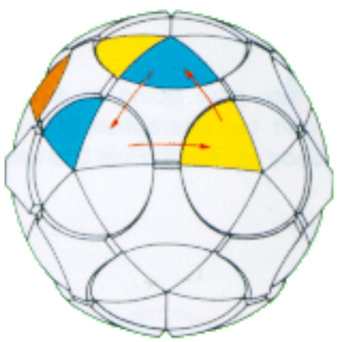


here

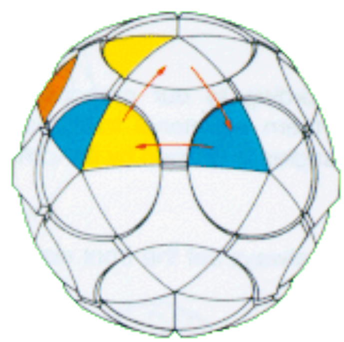
The color must not be here!

Now we want to set the front North Pole triangle. To this end we have to look for the triangle which has the colors Blue, Yellow, **Clockwise,** but does not have the third color ( )of the first triangle! Be careful! On this ingenious ball any combination of colors appears twice, once **Clockwise, once Counter-Clockwise.**

When you have found this triangle, bring it by some appropriate turns to the front place in the North Pole cap. If the yellow color hits the North Pole already, then you have completed this move. If you still have to orientate, there are two possibilities which you master by applying twice $\mathrm{L}^{\star\star}$ or $\mathrm{R}^{\star\star}$ respectively.



$\mathrm{L}^{\star\star} \cdot \mathrm{L}^{\star\star}$



$\mathrm{R}^{\star\star} \cdot \mathrm{R}^{\star\star}$

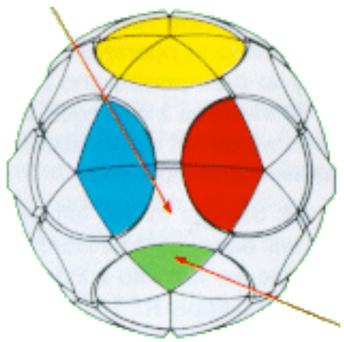
The idea to apply our operations twice is very simple! The first application exchanges the triangles at the front North Pole place and the front equatorial place. But the second operation exchanges them again! So these two triangles stay at their places and are just turned. The same holds true for the pair of triangles exchanged in the equatorial belt. By the way, for the moment, we do not bother about these latter turns, since we are still working at the North Pole cap.

Now, turn this newly set triangle to the left, and repeat the previous method. In this way you will easily set four North Pole triangles. Only with the last North Pole triangle we run into a little problem: Bring it in front underneath its final place and apply $\mathrm{R}^{\star\star}$. It then moves into the North Pole cap. If necessary, you orientate it by the previous method.

4. Setting the northern equatorial triangles

According to our proposed strategy, we are now going to set the northern equatorial triangles. The place and the orientation of these triangles is defined by the colors of the already set North Pole triangles.

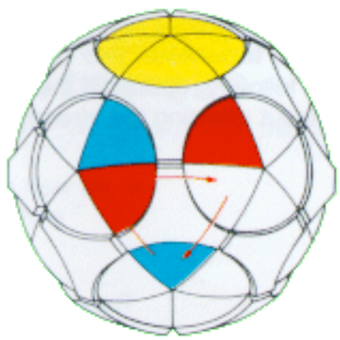
We always set the front equatorial triangle.

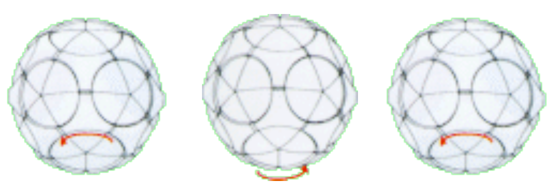


This color must not be yellow.

We have to look for the triangle having in the clockwise sense the colors , and not having ! By turning appropriate caps (not using however the North Pole cap), you can easily place it at the front equatorial place.

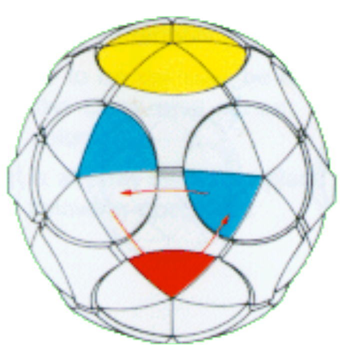
If the colors fit already, you are through with this step. If not,there are two possiblities which you treat by the following obvious operations:

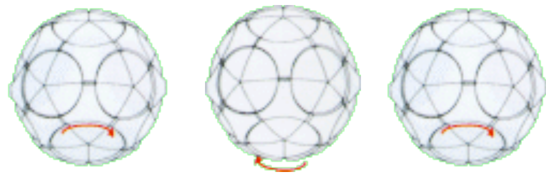




2 clicks **2 clicks**

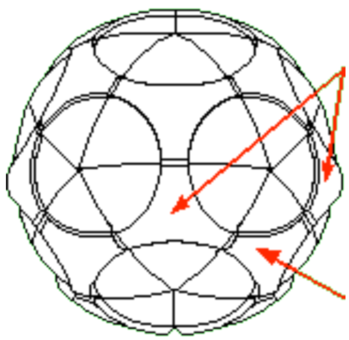
OR:

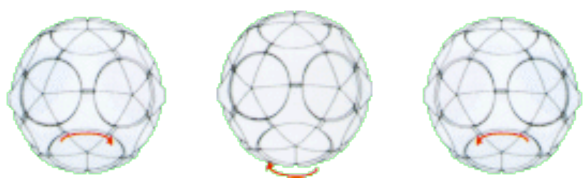




2 clicks **2 clicks**

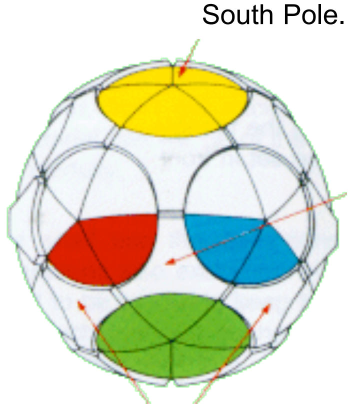
A little problem may just arise when a northern equatorial triangle is at a southern equatorial place **in between** two other already set northern equatorial triangles. The following operation then brings this triangle into the South Pole cap, without effecting the already set triangles.





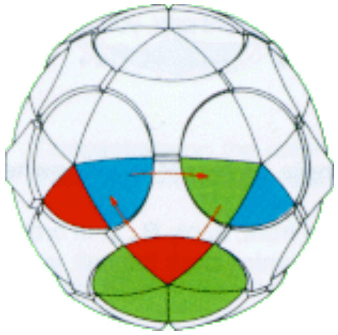
5. Setting the southern equatorial triangles

To this end we now turn over the ball such that the South Pole is on the top.

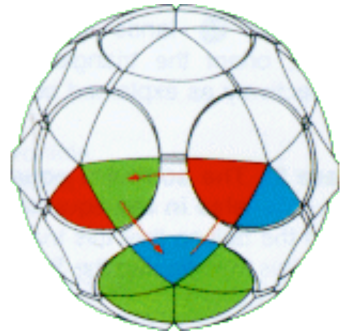


The colors of the triangle to be set must be , , (we don't have to worry anymore about the clockwise-sense, since there is only one triangle left with these colors).

If the triangle to be set is already at its final position and has to be orientated only, apply one of these operations:



$L_{\star\star} \cdot L_{\star\star}$

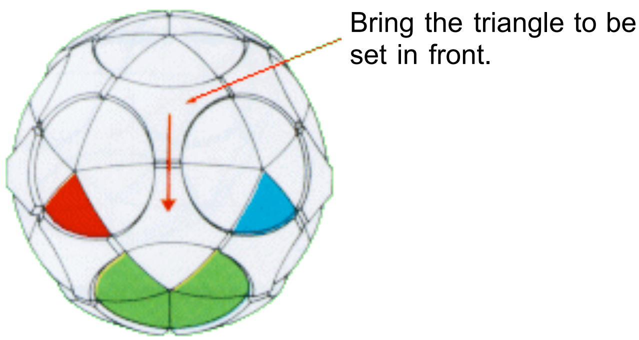


$R_{\star\star} \cdot R_{\star\star}$

If the triangle to be set is at another place, we have to distinguish between two cases:

Case 1: The southern equatorial triangle to be set is in the South Pole Cap.

Bring this triangle in front by turning the South Pole cap "Send" it down into the equatorial belt by applying $R_{\star\star}$.



$R_{\star\star}$

If the color "lands" immediately down in the front cap, you are ready. If not, orientate the triangle by applying $L_{\star\star}.L_{\star\star}$ or $R_{\star\star}.R_{\star\star}$ respectively as explained above.

Case 2: The southern equatorial triangle to be set is somewhere else in the equatorial belt.

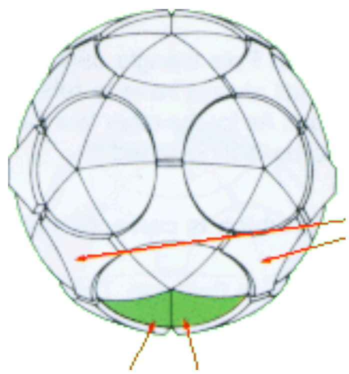
Turn the ball so that this triangle comes in front, apply $R_{\star\star}$, in order to "lift this triangle into the South Pole cap. Turning back the ball, you are back to Case 1.
You know that these operations turn, respectively exchange two triangles in the South Pole cap. But for the moment we won't bother about that.

6. Construction of the South Pole cap.

Naturally this is a difficult problem! It would be impossible to solve this problem by just turning the South Pole cap. We will have to "break into" the equatorial belt again.

Now, you might say, you are not telling me that we have to mess up the equatorial belt again which we have just so proudly finished? I am afraid, Yes! But don't get excited. The chaos we shall cause again in the equatorial belt will be of the controlled kind, and everything will turn out wonderfully.

We shall use the operations $\mathrm{L}^{\star\star}$ and $\mathrm{R}^{\star\star}$. As you know, these operations also exchange two northern equatorial triangles. Hence we want to be careful to keep the lower part of the ball fixed such that only two such triangles exchange continually. Now, choose any standard position and stick to it.

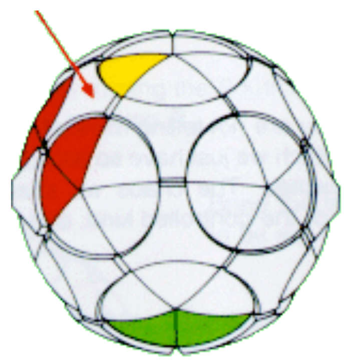


These two northern equatorial triangles exchange continually

Concentrate on the color of these two little triangles, which must always stay in front.

If there is already a little yellow triangle at the South Pole, turn the South Pole cap in such a way that the corresponding triangle is set, i.e. its colors match with those of the southern equatorial triangle underneath.

For example, this South Pole triangle is set



If there is no little yellow triangle at the South Pole, apply $\mathrm{L}^{\star\star}.\mathrm{L}^{\star\star}$ or $\mathrm{R}^{\star\star}.\mathrm{R}^{\star\star}$ (see page 18) to bring this about. Then turn the South Pole cap to set the corresponding triangle correctly.

This first correctly placed and orientated South Pole triangle defines the final position of the South Pole cap. We shall always have to refer to it again, and shall call it the **reference triangle.** As for the other four South Pole triangles, two different cases have to be distinguished.

Case 1: Orientating a South Pole triangle which is correctly placed but wrongly orientated.

Bring such a triangle in front by turning the South Pole cap. Then, orientate it correctly by applying $\mathrm{L}^{\star\star}.\mathrm{L}^{\star\star}$ or $\mathrm{R}^{\star\star}.\mathrm{R}^{\star\star}$ respectively (see page 18). Finally, turn back the South Pole cap in order to set correctly the reference triangle again.

Case 2: Setting a South Pole triangle which is wrongly placed.

Here we have to proceed by two steps.

First, bring such a triangle in front by turning the South Pole cap. Then "Park" it in the equatorial belt by applying $\mathrm{R}^{\star\star}$.

Finally, turn back the South Pole cap in order to set correctly the reference triangle again.

Secondly look for the place in the South Pole cap to which the "parked" triangle belongs. Bring this place in front by turning the South Pole cap. Then, "lift" the "parked" triangle to the South Pole cap again by applying $\mathrm{R}^{\star\star}$. If neccessary, orientate by applying $\mathrm{L}^{\star\star}.\mathrm{L}^{\star\star}$ or $\mathrm{R}^{\star\star}.\mathrm{R}^{\star\star}$ respectively. Finally, turn back the South Pole cap in order to set correctly the reference triangle again.

Let me give you an example for Case 2.

1. Suppose this is a South Pole triangle wrongly placed. We want to set this triangle correctly.

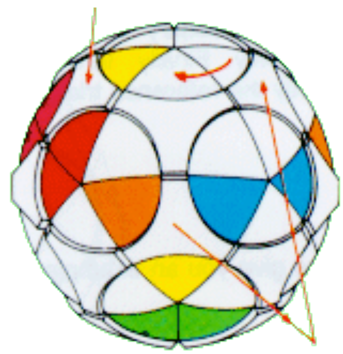
2. Our triangle to be set is now in front, the reference triangle has moved to the back.

Bring it in front by turning the South Pole cap by two clicks.

"Park" our triangle in the equatorial belt by applying $\mathrm{R}^{\star\star}$.

3. Our triangle is now "parked" in front equatorial place.

4. The reference triangle is correctly set again.



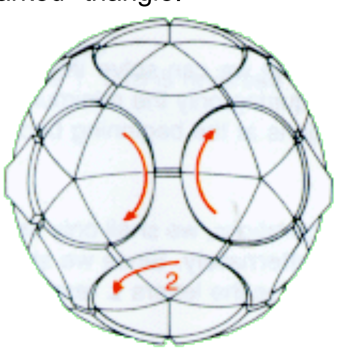
Our triangle belongs here!

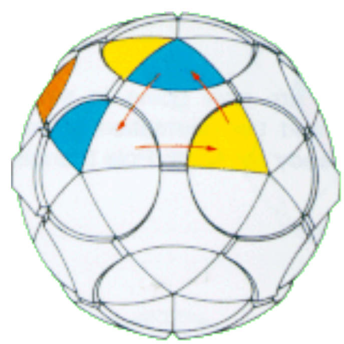
Turn back the South Pole cap by two clicks in order to set correctly the reference triangle.

Look now at which place our "parked" triangle belongs. In our example, it is the place to the right of the front North Pole place. Bring this place in front by turning the South Pole cap.

5. The fianl place of our triangle is now on top of our "parked" triangle.

6. Our triangle is now at its final place, however it still needs to be orientated.



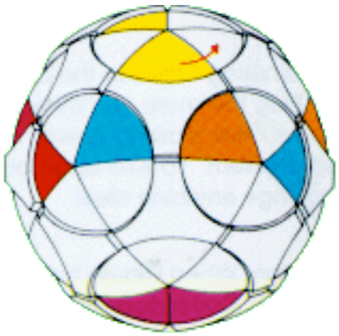


"Lift" it by applying $\mathrm{R}^{\star\star}$.

Orientate it by applying $\mathrm{R}^{\star\star}.\mathrm{R}^{\star\star}$.

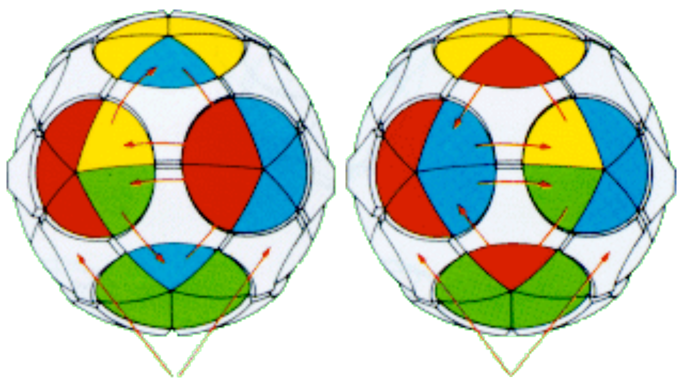
7. Our triangle is now at its final place and correctly orientated.

8. The reference triangle and our triangle are set.



Turn back the South Pole cap in order to correctly set the reference triangle again.

In general, repeating this procedure, you are ready. But sometimes, a very fancy thing happens! Either two or three triangles are not correctly placed or orientated. In this case, apply once or twice $\mathrm{R}^{\star\star}.\mathrm{R}^{\star\star}$ again, until the two northern equatorial triangles which were continually exchanged are set. Then you have run into the beautiful problem of IMPOSSIBALL. To orientate two neighbouring triangles which are correctly placed! There are two cases:



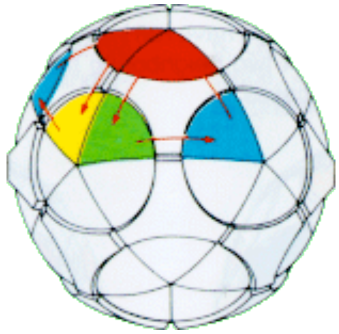
These northern equatorial triangles are set again.

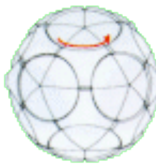
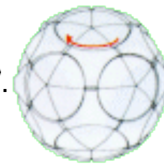
7. Impossiball's most beautiful problem

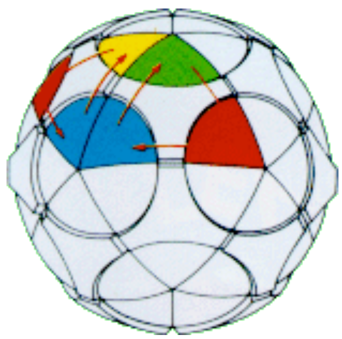
Solving a combinatorial game is a very fascinating affair! In the beginning, you storm from success to success. But, the more you think you are reaching the final solution, the more you slow down. And finally, you are stuck with a really difficult problem: To exchange or to orientate two moveable parts, without "destroying" anything else!

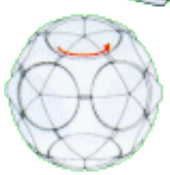
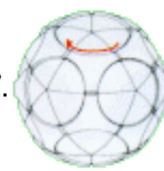
Here it took me quite some time to find a solution. When I finally had one, I was not satisfied, because it appeared too long. So I tried a little longer, and now I am glad to present to you a very elegant, solution.

We turn the ball to bring the two triangles in question into a new North Pole cap (whose corrseponding disc is of course not yellow anymore!). Then we can orientate both triangles by the following operations:



$L^{**}L^{**}$  $R^{**}L^{**}$ 



$R^{**}L^{**}$  $L^{**}L^{**}$ 

Everything O.K.? My congratulations, you are really on the ball, and for you IMPOSSIBALL has become POSSIBALL.

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