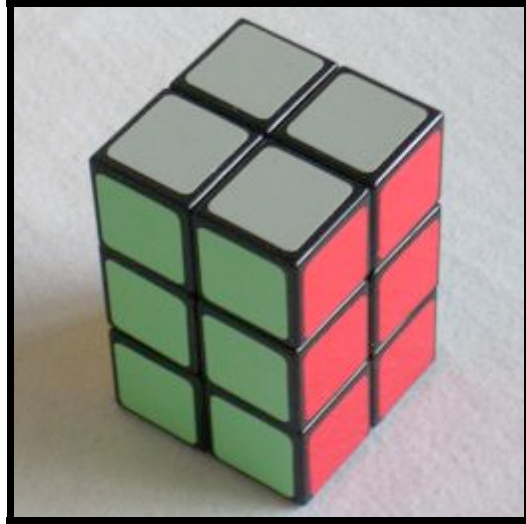# Tower Cube, 2×2×3

Tower Cube is a puzzle that is like a 2×2×3 Rubik's cube. The two 2×2 faces can be turned by quarter or half turns, but the four 2×3 faces only allow half turns. It is made by Gentosha.

The number of positions:

Taking one of the piece in the middle layer as a reference point, the three other middle layer pieces can be arranged in 3! ways, and the 8 corner pieces can be rearranged in 8! ways. This gives a total of 8!·3! = 241,920 positions.

A computer search gave the following result:

		Face turn metric															Total
		0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	
Q u a r t e r t u r n m e t r i c	0	1															1
	1		6														6
	2		2	22													24
	3			12	81												93
	4			1	64	276											341
	5				12	298	820										1,130
	6					96	1,140	2,100									3,336
	7					8	517	3,624	4,600								8,749
	8						50	1,652	8,964	8,156							18,822
	9							66	3,456	17,056	11,984						32,562
	10								68	6,296	23,956	14,512					44,832
	11									60	8,456	25,528	15,232				49,276
	12										308	6,480	27,648	9,792			44,228
	13											696	6,208	16,824	4,624		28,352
	14												704	2,096	6,000	576	9,376
	15													312	480		792
	Total	1	8	35	157	678	2,527	7,442	17,088	31,568	44,704	47,216	49,792	29,024	11,104	576	241,920

This shows that at most 14 moves are needed to solve the puzzle, or 15 if half turns of the square faces count as two moves.

Notation:

Hold the puzzle with the square faces at the top and bottom. Clockwise quarter turns of the top or bottom layer will be denoted by U and D respectively, half turns by U2 and D2, and finally counter-clockwise turns by U' and D'. For a half turn of a side the notation F, B, L, and R represent turns of the front, back, left and right face respectively.

Solution:

Phase 1: Solve the top and bottom layers.

a. First make the top face completely white. It is easy to make at least three of the top face squares white. To put the fourth white square in place, hold the puzzle so that the top face yellow square is at the front right, turn the bottom face so that its white square is at the front left, and then do R D R.
b. Now examine the side colours of the top layer to see which pieces need to be swapped to make the sides match. At most one pair needs to be swapped.
c. To swap adjacent corners, put the corners at the front right and front left, and do: F U' F U F R U R U' R
d. To swap diagonally opposite corners, put the corners at the front right and back left, and do: R U2 F U R U2 F U' R U2 F
e. The top layer should now be solved. Turn over the puzzle so that the bottom layer becomes the top layer, and do steps b-d to solve that as well.

Phase 2: Solve the middle layer.

a. Rotate the top layer so that it matches up with the bottom layer.
b. Rotate the middle layer so that as many of its pieces as possible match up with the top and bottom layers. If none of the pieces match up, then do R U2D2 L and try again. At least one piece can now match.
c. To swap two adjacent pieces of the middle layer, hold it so that the two pieces to be swapped are on the right hand side, and do R U2 R U2 R U2.
d. To swap two diagonally opposite pieces of the middle layer, hold it so that the two pieces to be swapped are on the front left and back right, and do F R U2 R U2 R U2 F.
e. To cycle around three pieces of the middle layer, hold it so that the correct piece is at the back left, and do R UD' F U'D. If that did not fix it, do it again.