## Panama Canal Puzzle


the great
Panama Canal

## PUZZLE

Uncle Josh takes a trip through the Panama Canal. He is writing home, but gets mixed on his spelling and writes: "I am having a fine time at the CANAMA PANAL.'

DIRECTIONS:
Arrange the red letters in the top row to spell PANAMA and the black letters in the lower row to spell CANAL. Now change the P and C as shown on the illustration bełow, and without removing any blocks from the box, arrange the words to spell PANAMA CANAL. This can be accomplished in about 20 moves.

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( 1.75 " $x 4.75$ " $x 7 / 16$ " cardboard box and 12 wood pieces with paper glued to top; directions are on the inside of the box lid)
Starting with the $\boldsymbol{P}$ and $\boldsymbol{C}$ exchanged, slide the pieces to put them back.
A parity argument (like that for the Fifteen puzzle) might lead one to think that a solution is not possible. However, the second and fourth squares of PANAMA can be exchanged without significantly changing the look of the picture formed by the pieces (if one ignores the background graphics, then shorter solutions are possible - see next page).

## Panama Canal Solution

Panama Canal: Here is a 32 move solution, where 1, 2, 3, 4,5 represent the five A's, and X, Y represent the two N's. If a piece can push other pieces, then this solution can be converted to 21 moves by combining steps $1 / 2,8 / 9 / 10,12 / 13 / 14,16 / 17,21 / 22,28 / 29 / 30 / 31 / 32$.

| C | 1 | X | 2 | M | 3 | C | 1 | X | 2 | M | 3 | C | 1 | X | 2 | M | 3 | C | 1 | X | M | 3 | C | 1 | X | M | 3 |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| P | 4 | Y | 5 | L |  | P | 4 | Y | 5 |  | L | P | 4 | Y |  | 5 | L | P | 4 | Y | 2 | 5 | L | P | 4 | Y | 2 | 5 | L |  |
| C | 1 | Y | X | M | 3 | C | 1 | Y | X | M | 3 | C | 1 | Y |  | M | 3 |  | C | 1 |  | Y | M | 3 | C |  | 1 | Y | M | 3 |
| P | 4 |  | 2 | 5 | L | P | 4 | 2 |  | 5 | L | P | 4 | 2 | X | 5 | L | P | 4 | 2 | X | 5 | L | P | 4 | 2 | X | 5 | L |  |
|  | C | 1 | Y | M | 3 | P | C | 1 | Y | M | 3 | P | C | 1 | Y | M | 3 | P | C | 1 | Y | M | 3 | P | C | 1 | Y | M | 3 |  |
| P | 4 | 2 | X | 5 | L |  | 4 | 2 | X | 5 | L | 4 |  | 2 | X | 5 | L | 4 | 2 |  | X | 5 | L | 4 | 2 | X |  | 5 | L |  |
| P | C | 1 |  | M | 3 | P | C |  | 1 | M | 3 | P |  | C | 1 | M | 3 | P | 2 | C | 1 | M | 3 | P | 2 | C | 1 | M | 3 |  |
| 4 | 2 | X | Y | 5 | L | 4 | 2 | X | Y | 5 | L | 4 | 2 | X | Y | 5 | L | 4 |  | X | Y | 5 | L |  | 4 | X | Y | 5 | L |  |
|  | 2 | C | 1 | M | 3 | 2 |  | C | 1 | M | 3 | 2 | C |  | 1 | M | 3 | 2 | C | X | 1 | M | 3 | 2 | C | X | 1 | M | 3 |  |
| P | 4 | X | Y | 5 | L | P | 4 | X | Y | 5 | L | P | 4 | X | Y | 5 | L | P | 4 |  | Y | 5 | L | P |  | 4 | Y | 5 | L |  |
| 2 |  | X | 1 | M | 3 |  | 2 | X | 1 | M | 3 | P | 2 | X | 1 | M | 3 | P | 2 | X | 1 | M | 3 | P | 2 | X | 1 | M | 3 |  |
| P | C | 4 | Y | 5 | L | P | C | 4 | Y | 5 | L |  | C | 4 | Y | 5 | L | C |  | 4 | Y | 5 | L | C | 4 |  | Y | 5 | L |  |
| P | 2 | X | 1 | M | 3 | P | 2 | X | 1 | M | 3 | P | 2 | X | 1 | M | 3 |  |  |  |  |  |  |  |  |  |  |  |  |  |
| C | 4 | Y |  | 5 | L | C | 4 | Y | 5 |  | L | C | 4 | Y | 5 | L |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

Panama Canal NB: For the simpler problem with no background, here is a 26 move solution. If a piece can push other pieces, then this solution can be converted to 17 moves by combining steps $1 / 2 / 3 / 4,8 / 9,15 / 16,22 / 23 / 24 / 25 / 26$.


Panama Canal H: For the even simpler problem with no background and "CANAL" can be right justified, Hordern's book gives a 23 move solution and Baxter's Page lets one search for a 21 move solution. If a piece can push other pieces, then this solution can be converted to 16 moves by combining steps $1 / 2 / 3 / 4,8 / 9,15 / 16$.

| C | 1 | X | 2 | M | 3 | C | 1 | X | 2 | M | 3 | C | 1 | X | 2 | M | 3 | C | 1 | X | 2 | M | 3 | C | 1 | X | 2 | M | 3 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| P | 4 | Y | 5 | L |  | P | 4 | Y | 5 |  | L | P | 4 | Y |  | 5 | L | P | 4 |  | Y | 5 | L | P |  | 4 | Y | 5 | L |
| C | X | 2 | M | 3 |  | C | X | 2 | M | 3 | P | C | X | 2 | M | 3 | P | C | X | 2 | M | 3 | P | C | X | 2 | M | 3 |  |
| P | 1 | 4 | Y | 5 | L | P | 1 | 4 | Y | 5 | L |  | 1 | 4 | Y | 5 | L | 1 |  | 4 | Y | 5 | L | 1 | 4 |  | Y | 5 | L |
| P | C |  | 2 | M | 3 | P |  | C | 2 | M | 3 | P | 4 | C | 2 | M | 3 | P | 4 | C | 2 | M | 3 |  | 4 | C | 2 | M | 3 |
| 1 | 4 | X | Y | 5 | L | 1 | 4 | X | Y | 5 | L | 1 |  | X | Y | 5 | L |  | 1 | X | Y | 5 | L | P | 1 | X | Y | 5 | L |
| 4 |  | C | 2 | M | 3 | 4 | C |  | 2 | M | 3 | 4 | C | X | 2 | M | 3 | 4 | C | X | 2 | M | 3 | 4 |  | X | 2 | M | 3 |
| P | 1 | X | Y | 5 | L | P | 1 | X | Y | 5 | L | P | 1 |  | Y | 5 | L | P |  | 1 | Y | 5 | L | P | C | 1 | Y | 5 | L |
|  | 4 | X | 2 | M | 3 | P | 4 | X | 2 | M | 3 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| P | C | 1 | Y | 5 | L |  | C | 1 | Y | 5 | L |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

## Another Version Of The Panama Canal Puzzle



Designed and made by J. A. Storer 2007.
(3"x 7 " x 1.4 " wood box, 11 wood pieces, and a black wood keeper piece)
Panama Canal S: In this version, the second and sixth squares of PANAMA can be exchanged without changing the picture. Here is a 48 move solution. If a piece can push other pieces, then this solution cab be converted to 36 moves by combining steps $6 / 7,11 / 12,16 / 17 / 18,20 / 21 / 22$, 24/25, 29/30, 36/37/38/39, 43/44.

| $\begin{array}{lllllll}\text { C } & 1 & X & 2 & M & 3 \\ P & 4 & \mathrm{Y} & 5 & \mathrm{~L} & \end{array}$ | $\begin{array}{lllllll}\text { C } & 1 & X & 2 & M & \\ \text { P } & 4 & \mathrm{Y} & 5 & \text { L } & 3\end{array}$ | $\begin{array}{lcccccc}\text { C } & 1 & X & 2 & & M \\ P & 4 & Y & 5 & L & 3\end{array}$ | $\begin{array}{llllrr}\text { C } & 1 & \mathrm{X} & 2 & \text { L } & \mathrm{M} \\ \mathrm{P} & 4 & \mathrm{Y} & 5 & & 3\end{array}$ | $\begin{array}{llllll} \text { C } & 1 & X & 2 & \text { L } & \text { M } \\ \text { P } & 4 & \mathrm{Y} & 5 & 3 & \end{array}$ |
| :---: | :---: | :---: | :---: | :---: |
| C $1 \times 2 \mathrm{~L}$ | C $1 \times 2 \mathrm{~L}$ | C 1 X 2 L | C $1 \times 52 \mathrm{~L}$ | C $1 \times 52 \mathrm{~L}$ |
| P 4 Y 53 M | P 4 Y 53 M | P 4 Y 53 M | P $4 \mathrm{Y} \quad 3 \mathrm{M}$ | P 4 Y 3 M |
| C 1 X 5 L | C 1 X 5 L | C 1 X 5 L | C 1 Y X 5 L | C 1 Y X 5 L |
| P 4 Y 3 3 M | P 4 Y 32 m | P 4 Y 32 m | P $4 \quad 3 \quad 2 \mathrm{M}$ | $\begin{array}{lllll}\text { P } & 4 & 3 & 2\end{array}$ |
| C 1 Y 5 L | C 1 Y 5 L | C $1 . \mathrm{Y} 5 \mathrm{~L}$ | C 1 Y 5 L | P C $1 . Y 5 \mathrm{~L}$ |
| P 4.3 X 2 2 M | P 4.3 X 2 2 M | P 4.3 X 2 2 M | P 4.3 X 2 M | 43 X 2 M |
| P C 1 Y 5 L | P C 1 Y 5 L | P C $1 . \mathrm{Y} 5 \mathrm{~L}$ | $\begin{array}{llll}\text { P C } & 1 & 5\end{array}$ | $\begin{array}{llll}\text { P C } & 1 & 5\end{array}$ |
| $4 \quad 3 \mathrm{X} 2 \mathrm{M}$ | 43 X 2 M | 43 X 2 M | 43 X Y 2 M | 43 X Y 2 M |
| P C 1 5 L | P 3 C 1 5 L | P 3 C 1 5 L | 3 C 15 L | 3 C 15 L |
| 43 X Y 2 M | 4 X Y 2 M | $4 \times \mathrm{Y} 2 \mathrm{M}$ | P $4 \times \mathrm{X}$ Y 2 M | P $4 \times \mathrm{X}$ Y 2 M |
| 3 C 15 L | 3 C X 1 5 L | 3 C X 15 L | 3 X 15 L | $3 \times 15 \mathrm{~L}$ |
| P $4 \times \mathrm{X}$ Y 2 m | P $4 \quad Y \quad 2 \mathrm{M}$ | $\mathrm{P} \quad 4 \mathrm{Y} 2 \mathrm{M}$ | P C 4 Y 2 M | P C 4 Y 2 M |
| P $3 \times 15 \mathrm{~L}$ | P 3 X 15 L | P $3 \times 15 \mathrm{~L}$ | P $3 \times 15 \mathrm{~L}$ | P $3 \times 15 \mathrm{~L}$ |
| C 4 Y 2 M | C 4 Y 2 M | C 4 Y 2 M | C $4 \mathrm{Y} \quad 2 \mathrm{M}$ | C 4 Y 2 M |
| P $3 \times 1 \times 15$ | P $3 \mathrm{X} \quad 1 \mathrm{~L}$ | P $3 \times 21 \mathrm{~L}$ | P 3 X 211 L | P 3 X 21 L |
| C 4 Y 25 M | C 4 Y 25 m | C $4 \mathrm{Y} \quad 5 \mathrm{M}$ | C 4 Y 5 M | C 4 Y 5 M |
| P $3 \times 21$ | P 3 X | P 33 X 22 M 1 | P 3 X 2 M 1 |  |
| C 4 Y 5 M L | C 4 Y 5 M L | C 4 Y 5 L | C 4 Y 5 L |  |

This version does a reasonable job of "exercising" the pieces. Since half of the possible positions are duplicates due to the two A's with the same background, there are $12!/ 2=239,500,800$ distinct positions. To find a solution, a computer program performing a simple breadth-first search visited $102,714,408$ positions ( 43 percent). In contrast, the program visited 3,611,235 positions ( 1.5 percent) to solve the standard version, and for the NB and $H$ versions, where the five A's are interchangeable and the two N's are interchangeable, giving 12! / 5! / $2=3,991,680$ distinct positions, to find a solution it visited 120,542 positions ( 3 percent) and 42,602 positions (1 percent) respectively.

## Related Puzzles

The 2-unit high shape has been used in similar puzzles. The 1923 Hartman patent shows the same puzzle but with the numbers 1 to 11 on the pieces; and the goal is to rearrange them so each column sums to 11 . The puzzle below, purchased in 2008, has a 2 by 7 array of pieces and an extra position (similar to the Sixteen puzzle):


## Further Reading

Baxter's Page, from: http://www.johnrausch.com/SlidingBlockPuzzles/classic.htm Hartman Patent, from: www.uspto.gov - patent no. 1,464,424

