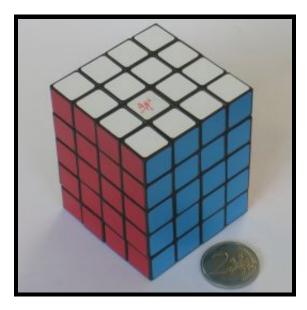# 4×4×5

This puzzle is like a 4×4×5 Rubik's cube. The two 4×4 faces and middle layers can be turned by quarter or half turns, but the four 4×5 faces and slices only allow half turns.

The number of positions:

The outer layers combined contain 8 centres, 8 corners, 8 left wing edges, and 8 right edges. Note that the two types of edges cannot be intermingled as flipping a pair of edges in place is impossible. Similarly the pair of inner slices contain 8 edge pieces and two sets of 8 centres. The middle layer contains 8 centres and 4 edges. None of these pieces can be twisted or flipped in place, except for the four middle layer edges which all change their orientation when the middle layer is given a quarter turn. This leads to a maximum of $2 \cdot 8!^8 \cdot 4!$ positions, but this maximum is not reached because:

- There are two sets of 4 indistinguishable outer layer centres ($4!^2$)
- There are 4 pairs of indistinguishable middle layer centres (2^4)
- The parity of the permutation of all the outer layer edges is even (2)
- There are 8 pairs of indistinguishable centres on the other inner layers (2^8)
- The orientation of the puzzle as a whole does not matter (8)

The last factor is due to there not being any fixed reference point to distinguish the sides, or the top and bottom. This gives a total of $2 \cdot 8!^8 \cdot 4! / (4!^2 \cdot 2^{13} \cdot 8) = 8{,}881{,}841{,}338{,}019{,}706{,}883{,}276{,}800{,}000{,}000$ positions.

Notation:

Hold the puzzle with the square faces at the top and bottom. Clockwise quarter turns of the top or bottom layer will be denoted by U and D respectively, half turns by U2 and D2, and finally counter-clockwise turns by U' and D'. Turns of the upper and lower inner layers will be denoted in the same way, except that the lower case letters u

and d are used. For the side faces the letters F, B, L, and R represent half turns of the front, back, left and right face respectively. Similarly, lower case letters f, b, l, and r represent half turns of the vertical inner layers adjacent to the F, B, L, and R face layers.

Solution:

Phase 1: Solve the outer layer centres.

a. Decide which of the two centre colours you want on the top. Hold the puzzle so that at least two top centres are already that colour.
b. Turn U until an incorrect centre lies at the front left.
c. Turn D until an incorrect centre lies at the front right.
d. Do r U' r to swap the two incorrect centres
e. Repeat steps b-d until all the centres are correct.

Phase 2: Pair up the outer layer edges.

a. Find any pair of adjacent edges in the outer layers that do not have matching colours. Hold the puzzle so that this mismatched pair lies at the top front.
b. Find the edge that has the same two colours as the left edge of your mismatched pair.
c. Using only outer layer moves that do not disturb the first mismatched pair (i.e. L, R, D, B), bring this second edge pair to the top back.
d. There must be another mismatched pair somewhere. Without disturbing the other two mismatched pairs (i.e. using L, R, and D), make sure that there is an unmatched pair at the top right location.
e. Match up the edges by doing the move sequence F rR U R U' rR.
f. Repeat steps a-e until all the edge pairs match.

Phase 3: Pair up the inner layer centres.

a. Find any pair of adjacent centres in the inner layers that do not have matching colours. Hold the puzzle so that this mismatched pair lies in the front face at the top.
b. Find another mismatched centre piece that matches the left centre of your mismatched pair.
c. Using only moves that do not disturb the first mismatched pair (i.e. L, R, d, B), bring this second edge pair to the top of the back face. Note that the centre matching the left one of the front pair must be the left one of the back pair (i.e. they both lie in the l layer). If that is not the case, do steps b-c again with a different mismatched centre of that colour.
d. Without disturbing the two mismatched pairs (i.e. using L, R, and d), make sure that at the top of the right face there is an unmatched pair, or a pair of the same colour as the centre at the right of the front mismatched pair.
e. Match up the centres by doing the move sequence F rR u R u' rR.
f. Repeat steps a-e until all the centre pairs match.

If we ignore the middle layer, the puzzle has now essentially turned into a 3×3×4 cuboid. All but the middle layer can be solved without breaking up the matched pairs, i.e. without using any vertical inner slice moves. The next 4 phases are exactly the same as the 3×3×4 solution.

Phase 4: Solve the top and bottom layer corners.

a. First put the corners in their correct layer. It is easy to get it so that there is only one corner in each layer that is out of place. To correct those final corners, hold the puzzle so that the in correct top corner is at the front right, and turn the bottom layer so that its wrong corner is at the bottom left, and then do R D R.
b. Now examine the side colours of the top layer corners to see which pieces need to be swapped to make the sides match. At most one pair needs to be swapped.
c. To swap adjacent corners, put the corners at the front right and front left, and do: F U' F U F R U R U' R
d. To swap diagonally opposite corners, put the corners at the front right and back left, and do: R U2 F U R

U2 F U' R U2 F
  e. The top layer corners should now be solved. Turn over the puzzle so that the bottom layer becomes the top
     layer, and do steps b-d to solve those corners as well.

Phase 5: Solve the top and bottom layer edge pairs.

  a. First put the edge pairs in their correct layer. Turn the top and bottom layers so that both have an incorrect
     edge pair at the front, and do U2 RFR U2 RFR. Repeat this until all the edges are in their correct layers.
  b. Now examine the side colours of the top layer edges to see which pieces need to be swapped to make the
     sides match the corners.
  c. To swap adjacent edge pairs, put the edges at the front and the right, and do: R D B R U2 R U2 R U2 B D'
     R
  d. To swap opposite edge pairs, put the edges at the front and the back, and do: R U2 R U2 R
  e. Repeat b-d until the top layer edge pairs are be solved.
  f. Turn over the puzzle so that the bottom layer becomes the top layer, and do steps b-e to solve those edge
     pairs at well.

Phase 6: Solve the edges of the two inner layers (not the middle layer).

  a. First put the edges in their correct layer. To recognize whether an edge is in the right layer or not, turn the
     layer until its colours match those of an adjacent outer layer corner. If the colours match exactly then it is in
     the correct layer, but if its colours are swapped then it is incorrect. Turn u until an incorrect edge is at the
     front right, turn d until an incorrect edge is at the front left, and then do R d R to swap them. Repeat this
     until all the edges are in their correct layers.
  b. Turn u to put at least two of its edges correct. At most one pair needs to be swapped.
  c. To swap adjacent edges, hold the puzzle so that the incorrect edges are at the front right and front left, and
     do: u' R u R d' R d R F d F
  d. To swap diagonally opposite edges, hold the puzzle so that the incorrect edges at the front right and back
     left, and do: R u2 F u' R u d R d' F u2 R u'
  e. The u layer edges should now be solved. Turn over the puzzle so that the bottom layer becomes the top
     layer, and do steps b-d to solve the other inner layer's edges as well.

Phase 7: Solve the centres of the two inner layer (not the middle layer).

  a. Choose two centre pairs that you would like to swap. Hold the puzzle so that one pair lies in the back face,
     and the other on the left or front.
  b. Do one of the following to swap the two pairs:
     1. To swap Bu and Lu, do: R F L d L F R u
     2. To swap Bd and Ld, do: R F L u L F R d
     3. To swap Bd and Lu, do: u R F L d L F R
     4. To swap Bu and Ld, do: d R F L u L F R
     5. To swap Bu and Fu, do: R F L d2 L F R u2
     6. To swap Bd and Fd, do: R F L u2 L F R d2
     7. To swap Bd and Fu, do: u2 R F L d2 L F R
     8. To swap Bu and Fd, do: d2 R F L u2 L F R
  c. Repeat a-b until the inner layer centres are all solved.

Phase 8: Solve the middle layer edges.

  a. Turn the middle layer until as many of its edges are solved as possible. If none can be solved, then do R
     M2 R M2 and try again.
  b. If two middle layer edges need to be swapped, then hold the puzzle so that one of them lies at the front
     right, and the other at the back (back left or back right).
  c. 1. To swap FR and BR, do Rr (Uu)2 Rr (Uu)2 Rr (Uu)2.
     2. To swap FR and BL, do B Rr (Uu)2 Rr (Uu)2 Rr (Uu)2 B.

   d. Repeat steps b-c until all the edges are solved.

Phase 9: Solve the middle layer centres.

   a. Hold the puzzle so that some of the incorrect middle layer centres are in the front and back faces.
   b. If both of the front middle layer centres are incorrect, do r2 M2 r2 M2.
   c. If only one of the front and one of the back centres are incorrect, then
      1. If the front incorrect centre is on the right, then do an F move to bring it to the left, the Fl location.
      2. If the back incorrect centre is on the left, then do a B move to bring it to the right, the Br location diagonally opposite the front incorrect centre.
      3. Do (Uu)2 Rr (Uu)2 F (Uu)2 Rr (Uu)2 to swap them.
      4. If necessary, do F and/or B to return those layers to their correct locations.
   d. If necessary repeat a-c for the other pair of faces.

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