



Membership

**Publications** 

Professional Development

Meetings

Organization

Competitions

## Math Games

## sliding-block Puzzles

Ed Pegg Jr., December 13, 2004

The December issue of *The Economist* contains an <u>article</u> with a prominent question. **Has an inventor found the hardest possible simple sliding-block puzzle?** It goes on to describe the *Quzzle* puzzle, by Jim Lewis. The article finishes with "Mr Lewis claims that *Quzzle*, as he dubs his invention, is 'the world's hardest simple sliding-block puzzle.' Within the terms of his particular definition of 'simple,' he would seem to have succeeded."

The claim is wrong. Quzzle is *not* the world's hardest simple sliding-block puzzle, no matter how you define "hard."

Sometimes, hard means *lot and lots of moves*. Junk Kato has succinctly described a record setting series for the most moves required with *n* pieces. The basic underlying principle is <u>Edouard Lucas</u>' <u>Tower of Hanoi</u> puzzle. Move the red piece to the bottom slot.

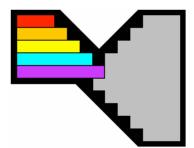


Figure 1. Junk's Hanoi by Junk Kato. From Modern sliding-block Puzzles.

Often, "hard" maximizes complexity with a sparse number of pieces. Usually, the fewer the pieces, the better the puzzle. Two people at the forefront of making really hard sliding-block puzzles with just a few pieces are Oskar van Deventer and James Stephens. James started with <u>Sliding-Block Puzzles</u>. The two of them collaborated to create the <u>ConSlide Puzzle</u> and the <u>Bulbous Blob Puzzle</u>. Oskar even went so far as to make a prototype of the excellent <u>Simplicity</u> puzzle. If any sliding-block puzzle deserves the label "hardest simple puzzle," it might be this one.

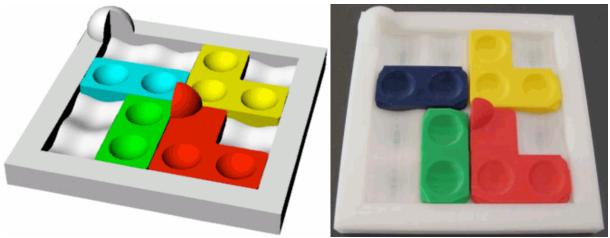


Figure 2. Simplicity by James Stephens, prototyped by Oskar. Move the red piece to the upper corner.

Oskar's prototype for <u>Bulbous Blob</u> is a thing of beauty, both aesthetically and puzzlistically. I hope it gets sold in stores eventually. These pieces are the diominos and triominos from the set of <u>rounded polyominos</u>.



Figure 3. The <u>Bulbous Blob</u> puzzle by James Stephens and Oskar van Deventer.

For puzzles with a small number of pieces, a high number of moves, and difficult solutions, my own puzzles used to be record setters. <u>F-penta</u> and <u>Centrifuge</u> are very difficult 4 and 5 piece sliding-block puzzles, requiring 41 and 63 moves. While writing this column, I tried a tiny modification to Simplicity (which I will call Simplicity 2), and found it requires 68 moves. (Hopefully, someone will topple this new record for 4 piece puzzles.)

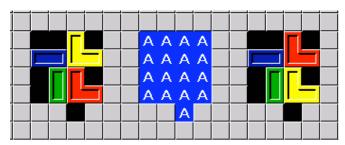


Figure 4. Simplicity 2, "solution found" display for the Taniguchi solver.

I created <u>my puzzles</u> with <u>Taniguchi's sliding-block Puzzle Solver</u>. It's an excellent free program that I regularly mention it on <u>mathpuzzle.com</u>, in the hopes someone will create a fantastic new series of puzzles. Is Simplicity 2 the hardest 4 piece puzzle? <u>Let me know</u>.

An old-style sliding-block puzzle fits into a 4x5 rectangle, has all pieces 1x1, 1x2, or 2x2, and has goal "move the large piece to position B." The most famous puzzle of this type is <u>Dad's puzzle</u>. For comparison purposes, Nick Baxter has put together a <u>page of 4x5 puzzles</u>. In <u>Winning Ways</u>, John Conway published the Century puzzle, which has long been the "hardest" old-style puzzle. Here's the solution:

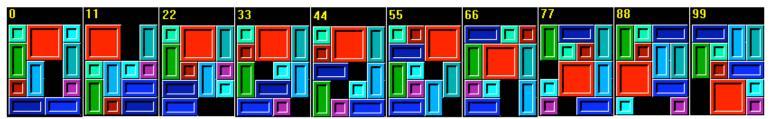


Figure 5. Steps in the solution of John Conway's Century puzzle.

He was later bested by Junk Kato, with the 123-move puzzle <u>Supercompo</u>. Cut to last week. Gil Dogon, a software Engineer and part time puzzler/mathematician/hacker, was a "bit irked" by the claims in *The Economist* that Jim Lewis had created the "Hardest 4x5 sliding puzzle problem." He proceeded to find a 138 move puzzle which he calls <u>Super-Century</u>. Quzzle requires 84 moves.

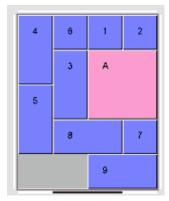


Figure 6. Super-Century by Gil Dogon. Move piece A to the lower middle of the board.

Another old-style puzzle has goal "go from board position A to board position B." Conway's <u>Century and a Half</u> requires flipping the board position, and needs 150 moves. Junk Kato's <u>HiFi</u> requires swapping 2 pieces, and needs 200 moves. Gil Dogon's position above requires 229 moves to flip the board position.

If L-trominoes or other polyforms are tossed into the mix, greater complexities appear. Michael McKee has made very, *very* difficult puzzles with the pentominoes and a 9×9 board-- see <u>Pentomino Mosaics</u>. So -- just how difficult can sliding-block puzzles be?

In February 1964, Martin Gardner devoted his *Mathematical Games* column to sliding-block puzzles. He said: "These puzzles are very much in want of a theory. Short of trial and error, no-one knows how to determine if a given state is obtainable from another given state, and if it is obtainable, no-one knows how to find the minimum chain of moves for achieving the desired state."

Recently, Erik Demaine and Robert Hearn proved that sliding-block puzzles are PSPACE-complete. <u>In their paper</u>, you can see the AND-gates, OR-gates and other wiring necessary to build a sliding-block puzzle that emulates a computer. In fact, a puzzle made just with dominoes is PSPACE-complete. Since sliding-block puzzles are PSPACE-complete, "hardest" is meaningless. For any question a computer can answer with YES or NO, an equivalent sliding-block puzzle can be set up whose solvability depends on the answer.

What is the hardest sliding-block puzzle with *n* pieces, fitting within a board containing *k* squares? We don't know. For a related problem, Sokoban, Erich Friedman has <u>compiled some results</u>.

What is the most elegant sliding-block puzzle? There are two that I particularly like. Serhiy Grabarchuk's <u>Sliding Digits</u> is a twist on the Sam Loyd <u>15 puzzle</u>. Just swap the positions of the 7 and 8. In Rectangular Jam, the pieces may be rotated around, if enough room exists. The size of the pieces is very delicately calculated, to produce a puzzle with much more complexity than is apparent at first glance. <u>Rightangular Jam</u> and <u>Triangular Jam</u> are similarly elegant.

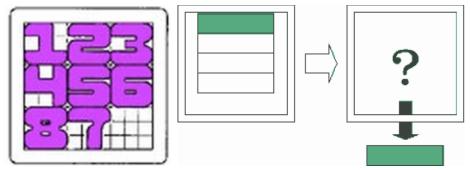


Figure 7. Sliding Digits by Serhiy Grabarchuk, and Rectangular Jam by Hirokazu Iwasawa.

Many good sliding-block puzzles were discovered *this year*. Worse, some of the nice puzzles were discovered by *me*, just bumbling around. Both of those are good signs that there are lots of elegant, undiscovered sliding-block puzzles, waiting for someone to careen into them.

Incidentally, if you have a good mechanical puzzle, you might want to enter the <u>Nob Yoshigahara Puzzle Design Competition</u> for 2005. Any puzzle designer, anywhere in the world, is welcome to enter. Quzzle was an entry in the <u>2004 competition</u>, along with many other wonderful puzzles.

## References:

Nick Baxter, "The sliding-block Puzzle Page," <a href="http://www.puzzleworld.org/SlidingBlockPuzzles/default.htm">http://www.puzzleworld.org/SlidingBlockPuzzles/default.htm</a>.

Nick Baxter, "IPP 24 Competition Entries," <a href="http://www.puzzleworld.org/DesignCompetition/2004/entries.htm">http://www.puzzleworld.org/DesignCompetition/2004/entries.htm</a>.

Elwyn Berlekamp, John Conway, Richard Guy, Winning Ways for Your Mathematical Plays, A K Peters, 2004, p 877-884.

Barry Cipra, Erik Demaine, Martin Demaine, Tom Rodger, Tribute to a Mathemagician, A K Peters, 2004.

Pierre-François Culand, "SBPSolver," <a href="http://www.culand.ch/dev/SBPSolver.htm">http://www.culand.ch/dev/SBPSolver.htm</a>.

Economist.com, "A hard, simple problem" <a href="http://www.economist.com/science/displayStory.cfm?story\_id=3445734">http://www.economist.com/science/displayStory.cfm?story\_id=3445734</a>.

Martin Gardner, "The hypnotic fascination of sliding-block puzzles," Scientific American, 210:122-130, 1964.

Robert A Hearn, "The Complexity of Sliding-Block Puzzles and Plank Puzzles," <a href="http://www.swiss.ai.mit.edu/~bob/sliding-blocks.pdf">http://www.swiss.ai.mit.edu/~bob/sliding-blocks.pdf</a>. (Also published in *Tribute to a Mathemagician*, above.)

Mike Henkes, "Slide Puzzle," <a href="http://home.wanadoo.nl/mike.henkes/index.html">http://home.wanadoo.nl/mike.henkes/index.html</a>.

Edward Hordern, *Sliding Piece Puzzles*, Oxford Univ Press, 1987.

Hirokazu Iwasawa, "Iwahiro's Puzzles," <a href="http://home.r01.itscom.net/iwahiro/main/eng">http://home.r01.itscom.net/iwahiro/main/eng</a> contents/eng intro.html.

Jim Lewis, "Quzzle," <a href="http://www.guirkle.com/puzzle/index.htm">http://www.guirkle.com/puzzle/index.htm</a>.

Ed Pegg Jr, "Modern Burr Puzzles," http://www.maa.org/editorial/mathgames/mathgames 08 02 04.html.

Ed Pegg Jr, "Nob Yoshigahara," http://www.maa.org/editorial/mathgames/mathgames 06 28 04.html.

Slashdot, "Programming Puzzles," <a href="http://developers.slashdot.org/article.pl?sid=04/12/04/0116231">http://developers.slashdot.org/article.pl?sid=04/12/04/0116231</a>.

James Stephens, sliding-block Puzzles, <a href="http://www.puzzlebeast.com/slidingblock/">http://www.puzzlebeast.com/slidingblock/</a>.

"Taniguchi's Programs," <a href="http://homepage2.nifty.com/yuki-tani/index">http://homepage2.nifty.com/yuki-tani/index</a> e.html.

Eric W. Weisstein. "15 Puzzle, Tower of Hanoi" From MathWorld--A Wolfram Web Resource. http://mathworld.wolfram.com/.

Nob Yoshigahara, Rush Hour, <a href="http://www.thinkfun.com/RUSHHOUR.ASPX?PageNo=RUSHHOUR">http://www.thinkfun.com/RUSHHOUR.ASPX?PageNo=RUSHHOUR</a>.

## Math Games archives.

Comments are welcome. Please send comments to Ed Pegg Jr. at ed@mathpuzzle.com.

Ed Pegg Jr. is the webmaster for <u>mathpuzzle.com</u>. He works at Wolfram Research, Inc. as an associate editor of <u>MathWorld</u>, and as administrator of the <u>Mathematica Information Center</u>.

Copyright ©2007 The Mathematical Association of America

Please send comments, suggestions, or corrections for this page to <a href="webmaster@maa.org">webmaster@maa.org</a>.

MAA Online disclaimer Privacy policy