Jaap's Puzzle Page

Rubik's Fifteen

Rubik's Fifteen is a type of sliding piece puzzle. It consists of 17 tiles in a 5x5 square. On the middle three columns there are levers which can move the 4 tiles in a column up or down one square. On the middle three rows there is one large lever that moves the 3x4 tiles left or right. On the top and bottom row there are also levers, but these will only work if there are 4 tiles in the row, i.e. only if all the column levers are up does the top row lever work, only if they are all down does the bottom row lever work.

The 15 tiles are numbered 1-15 in Roman numerals, and of the remaining two tiles one has an asterisk, one is black. The aim is to build a 4x4 square of the numbers in order with the asterisk where 16 would be.

On the back of the puzzle is a window showing the middle 3x3 square of tiles. The back of the 15 numbered tiles have the same numbers as the front, the other two tiles have a zero on the back. As a separate puzzle you can make a magic square on the back. It is clearly impossible to do this if the front is solved, but if you can solve the front the same method suffices to solve the back.

The puzzle was patented by Ernö Rubik on 18 September 1984, US 4,471,959, but there is an earlier Hungarian patent from 1 September 1980, HU2154/80.

A related but much more difficult puzzle is 'It' / Tsukuda's Square. Rubik's Fifteen is easier, because it has more levers and therefore less restrictive movement of the tiles.

If your browser supports JavaScript, then you can play Rubik's Fifteen by clicking the link below:

JavaScript Rubik's Fifteen

The number of positions:

Assume the levers are in one particular position. There are 17 tiles, so there are at most 17! positions. This limit is not reached because only even permutations of the tiles are possible. This leaves 17!/2 = 177,843,714,048,000 positions for any particular position of the levers. There are 6 levers with 2 possible states each, so there are 2^6 possible lever positions. There are therefore $17! \cdot 2^5 =$ 11,381,997,699,072,000 positions all together.

Links to other useful pages:

Raymond Penner's page A Java simulation of the puzzle.

Notation:

Let M mean the pushing right of the large main lever, and let m mean pushing it back to the left. Similarly, let A, B, C denote the column levers from left to right, with a capital letter meaning pushing it down and a small letter pushing it up. The top row lever is T and t for right and left, and the bottom row D and d.

Solution:

Phase 1: Solve the black tile.
The solution given here will solve the 4x4 square on the bottom four rows. The black tile therefore has to go in the top row, at either the left or the right side. Unfortunately it is only possible to solve the puzzle with the tile at the right - in the other you are left with two swapped tiles which is impossible to solve (the puzzle would have to be opened to swap two tiles).

It is also possible to solve the puzzle on the top 4 rows, and in that case the black piece has to go on the same side on the bottom row, the bottom right.

To place the black piece on the top right, follow this method:

a. Push abcT to get the top lever to the right.
b. Push all the other levers to the right and down, i.e. ABCMD.
c. In the unlikely case that the black tile is in the bottom right, then do the sequence dcmCDM to bring it out.
d. If it lies in column 2 (under lever A) then do amAM (or maMA) until it lies in column 3.
e. If it lies in column 3 (under lever B) then do bmBM (or mbMB) until it lies in column 4.
f. If it lies in column 4 or 5 (under lever C) then do cmCM (or mcMC) until it lies at the top of column 5 (or rather in the second row, just below the spot where you want to put it).
g. Remember the method used here to move a tile from column to column. It will be used often, and will not be explained again.
h. To bring the black tile up one row to its final position, do moves abct CmcM TABC.

From this point on the T lever is never used, and the other levers will generally be kept pushed down and to the right.

Phase 2: Solve column A.
The first column is solved by slotting in the tiles XIII, IX, V, and I one by one from the top until the column is correct.

a. Find piece XIII.
b. Bring it to column A, and if necessary do amAM until it lies in row 2 (i.e. until it is the top of the 4 tiles in column A).
c. Find piece IX. If it already lies in column A then do maMA till it is in column B, then do mBMb, and do AmaM until IX is back in row 4.
d. Bring IX to column B, do bmBM till it is in row 4, and do maMA to slot it into column A.
e. Find piece V. If it already lies in column A then do maMA till it is in column B, then do mBMb, and do AmaM until V is back in row 4.
f. Bring V to column B, do bmBM till it is in row 4, and do maMA to slot it into column A.
g. Find piece I. If it already lies in column A then do maMA till it is in column B, then do mBMb, and do AmaM until I is back in row 4.

h. Bring I to column B, do bmBM till it is in row 4, and do maMA to slot it into column A.

The A column should now correctly read I, V, IX, XIII from top to bottom.

Phase 3: Solve column B.
This is done in exactly the same way as column A, by slotting the tiles XIV, X, VI, II into it from the top.

Phase 4: Solve the final two columns.

The sequence cmCM or its inverse mcMC rotate around in a loop all the tiles in the two columns except for the bottom right tile. To solve the two columns we will rotate the loop, and use D (and d) to swap one of the tiles in the loop with another one. This is repeated, each time replacing one of the tiles in the loop with another until the loop contains the pieces in the order IV, VIII, XII, XV, XI, VII, III clockwise from the top. The column has then been solved. The bottom lever will automatically end up correct, with tile marked * at the bottom right.

Lets call the tile that is not part of the loop the spare tile. If the bottom lever has been pushed to the right then the spare tile is the one in the bottom right hand corner. If the bottom lever has been pushed to the left, then the spare tile is the tile at the bottom of column B.

In more detail, the method is as follows:

a. If the spare tile is XV then move the bottom lever. Otherwise repeat cmCM until tile XV is in position at the bottom of column C.
b. If the spare tile is * and the puzzle is not solved then find an incorrect tile, repeat cmCM until that incorrect tile is at the bottom of column C, move the bottom lever, and repeat cmCM to bring XV to the bottom of column C.
c. If the spare tile is not XV or *, then look it up in this list:
   III, 4
   IV, 3
   VII, 5
   VIII, 2
   XI, 6
   XII, 1
   Repeat the sequence cmCM as many times as shown in the list, move the bottom lever, and repeat cmCM until XV lies at the bottom of column C.
d. Repeat this phase until the puzzle has been solved.

If you want to put a magic square on the back, here is a list of all possible magic squares you can have:

5 0 7 6 4 2 1 8 3	7 0 8 6 5 4 2 10 3	6 0 9 8 5 2 1 10 4	10 0 11 8 7 6 3 14 4	7 0 11 10 6 2 1 12 5	9 0 12 10 7 4 2 14 5	8 0 13 12 7 2 1 14 6
6 1 8 7 5 3 2 9 4	8 1 9 7 6 5 3 11 4	7 1 10 9 6 3 2 11 5	11 1 12 9 8 7 4 15 5	8 1 12 11 7 3 2 13 6	10 1 13 11 8 5 3 15 6	9 1 14 13 8 3 2 15 7
7 2 9 8 6 4 3 10 5	9 2 10 8 7 6 4 12 5	8 2 11 10 7 4 3 12 6		9 2 13 12 8 4 3 14 7		
8 3 10 9 7 5 4 11 6	10 3 11 9 8 7 5 13 6	9 3 12 11 8 5 4 13 7		10 3 14 13 9 5 4 15 8		

9 4 11 10 8 6 5 12 7	11 4 12 10 9 8 6 14 7	10 4 13 12 9 6 5 14 8
10 5 12 11 9 7 6 13 8	12 5 13 11 10 9 7 15 8	11 5 14 13 10 7 6 15 9
11 6 13 12 10 8 7 14 9		
12 7 14 13 11 9 8 15 10		

These can also be rotated, or reflected.

Home **Links** **Guestbook**