Rubik's Rings / Hungarian Rings

This puzzle consists of two intersecting rings made up of a number of coloured balls. The rings of balls intersect at two places, so they share two of the balls. Each ring of balls can be turned, so the balls can be mixed.

The Rubik's Rings version has 34 balls of three colours. The intersections divide each ring into sections; there are 5 balls on the inner sections between the two intersection points, and an outer section with 11 balls. In its solved state the 11 blue, 11 red, 12 yellow balls are arranged so that the outer sections are red or blue, and the inner sections and intersections are yellow.

The Hungarian rings puzzle has 38 balls of four colours, intersections which lie 5 apart (i.e. 4 balls in between them). Two colours have 9 balls (yellow and blue) and two colours have 10 balls (black, red). When solved, the balls of each colour must form a continuous row.

Rubik's Rings has the two rings intersecting at an angle, making a 3-dimensional shape. This allows for a neat ratchet system to stop the balls from moving unintentionally.

It might be interesting to quote from the afterword of the Rubik's Cubic Compendium [p212] here. It has a picture of the Hungarian rings and the following text by David Singmaster:

> Closer to Rubik's Magic Cube are 'interlocking cycle' puzzles where several rings of pieces cross each other. Endre Pap, a Hungarian engineer, invented a flat version with two rings which was marketed as the Hungarian Rings. The idea was not entirely new, as there is an 1893 patent for it.

That patent is US 507,215 by William Churchill, filed on May 28 1891, granted on October 24, 1893.

The number of positions of Rubik's Rings:
There are 34 balls, which can be arranged in at most 34! ways. This limit is not reached because:

- The yellow balls are indistinguishable (12!)
- The red balls are indistinguishable (11!)
- The blue balls are indistinguishable (11!)
- The red and blue balls are equivalent (2)

The last point is because the puzzle can be solved both with red or with blue on the left hand side. The total number of positions is therefore 34!/(2·11!·11!·12!) = 193,413,243,572,640.

The number of positions of Hungarian Rings:
There are 38 balls, which can be arranged in at most 38! ways. This limit is not reached because:

- The yellow balls are indistinguishable (9!)
- The blue balls are indistinguishable (9!)
- The red balls are indistinguishable (10!)
- The black balls are indistinguishable (10!)
- The yellow and blue balls are equivalent (2)
- The red and black balls are equivalent (2)

The total number of positions is therefore $38!/(2 \cdot 9! \cdot 10!)^2$ = 75,406,424,215,922,599,800. Note however that (even when taking into account the colour equivalences) there are still 8 possible solutions.

If your browser supports it, you can click on the links below to see a Javascript version of either of these puzzles.

Javascript Rubik's Rings Puzzle

Javascript Hungarian Rings Puzzle

Links to other useful pages:
Hungarian Rings in Java with a different colour scheme

Solution to Rubik's Rings:
This solution is different to the one in the booklet supplied with the puzzle. I think this solution is quicker. It is also more intuitive and therefore easier to remember.

Phase 1: Solve the yellow balls (the inner area)
In this phase all the yellow balls will be placed in position, in the inner sections of the rings including the intersection points.

a. First construct a row of 5 yellow balls in the left ring. This is quite easy to do as follows:
   1. Find a yellow ball in the ring on the right that does not lie at an intersection.
   2. Rotate the left ring to bring whatever row of yellow balls you already have into the outer section of the ring, but adjacent to the intersection point nearest your chosen yellow ball.
   3. Turn the right ring to bring the yellow ball to the intersection, joining it up with the row of balls in the left ring. Repeat this process until you have a row of 5.
b. Turn the puzzle upside down so that the row of 5 balls is in the right ring.
c. Now we will put the 7 remaining yellow balls in a row in the left ring. Nearly the same method can be used as in step a, as long as we make sure that the row of 5 remains in the outer section if the right ring and so takes no part in the action. It is however possible that while building your row of 7, all the remaining loose yellow balls also lie in the left ring so that you get stuck at step a1 above. In this case move the left ring so that the yellow row is still in the outer section, but one of the loose yellow balls lies at an intersection point. You can now move

the right ring a little and continue with step a2.

d. You now have a row of 5 and a row of 7 yellow balls. First make sure both rows are away from the intersection points, next turn the right ring to put the row of 5 in place between the intersection points, and finally turn the left ring to put the row of 7 in place.

Phase 2: Separate the red/blue balls.
In this phase the red and blue colours are separated, thus solving the puzzle. Before you start this phase however, you have to decide which colours the outer sections should be. It is usually best simply to find out which colour dominates a section, red or blue, and then consider that to be the colour it is to be when solved. Balls of the wrong colour in each ring will be called simply 'wrong balls', and balls of the correct colour in each ring 'correct balls'. This phase will attempt to swap a wrong ball on the left with a wrong ball on the right, and thus make them correct.
Instead of clockwise or counter-clockwise turns, I will use the terms 'inwards' and 'outwards' turns. An inwards turn of a ring brings some balls from the outer section of the ring inwards towards the top intersection point. In other words, an inwards turn of the left ring will be clockwise but an inwards turn of the right ring will be counter-clockwise. Outwards turns are the opposite of inwards turns.

a. If there is a wrong ball in the top half of each ring, then the following sequence will correct them:
1. Turn the right ring inwards to bring a correct ball to the top intersection.
2. Turn the left ring inwards to bring the left wrong ball to the top intersection.
3. Turn the right ring to bring the right wrong ball to the top intersection.
4. Turn the left ring back (outwards) into position.
5. Turn the right ring back (outwards) into position.
Note how at the bottom intersection there will always be a yellow ball, so no changes occur there.

b. Repeat step a as often as you can. Note that as it stands, the wrong ball in the right hand ring can even be in the exact middle of the outer section (6 balls away from the intersection) and the sequence will still work, but the left wrong ball must be above the middle. If the left ball is in the middle and the right hand one is not, then you can use the same sequence but with left and right interchanged in each of the steps a1 to a5.

c. If there are wrong balls in the lower halves of both rings then turn the puzzle upside down and go back to step a to correct them.

d. If there are wrong balls in the top half of one ring, and in the bottom half of another, then this can be solved by bringing one wrong ball to the middle, and going back to step a to correct it. To bring a wrong ball in the top half to the middle, do the following sequence:
1. Turn the ring with the wrong ball 6 steps inwards, bringing the ball that was in the middle to the top intersection.
2. Turn the other ring inwards till a correct ball lies at the top intersection.
3. Turn the first ring outwards till the wrong ball lies at the top intersection.
4. Turn the other ring back (outwards) into position.
5. Turn the first ring back (outwards) into position.

e. The only case that can not be handled by the previous steps is when there are only two wrong balls, both in the middle. This can be solved by the following sequence:
1. Turn one ring inwards 6 steps.
2. Turn the other ring inwards 6 steps.
3. Turn the first ring outwards 6 steps.
4. Turn the other ring outwards 6 steps.

Note: Very often you can run various instances of the sequence in phase 2 step a together. The first turn brings a correct ball to the top intersection, and from then on each turn brings a wrong ball to the intersection. You must take care not to turn too far outwards, as the bottom intersection must always have a yellow ball. When you have no wrong balls within reach, turn the rings back to their original positions. This will speed up the process considerably.

Solution to the Hungarian Rings:

Phase 1: Solve one outer colour
In this phase all ten balls of one colour, for example black, will be placed together in the outer sections of one of the

rings.

 a. Find a black ball in the ring on the right that does not lie at an intersection. If there are none, then go to step e below.
 b. Rotate the left ring to bring whatever row of black balls you already have into the outer section of the ring, but adjacent to the intersection point nearest your chosen black ball.
 c. Turn the right ring to bring the black ball to the intersection, joining it up with the row of black balls in the left ring.
 d. Repeat steps a-c until you have a row with all 10 black balls.
 e. It sometimes happens in step a that all the remaining loose black balls also lie in the left ring so that you get stuck at step a above. In this case move the left ring that the black row is still in the outer section, but one of the loose black balls lies at an intersection point. You can now move the right ring a little and continue with step b.

Phase 2: Solve the other outer colour
In this phase all the balls of the second colour with ten balls, for example red, will be placed together in the outer section of the other ring.

 a. Turn over the puzzle so that the black balls you solved in phase 1 are now in the outer section of the right ring.
 b. Solve the red balls in the left outer section using the exact same method as phase 1.

Phase 3: Solve the remaining two colours.
In this phase the yellow and blue colours are separated, thus solving the puzzle.

 a. Turn the left ring so that its previously solved outer section (red or black) is adjacent to the top intersection. Also turn the right ring to bring its previously solved outer section down until it is next to the bottom intersection. Decide which ring should get which colour. I assume that the blue balls will all belong all be in the left ring, and the yellow balls in the right ring. Of course, you can solve them the other way around, but the description is easier if it is limited to one case.
 b. Below I will give a sequence of moves which swap two balls. Using this to solve all the yellow/blue balls can be a bit slow, so if you see any way to solve some of them with a few moves (moves which never bring a red or black ball into an intersection) then by all means do so.
 c. Find a yellow ball and a blue ball which you want to swap. The yellow ball will be on the left ring, and the blue one on the right ring.
 d. If the yellow ball in the left ring lies below the bottom intersection, then the two balls can be swapped using the following sequence:
  1. Turn the right ring until the ball you want to swap lies at the top right, with four balls between it and the top intersection.
  2. Turn the left ring until the ball you want to swap lies at the top left, again with four balls between it and the top intersection.
  Note that the two intersections should now be the same colour.
  3. Turn the left ring 5 steps clockwise (the left ball to swap is now at the top intersection).
  4. Turn the right ring 5 steps anti-clockwise (the two balls to swap are now at the intersections).
  5. Turn the left ring 5 steps anti-clockwise.
  6. Turn the right ring 5 steps clockwise.
  Note that steps 3-6 swapped the two balls, but also swapped the two balls at the intersections but since they had the same colour this is not noticeable.
  7. Turn the left ring back to its original position, with its outer section next to the top intersection.
  8. Turn the right ring back to its original position, with its outer section next to the bottom intersection.
 e. The above worked only if the ball to be swapped from the left ring was below the bottom intersection. If the ball in the right ring lies between the intersections, we can use nearly the same method except that we bring the balls to the top in the opposite order:
  1. Turn the left ring until the ball you want to swap lies at the top left, with four balls between it and the top intersection.
  2. Turn the right ring until the ball you want to swap lies at the top right, again with four balls between it and the

top intersection.
Note that the two intersections should now be the same colour.
3-6. Exactly the same as steps d3-6 above.
7. Turn the right ring back to its original position, with its outer section next to the bottom intersection.
8. Turn the left ring back to its original position, with its outer section next to the top intersection.

f. If either the ball to swap on the left ring lies between the intersections, or the ball to swap on the right ball lies above the top intersection, then turn the puzzle around. You can then use one of the sequences above.

g. The only case that is not covered by the above sequences is the case where you want to swap the two balls at the intersections. This case can usually be avoided by solving the intersections as soon as possible with swaps involving the other balls. If however these are the only two balls left to be swapped, then you will have to disturb a few solved balls first. For example, turn right ring anti-clockwise, left ring anti-clockwise, right ring clockwise to original position, left ring clockwise to original position. You now have 4 balls which are wrong, of which only one is at an intersection, and so it can be solved using two swaps.

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