## Octahedron



This is a regular octahedron, of which each triangular face is divided into 9 identical smaller triangles, three to a side. A move consist of rotating a vertex; either just a tip (one triangle to a side) or a larger part (two triangles to a side). Christoph's Jewel is essentially the same puzzle except that the tips are missing, much like the Tetraminx is a Pyraminx without tips.

It is equivalent to a Rubik's cube puzzle without corners, but with the face centres marked to show their orientations. If you understand the solution for the Pyraminx, then you can probably solve much of this puzzle too, because it acts very similar.

## The number of positions:

There are 6 vertex pieces with 4 orientations, 12 edge pieces with 2 orientations giving a maximum of 12! $\cdot 2^{12} \cdot 6^{4}$ positions. This limit is not reached because:

- Only half the vertex orientations are possible given a certain edge position because a quarter turn is an odd permutation (2)
- Only and even number of flipped pieces are possible (2)

This leaves $12!\cdot 2^{10} \cdot 4^{6}=2,009,078,326,888,000$ or $2 \cdot 0 \cdot 10^{15}$ positions.
If you include the trivial vertex tips then this has to be multiplied by a further $4^{6}$.

## Links to other useful pages:

'Simplest Solutions' page A text based solution.

## Notation:

The vertices are denoted by the letters L, R, F, B, U, D (Left, Right, Front, Back, Up and Down). Clockwise quarter turns are denoted by the appropriate letter, ant-clockwise turns by the letter followed by an apostrophe (e.g. L' means turn the Left vertex 90 degrees anti-clockwise), and half turns by the letter followed by a 2 (e.g.. F2 means turn Front 180 degrees). Since the vertex tips are solved independently from the rest of the puzzle (see phase 1 below) this notation is not applied to tip moves but only to the larger moves.

## Solution:

Phase 1: Solve tips.
a. Rotate the tips on each vertex to match the adjacent vertex pieces.

Phase 2: Solve vertex pieces.
a. Rotate the F, B, L, R and U vertices so that their vertex pieces match, and only the edge pieces have the wrong colour. The $D$ vertex will be done later.

Phase 3: Place the U edges correctly.
a. First find any edge that belongs next to the $U$ vertex.
b. If the edge is not at the $D$ vertex, then put it there by using this sequence:

Move FR->DF: Do R'D'R.
This moves the edge between the $F$ and $R$ vertices to a position between the $D$ and $F$ vertices.
c. Hold the puzzle so that the edge belongs at the front, i.e. at FU.
d. Rotate $D$ to place the piece at the front, i.e. at DF.
e. Use one of the following sequences to place the edge in its correct position:

To move FD->FU do D F2 D' F2.
To move DF->FU do D F' L D2 L' F.
f. Repeat the above until all $U$ edges are at the $U$ vertex. Note that if all $U$ edges are at the $U$ vertex but they are not all correctly positioned, then you can bring one down to the $D$ vertex by using one of the above sequences to displace a $U$ edge.

Phase 4: Place middle edges correctly.
a. Find a middle layer edge that lies at the $D$ vertex.
b. Hold the puzzle so that the edge belongs at FR.
c. Rotate $D$ to bring the edge to the DF position.
d. Place the edge piece by using one of the following:

To move FD->FR, do R' D R.
To move DF->FR, do D L D' L'.
e. Repeat the above until all middle edges are correct. Note that if a middle edge is incorrectly positioned, it can be brought down to the D vertex by using one of the above sequences to displace it.

Phase 5: Solve D vertex piece.
a. Rotate $D$ until the vertex piece matches the others. Now only the four $D$ edge pieces remain unsolved.

Phase 6: Place D edges in position.
a. Hold the puzzle so that one of the following sequences will place the remaining edges in the correct position:

1. To move DR->DF->DL->DR, do R' D R D R' D2 R
2. To move DR->DL->DF->DR, do R' D2 R D' R' D' R.
3. To move DR-DL, DF-DB, do R' D2 R D2 R' D R D2 R' D2 R D'
4. To move DR-DF, DL-DB, do R' D R D' R' D2 R D R' D' R D2

Phase 7: Orient the D edges.
a. Hold the puzzle so that one of the following sequences will solve the puzzle completely:

1. To flip DR, DF, do R D' F D F' D R' D'
2. To flip DR, DL, do R D' F D2 F' D R' D2
3. To flip DR, DL, DF, DB, use b or $c$ to flip them two at a time.

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