Jaap's Puzzle Page

Rashkey

This puzzle consists of two overlapping circular disks. The distance between the centres is exactly one radius length. The disks are made up of many pieces with curved sides, which allow them to be rotated any number of quarter turns. Each disk has 4 squares, 4 diamonds, and 12 triangles. As some pieces are shared between the two disks, there are all together 6 squares, 7 diamonds, and 20 triangles.

The puzzle comes in three possible colour schemes:

1. The central 4 triangles of one disk are red, those of the other disk are blue, and all the rest is yellow.
2. One disk is completely yellow, and the remaining parts of the second disk are red.
3. One disk is red and the other is blue, except that the pieces which are shared between them which are yellow.

These are in order of difficulty - 1 is easiest and 3 is hardest.

Rashkey was invented by Oleg Raschkov, and patented on 2 September 1999, DE 29,904,348 U.

If your browser supports it, you can click on the link below to see a Javascript version of the Rashkey puzzle. It can play all three types. The solver for types 1 and 2 will always find the shortest method, but the solver for type 3 will do the triangles, squares, and diamonds in separate phases (though each of those phases is done in shortest way, the overall solution will usually not be optimal).

Javascript Rashkey Puzzle

The number of positions:

The triangles lie in two sets of 10 which cannot intermingle. The 33 pieces fall into sets (orbits) of 10, 10, 7, and 6 pieces which can each be mixed. There are some parity restrictions, but since all colour schemes have many identical pieces, these restrictions have no discernible effect. There is a restriction on the set of squares (similar to Turn'Push), but only the colouring used in Rashkey 3 is detailed enough for it to be noticeable.

	Triangles 1	Triangles 2	Diamonds	Squares	Positions	
Rashkey 1	4, 6	4, 6	7	6	$(10!/4!6!)^2$	44,100
Rashkey 2	2, 8	4, 6	3,4	2,4	10!/2!8! 10!/4!6! 7!/3!4! 6!/2!4!	4,961,250
Rashkey 3	2, 2, 6	2, 2, 6	1, 3, 3	2, 2, 2	$(10!/2!2!6!)^2$ 7!/1!3!3! 60	13,335,840,000

I have calculated through all positions of Rashkey types 1 and 2, and the results are shown in the tables below. They show the number of positions for each number of moves from the start. There are two ways of counting moves - a turn of a disc by any amount is a single move (face turn metric), or each quarter turn of a disc is counted as a move (quarter turn metric). It shows that Rashkey 1 needs at most 12 face turns (9.1118 on average) or 15 quarter turns (11.190 on average), and that Rashkey 2 needs at most 19 face turn (14.356 on average) or 23 quarter turns (17.573 on average). Rashkey 3 has too many positions to calculate fully.

Rashkey 1:

		Face turn metric													
		0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**Total**
Quarter turn metric	**0**	1													**1**
	1		4												**4**
	2		2	8											**10**
	3			8	16										**24**
	4			2	24	32									**58**
	5				12	64	60								**136**
	6				1	44	152	96							**293**
	7					8	140	336	168						**652**
	8						56	408	744	266					**1,474**
	9						8	236	1,160	1,400	372				**3,176**
	10							64	800	2,836	2,436	312			**6,448**
	11							4	180	2,130	5,352	2,740	112		**10,518**
	12								8	471	4,090	6,662	1,332	8	**12,571**
	13									32	693	4,028	2,724	104	**7,581**
	14										16	308	674	132	**1,130**
	15												16	8	**24**
	Total	**1**	**6**	**18**	**53**	**148**	**416**	**1,144**	**3,060**	**7,135**	**12,959**	**14,050**	**4,858**	**252**	**44,100**

Rashkey 2:

Quarter turn metric \ Face turn metric	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	T
0	1																				
1		2																			
2		1	4																		
3			4	8																	
4			1	12	16																
5				5	30	32															
6					20	76	64														
7					5	70	184	126													
8						30	218	424	248												
9						5	132	614	960	480											**2,**
10							37	488	1,634	2,104	928										**5,**
11							2	192	1,564	4,181	4,547	1,742									**12,**
12								27	822	4,679	10,202	9,623	3,048								**28,**
13									196	3,011	13,122	24,281	19,328	4,888							**64,**
14									15	997	9,943	35,211	55,056	35,533	6,785						**143,**
15										133	4,211	30,296	89,665	115,258	56,542	7,066					**303,**
16										6	829	14,661	84,917	209,467	208,281	65,413	4,453				**588,**
17											71	3,503	42,943	208,527	410,860	270,493	43,258	1,068			**980,**
18												277	10,195	99,963	397,330	549,213	188,712	10,949	54		**1,256,**
19												9	931	19,195	153,871	462,995	364,628	47,233	544		**1,049,**
20													24	1,252	19,387	119,242	229,555	77,695	2,247		**449,**
21														22	661	7,104	31,605	30,743	2,825	8	**72,**
22															4	105	678	1,577	516	8	**2,**
23																	4	4	6		
Total	**1**	**3**	**9**	**25**	**71**	**213**	**637**	**1,871**	**5,439**	**15,591**	**43,853**	**119,603**	**306,107**	**694,105**	**1253,721**	**1481,631**	**862,893**	**169,269**	**6,192**	**16**	**4,961,**

In Sloane's On-Line Encyclopedia of Integer Sequences these are included as sequences A079843, A079844, A079857, and A079864.

Links to other useful pages:

PuzzleAtomic is Douglas Engel's site, where he sells many different types of circle puzzle, including Dancing Gears which is equivalent to Rashkey.

Notation used in solutions:

Let a clockwise quarter turn of the left disk be denoted by L. Rotations of 180, or 270 degrees are then denoted by L2, and L3. Turns of the right disk are denoted in the same way as R, R2 or R3.

Rashkey 1

I will use the 'the left centre' to mean the four central triangles of the left disk, and similarly 'right centre' are those of the right disk. I will assume that the red pieces belong in the left centre when it is solved, and the blue pieces in the right centre. Thus in the mixed puzzle the red pieces will be in the left centre and the rim of the right disk, whereas the blue lie in the right centre and the rim of the left disk. You may have to turn the puzzle upside down to achieve this.

It is actually remarkably tricky to find a short straightforward solution to this puzzle. Much of the solution below is intuitive once you see what you are trying to do, but looks complicated when written out in such detail.

Phase 1: Place all the red pieces in two pairs at the top and right of the right disk.

a. If there is not yet a pair of red pieces next to each other, then make such a pair. Now bring that pair to the top of the right disk. This is easy so I will not elaborate on it.
b. There are several possibilities for the other two red pieces. If they are together at the right hand side of the right disk, then continue with phase 2.
c. If they are together at the bottom of the right disk, then do R L2 R L2 R2 to put them on the right, and then continue with phase 2.
d. If there is no red piece at the left centre, then do R L2 R3 to bring (at least) one such piece there.
e. Bring the other loose red piece to the left centre as well, by turning the right disk.
f. If the two red pieces in the left centre are not adjacent, then do the following steps to make put them next to each other:
   1. Turn the left disk so that the two red pieces are at the bottom left and top right of the left centre.
   2. Turn the right disk so that there are no red pieces that lie in both disks.
   3. Do L.
   4. Turn the right disk to bring the two loose pieces together.
g. Turn the left disk so that its adjacent red pair lies on the left.
h. Turn the right disk to bring its pair to the bottom.
i. Do L2 R2, and the red pieces will be at the top and right of the rim of the right disk.

Phase 2: Make the two blue piece pairs on the rim of the left disk.
The red piece are out of the way, so we can move the blue pieces without disturbing the reds.

a. If there is a blue piece at the top right of the right centre, then you need to remove it as follows:
   1. Make sure there is a yellow piece immediately to the left of the blue piece, by turning the left disk.
   2. Do R3 L2 R2 L2 R2.
   This swaps the order of the red pairs, dislodging the blue piece in the process.
b. Turn the left disk through 360 degrees, and as you do so check out how many of the loose blue pieces visit the top left position of the right centre (suppose there are x of these) and also how many visit the bottom left position of the right centre (suppose there are y of these).
c. If x is at least 2, then you can make a blue pair as follows:
   1. Turn the left disk to bring one of the loose blue pieces to the top left of the right centre.
   2. Do R.
   3. Turn the left disk to bring another loose blue to the top left of the right centre.
   4. Do R3.
   Now go back to step b.
d. If y is at least 2, then you can bring together two of those pieces to make a blue pair as follows:
   1. Turn the left disk so that the two loose blue pieces do not lie at the right centre, and nor does any previously solved blue pair.
   2. Do R.
   3. Turn the left disk to bring one of the loose blue pieces to the bottom left of the right centre.
   4. Do R3.

5. Turn the left disk to bring another loose blue to the bottom left of the right centre.
6. Do R.
7. Turn the left disk so that the right centre is completely yellow.
8. Do R3 to bring the red pairs to the top and right as before.
Now go back to step b.

e. If x and y are both 1, then the two loose pieces cannot be paired up immediately. One of them has to be shifted as follows:
1. Turn the left disk to bring one of the loose blue pieces to the bottom left of the right centre.
2. Do R.
3. Turn the left disk so that the right centre is completely yellow.
4. Do R3.
Now go back to step c to pair them up.

Phase 3: Bring the pairs together.

a. Turn the left ring to bring one blue pair to the right centre, and the other at the left or top of the left disk. The red pairs will still be at the right and top of the right disk.
b. If the second blue pair is on the left side of the left disk, then do the move sequence:
R2 L2 R
c. If the second blue pair is at the top of the left disk, then do the move sequence:
R L3 R3 L R L3 R L2 R

Pretty Patterns for Rashkey 1

1. Wheels 1: R2 L R3 L3 R2 L2 R L R L2 R3 L3
2. Wheels 2: R2 L R L3 R2 L2 R3 L R3 L2 R L3

Rashkey 2

I will assume that the red pieces belong in the left disk when it is solved. Thus in the mixed puzzle there should be more than two red triangles on the rim of the left disk. If this is not so, you have to turn the puzzle upside down.

Phase 1: Solve the 6 red triangles on the outside of the left disk.
Note that you can equally well think of this phase as solving the 4 yellow triangles in the centre of the right disk.

a. Turn the left disk until there are (at least) two adjacent yellow triangles in the centre of the right disk, and then turn the right disk so that the two yellow triangles lie on the right of the centre.
b. Turn the left disk so that there are at least three yellow pieces at the right centre. If the right centre is completely yellow, continue with phase 2.
c. Turn the right disk so that its single red centre triangle is at the top right.
d. Turn the left disk so that the right centre has two red triangles.
e. If the two red triangles in the right centre are not adjacent, then
1. Turn the left disk so that there is only one red triangle in the right centre.
2. Turn the right disk a quarter turn clockwise (so the centre red triangle is at the bottom right)
3. Turn the left disk back, so the right centre has two adjacent red triangles.
f. Turn the right disk so that its red centre triangles are on the left.
g. Turn the left disk so that there are no red pieces in the right centre.

Phase 2: Solve the 2 red triangles in the left half of the centre of the left disk.

a. If the two remaining red triangles are already correct, i.e. on the left half of the left centre, then go directly to the next phase.
b. Turn the right disk to bring one of the unsolved red triangles to the left centre.
c. If that loose red is at the top right of the left centre, then do L R2 L3 R2 L, but if it lies at the bottom right of the left centre then do L3 R2 L R2 L3. These sequences will move the loose red into the left half of the left centre.
d. Turn the right disk to bring the last unsolved red triangle to the left centre.
e. There are 4 cases for the last two reds, depending on where they lie in the left centre.
Bottom left and top right: L R2 L3 R2 L
Bottom left and bottom right: R2 L3 R2 L3 R2 L2 R2 L3
Top left and bottom right: L3 R2 L R2 L3
Top left and top right: R2 L R2 L R2 L2 R2 L

Phase 3: Solve the 2 red squares

a. If the top left square is not red, then turn the right disc to bring a red square to the bottom centre of the puzzle, and do L R3 L3 R L R3 L3 R L.
b. If the bottom left square is not red, then turn the right disc to bring a red square to the top centre of the puzzle, and do L3 R L R3 L3 R L R3 L3.
c. Repeat steps a or b until both red squares are correct.

Phase 4: Solve the 3 red diamonds

a. If the diamond on the far left of the puzzle is not red, then turn the right disc to bring a red diamond to the far right, and do L R3, followed by L R3 L3 R ten times, and then R L3.
b. If the diamond on the top left of the puzzle is not red, then turn the right disc to bring a red diamond to the top right, and do L R3 L3 R ten times.
c. If the diamond on the bottom left of the puzzle is not red, then turn the right disc to bring a red diamond to the bottom right, and do L3 R L R3 ten times.

Rashkey 3

I will assume that the red pieces belong in the left disk when it is solved. Thus in the mixed puzzle there should be more than two red triangles on the rim of the left disk. If this is not so, you have to turn the puzzle upside down.

Phase 1: Solve all the red triangles
This is exactly the same as phases 1 and 2 of Rashkey 2 (but where yellow is mentioned, it should say "non-red", i.e. "yellow or blue").

Phase 2: Solve the 6 blue triangles on the rim of the right disc.
Note that you can equally well think of this phase as solving the 2 yellow triangles in the centre of the left disk.

a. Turn the right disc until there is at least one yellow triangle in the left centre.
b. If the yellow triangle is at the top right of the left centre, then bring the other yellow triangle in the left centre as well by doing the appropriate sequence below:
Top left: L3 R3 L3 R L3 R2 L2 R L2 R L3
Top right: L2 R2 L3 R L2 R3 L R2 L2

Right top: L3 R2 L3 R2 L R2 L
Right bottom: L3 R2 L2 R L R L3 R2 L3 R2 L3 R2
Bottom right: L3 R L3 R3 L3 R2 L2 R3 L2 R3 L3
Bottom left: L2 R2 L3 R3 L2 R L R2 L2

c. If the yellow triangle is at the bottom right of the left centre, then bring the other yellow triangle in the left centre as well by doing the appropriate sequence below:
Top left: L2 R2 L R L2 R3 L3 R2 L2
Top right: L R3 L R L R2 L2 R L2 R L
Right top: L R2 L2 R3 L3 R3 L R2 L R2 L R2
Right bottom: L R2 L R2 L3 R2 L3
Bottom right: L2 R2 L R3 L2 R L3 R2 L2
Bottom left: L R L R3 L R2 L2 R3 L2 R3 L

Phase 3: Solve the triangles in the right centre

a. Do one of the following sequences, depending on the position of the two blue triangles in the right centre:
Right: Do nothing as the triangles are solved already.
Bottom: R L R L3 R3 L R L3 R3 L R3
Left: R L2 R2 L2 R2 L2 R3
Top: R3 L3 R3 L R L3 R3 L R L3 R
Top left and bottom right: R2 L3 R3 L3 R L R L R3 L3 R3 L R
Top right and bottom left: R2 L R L R3 L3 R3 L3 R L R L3 R

Phase 4: Solve the squares

a. If the top left square is not red, then turn the right disc to bring a red square to the top right, do L R3 L3 R five times, and turn the right disc back to its original position.
b. If the bottom left square is not red, then turn the right disc to bring a red square to the bottom right, do L3 R L R3 five times, and turn the right disc back to its original position.
c. Depending on the positions of the two blue squares, do one of the following move sequences:
Top right, bottom right: Do nothing as they are solved already.
Bottom centre, bottom right: Do L2, then L R3 L3 R five times, and L2.
Top centre, top right: Do L2, then L3 R L R3 five times, and L2.
Bottom centre, top centre: Do L3 R3, then L R3 L3 R five times, and R L.

Note that phase 4 uses the sequence $(L\ R3\ L3\ R)^5$, which swaps the centre squares as well as the top left and top right squares. It possible to solve the squares more quickly than the above method by using different conjugates of that sequence to solve more pieces at the same time. In fact you need to apply that sequence at most twice and often only once is enough.

Phase 5: Solve the diamonds

a. If the left diamond (of the left disc) is not red, then turn the right disc until a red diamond is at the top, and do L, repeat L R3 L3 R ten times, do L3, and turn the right disc back to its original position.
b. If the top left diamond is not red, then turn the right disc until a red diamond is at the top, repeat L R3 L3 R ten times, and turn the right disc back to its original position.
c. If the bottom left diamond is not red, then turn the right disc until a red diamond is at the bottom, repeat L3 R L R3 ten times, and turn the right disc back to its original position.
d. Depending on where the yellow diamond is, do one of the following sequences:
Centre: Do nothing as the puzzle is solved already.
Top right: Do L3 R3, then R L3 R3 L ten times, and R L.
Far right: Do L3 R3, then L R3 L3 R ten times, and R L.
Bottom right: Do L R, then R3 L R L3 ten times, and R3 L3.

Note that phase 5 uses the sequence $(L\ R3\ L3\ R)^{10}$, which cycles the centre, top left and top right diamonds. It possible to solve the diamonds more quickly than the above method by using different conjugates of that sequence to solve more pieces at the same time.

Pretty Patterns for Rashkey 3

1. Batman: R2 LRLR L R3 L2 R L3 R3 L2 R L2 R3 LRL R3 L3 R3 L3
2. Butterfly: L3 R L2 R3 L R L R2 L3 R2L2R2L2R2 L R3 L R2 L R3 L R2 L R2

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