The first steps...

A little history

Once upon a time I bought a Square One. It took me some time to live happily everafter. This is a much abbreviated account (I left out (most?) of my mistakes) of my search for happiness, i.e. a solution. Since I don't want to spoil all your fun, I have only written down a few ideas (the important ones), the basic concepts and leave to you to fill in some of the details. I hope this will stimulate you to invent your own solution and also be very happy. <-:
Happy reading and hunting !

The middle layer

The middle layer consists of two pieces which form either a square or a irregular hexagon:

The main reason I want to talk about it now is that after this you can (almost) ignore it, i.e. theoretically. The reason being that the two shapes can be obtained without disturbing the rest of the layers.

Proof :

1. Irregular hexagon to square: Do the following two moves three times
    - Turn the right half of your 'cube'
    - Rotate the upper layer 180 degrees
2. Square to hexagon: do the inverse of 1.

foorP

Odds and ends

This section will (in time) explain some group theoretical important concepts (for really understanding any solution). At this time I am still searching what (and how) to include here and if it should be placed here or in a more general context (and hence in a seperate file). Come back later to see if I've made my mind up.

(permutations, odd/even, conjugation, Theorem of Lagrange....)

In the mean time read the lecture notes of W.D. Joyner, in particular about permutations and the definition of odd and even permutations.

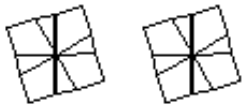
Getting back into shape

Well, the nice thing about this section is that it is short. Since I redirect you to my list where you can get your mangled 'Cube' back into shape.

Poëtry in motion

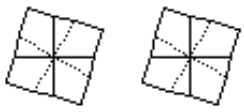
So, I presume we have managed to get into the Cube-shape. Most people are now terrified to lose this dull, but sometimes long searched for, shape. So did I at a time. So I started with finding nice moves. Of course I already had some but this time I wanted nice moves for Square One.

Since we don't want to loose the Cube-shape at this stage... there are only a few *basic* moves we admit. They are all obtained by the following two moves (and 90 degrees turns/rotation of the top and bottom layer):

1.  , and
2. 

So what is the group that we get ? Well, by looking at it from a slightly different angle I saw it immediately. Of course I had to check it. For those which already solved the 2x2x2-cube let me sketch the way I knew what this (sub)group is.

First we glue a few pieces together (well not really, let's just pretend). Then it looks (inside our head(s)) as:



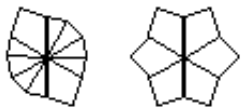
Note that we can also do this in the mirror image. So now we have a "2x2x2 cube" (we are still ignoring the middle layer).
We don't have to worry about orientations. Then we see, by using the 2x2x2-permutation group (S8), that the glued blocks can be permutated as we please. Then note that for all (basic and hence all cube-preserving) moves we have that:

> *The permutations of the narrow and wide parts are either both odd or both even.*

And by using (if you want to both) 2x2x2-cube-group(s) we can easily obtain all these.
Hence we are done (you have to check all this of course).

Getting even

To be totally honest. Finding the results of the former paragraaf, I thought that I solved Square One. So when I turned and turned ... Oeps! I suddenly was left with only a single swap of two (narrow) parts. Which of course posed me with a problem. Since we know that the Cube-preserving motions are all even permutations, I had to look at the total group to solve this puzzle.
After some thought I finally found a solution. I will however show here only that it is indeed possible to obtain a single swap of two wide parts (check that this also implies that we can obtain a single swap of two narrow parts). For this look at the following shape (obtainable for instance by using my list)



By turning the eye (left) a 180 degrees. We see that we get four single swaps of narrow parts and a single swap of two wide parts. So if we had at this moment the Cube-shape and went to the shape above, turned the eye 180 degrees and then going exactely the same way back. Then we have obtained an even permutation of narrow parts and a single swap of two wide parts. So by using our knowledge of the previous sections we can 'cancel' the even permutation of narrow parts with the help of Cube-preserving motions and hence have obtained a single (odd) permutation/swap.

Of course this is not the most simple way to achieve a single swap of two wide (resp. narrow) parts. But it does prove that they are possible. A shorter sequence of moves to achieve this is left to the reader (you that is). If you find a nice (short) one, please let me know. I plan to make a short list of usefull moves.

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