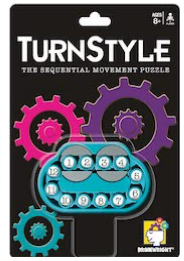
Turnstyle





a.k.a. *Tom's Turnstile*
Copyright Brainwright, a division of Ceaco, 2016;
U.S. Patent 7,604,234 of *Tom Cutrofello, 2009.*
(plastic: 2 x 2.25 x 5/8 inches;
brass version: 2 x 2.25 x 5/8 inches, 6.1 ounces; box is 4 x 4 x 2 inches)

Two discs connected by a gear on the back, that spin together by turning the knob; similar but different from the *Top Spin* puzzle. Easy to rotate the discs, a bit slow to slide the numbers due to the small size (but it fits nicely into a pocket). The basic moves are:

Shift all pieces one position right (clockwise) or left (counter-clockwise).

Flip the discs to change four consecutive pieces from ABCD to BADC.

It is tempting to argue parity for the number of exchanges in a solution, but since the pieces lie on a cycle and not a line, every piece is both ahead of and behind every other, and it is possible to exchange just two adjacent pieces and leave the rest of the puzzle the same (as shown in the photo above); call that transformation **SWAP**. Hence, solving is always possible from any permutation (although from any given permutation, faster solutions may be possible):

Phase 1: Consider 12 solved. Then, starting with 11 in the rightmost position, successively move 11 right one position with a shift left followed by a flip, until 11 is next to 12. Then repeat this for 10, 9, ... 5.

Phase 2: The final four can always be rearranged using successive swaps. For example, starting with 4321, swap(43), swap(42), swap(41), swap(32), swap(31), swap(21) results in 1234 (or, using only two swaps do flip, swap(41), flip, swap(32)).

SWAP Sequence

Computer search found this **SWAP** sequence; **R** denotes move right (clockwise), **L** denotes move left (counterclockwise), and **F** denotes flip, and the letters 0,A,B stand for 10,11,12:

0: 1 2 3 4 5 6 7 8 9 0 A B	**11-F:** 0 8 1 9 A B 2 3 5 4 6 7	**22-F:** 8 4 9 0 A B 2 3 5 6 7 1
1-L: 2 3 4 5 6 7 8 9 0 A B 1	**12-R:** 7 0 8 1 9 A B 2 3 5 4 6	**23-R:** 1 8 4 9 0 A B 2 3 5 6 7
2-F: 3 2 5 4 6 7 8 9 0 A B 1	**13-F:** 0 7 1 8 9 A B 2 3 5 4 6	**24-R:** 7 1 8 4 9 0 A B 2 3 5 6
3-R: 1 3 2 5 4 6 7 8 9 0 A B	**14-R:** 6 0 7 1 8 9 A B 2 3 5 4	**25-F:** 1 7 4 8 9 0 A B 2 3 5 6
4-R: B 1 3 2 5 4 6 7 8 9 0 A	**15-R:** 4 6 0 7 1 8 9 A B 2 3 5	**26-R:** 6 1 7 4 8 9 0 A B 2 3 5
5-F: 1 B 2 3 5 4 6 7 8 9 0 A	**16-F:** 6 4 7 0 1 8 9 A B 2 3 5	**27-F:** 1 6 4 7 8 9 0 A B 2 3 5
6-R: A 1 B 2 3 5 4 6 7 8 9 0	**17-L:** 4 7 0 1 8 9 A B 2 3 5 6	**28-R:** 5 1 6 4 7 8 9 0 A B 2 3
7-R: 0 A 1 B 2 3 5 4 6 7 8 9	**18-F:** 7 4 1 0 8 9 A B 2 3 5 6	**29-F:** 1 5 4 6 7 8 9 0 A B 2 3
8-R: 9 0 A 1 B 2 3 5 4 6 7 8	**19-L:** 4 1 0 8 9 A B 2 3 5 6 7	**30-R:** 3 1 5 4 6 7 8 9 0 A B 2
9-F: 0 9 1 A B 2 3 5 4 6 7 8	**20-F:** 1 4 8 0 9 A B 2 3 5 6 7	**31-F:** 1 3 4 5 6 7 8 9 0 A B 2
10-R: 8 0 9 1 A B 2 3 5 4 6 7	**21-L:** 4 8 0 9 A B 2 3 5 6 7 1	**32-R:** 2 1 3 4 5 6 7 8 9 0 A B

The existence of a SWAP sequence means that all permutations are possible and the solution method proposed on the previous page is well defined. Phase 1 placement of numbers 5 through 12 is straightforward, although it could be shorter for a given permutation. An interesting general question is to improve Phase 2 to fix the final 4 in a way faster than "brute force" use of **SWAP**. For example, the previous page noted that a final 4 of 4321 can be fixed with 2 flips and two **SWAP**s, for a total of 66 moves. However, it can be done in only 18 moves:

0: 4 3 2 1 5 6 7 8 9 0 A B	**7-F**: B 4 2 3 1 5 6 7 8 9 A 0	**13-F:** 0 A 2 B 4 1 3 5 6 7 8 9
1-R: B 4 3 2 1 5 6 7 8 9 0 A	**8-L**: 4 2 3 1 5 6 7 8 9 A 0 B	**14-L**: A 2 B 4 1 3 5 6 7 8 9 0
2-R: A B 4 3 2 1 5 6 7 8 9 0	**9-F:** 2 4 1 3 5 6 7 8 9 A 0 B	**15-L:** 2 B 4 1 3 5 6 7 8 9 0 A
3-R: 0 A B 4 3 2 1 5 6 7 8 9	**10-R**: B 2 4 1 3 5 6 7 8 9 A 0	**16-F:** B 2 1 4 3 5 6 7 8 9 0 A
4-F: A 0 4 B 3 2 1 5 6 7 8 9	**11-R:** 0 B 2 4 1 3 5 6 7 8 9 A	**17-L:** 2 1 4 3 5 6 7 8 9 0 A B
5-L: 0 4 B 3 2 1 5 6 7 8 9 A	**12-R:** A 0 B 2 4 1 3 5 6 7 8 9	**18-F:** 1 2 3 4 5 6 7 8 9 0 A B
6-L: 4 B 3 2 1 5 6 7 8 9 A 0		

Computer search for all 24 permutations found that on average about 22.12 moves are needed to fix the final 4; here is a table of the lengths found for each permutation:

1234 - 0	2134 - 32	3124 - 16	4123 - 27
1243 - 32	2143 - 1	3142 - 32	4132 - 17
1324 - 33	2314 - 16	3214 - 28	4213 - 17
1342 - 16	2341 - 27	3241 - 17	4231 - 32
1423 - 16	2413 - 32	3412 - 17	4312 - 30
1432 - 28	2431 - 17	3421 - 30	4321 - 18

Reversing The Turnstyle

A natural task for this puzzle is to reverse the the numbers. After doing your best, compare the number of steps used to the sequence below, found by computer search, to convert the reversed sequence back to the original one. Note that this sequence is only 9 steps more than the minimum number of steps for a single **SWAP**.

Step 0: B A 0 9 8 7 6 5 4 3 2 1	**Step 14, flip:** 1 6 4 5 3 A 9 B 0 8 7 2	**Step 28, right:** 9 8 0 7 A B 2 1 4 6 3 5
Step 1, flip: A B 9 0 8 7 6 5 4 3 2 1	**Step 15, left:** 6 4 5 3 A 9 B 0 8 7 2 1	**Step 29, flip:** 8 9 7 0 A B 2 1 4 6 3 5
Step 2, right: 1 A B 9 0 8 7 6 5 4 3 2	**Step 16, flip:** 4 6 3 5 A 9 B 0 8 7 2 1	**Step 30, right:** 5 8 9 7 0 A B 2 1 4 6 3
Step 3, flip: A 1 9 B 0 8 7 6 5 4 3 2	**Step 17, left:** 6 3 5 A 9 B 0 8 7 2 1 4	**Step 31, flip:** 8 5 7 9 0 A B 2 1 4 6 3
Step 4, right: 2 A 1 9 B 0 8 7 6 5 4 3	**Step 18, left:** 3 5 A 9 B 0 8 7 2 1 4 6	**Step 32, right:** 3 8 5 7 9 0 A B 2 1 4 6
Step 5, right: 3 2 A 1 9 B 0 8 7 6 5 4	**Step 19, left:** 5 A 9 B 0 8 7 2 1 4 6 3	**Step 33, right:** 6 3 8 5 7 9 0 A B 2 1 4
Step 6, flip: 2 3 1 A 9 B 0 8 7 6 5 4	**Step 20, left:** A 9 B 0 8 7 2 1 4 6 3 5	**Step 34, flip:** 3 6 5 8 7 9 0 A B 2 1 4
Step 7, right: 4 2 3 1 A 9 B 0 8 7 6 5	**Step 21, flip:** 9 A 0 B 8 7 2 1 4 6 3 5	**Step 35, left:** 6 5 8 7 9 0 A B 2 1 4 3
Step 8, flip: 2 4 1 3 A 9 B 0 8 7 6 5	**Step 22, left:** A 0 B 8 7 2 1 4 6 3 5 9	**Step 36, flip:** 5 6 7 8 9 0 A B 2 1 4 3
Step 9, right: 5 2 4 1 3 A 9 B 0 8 7 6	**Step 23, flip:** 0 A 8 B 7 2 1 4 6 3 5 9	**Step 37, right:** 3 5 6 7 8 9 0 A B 2 1 4
Step 10, flip: 2 5 1 4 3 A 9 B 0 8 7 6	**Step 24, left:** A 8 B 7 2 1 4 6 3 5 9 0	**Step 38, right:** 4 3 5 6 7 8 9 0 A B 2 1
Step 11, right: 6 2 5 1 4 3 A 9 B 0 8 7	**Step 25, flip:** 8 A 7 B 2 1 4 6 3 5 9 0	**Step 39, right:** 1 4 3 5 6 7 8 9 0 A B 2
Step 12, flip: 2 6 1 5 4 3 A 9 B 0 8 7	**Step 26, right:** 0 8 A 7 B 2 1 4 6 3 5 9	**Step 40, right:** 2 1 4 3 5 6 7 8 9 0 A B
Step 13, left: 6 1 5 4 3 A 9 B 0 8 7 2	**Step 27, flip:** 8 0 7 A B 2 1 4 6 3 5 9	**Step 41, flip:** 1 2 3 4 5 6 7 8 9 0 A B

Turnstyle Puzzles of Smaller Sizes

One can imagine Turnstyle with a smaller number of pieces, and consider what sizes have the property that any permutation can be reached; i.e., for which a **SWAP** is possible.

Computer search shows that a **SWAP** is not possible for sizes less than 8, and also that it is not possible for puzzles of size 9 and 11.

Here are **SWAP** sequences for puzzles of sizes 8 and 10, where **R** denotes move right, **L** denotes move left, and **F** denotes flip, and the letters 0,A,B stand for 10,11,12.

Puzzle of size 8:

0: 1 2 3 4 5 6 7 8	**7-R**: 4 5 6 1 7 3 8 2	**14-L:** 3 2 5 1 4 6 7 8
1-R: 8 1 2 3 4 5 6 7	**8-F:** 5 4 1 6 7 3 8 2	**15-F:** 2 3 1 5 4 6 7 8
2-F: 1 8 3 2 4 5 6 7	**9-R:** 2 5 4 1 6 7 3 8	**16-L:** 3 1 5 4 6 7 8 2
3-R: 7 1 8 3 2 4 5 6	**10-F:** 5 2 1 4 6 7 3 8	**17-F:** 1 3 4 5 6 7 8 2
4-F: 1 7 3 8 2 4 5 6	**11-R:** 8 5 2 1 4 6 7 3	**18-R:** 2 1 3 4 5 6 7 8
5-R: 6 1 7 3 8 2 4 5	**12-R:** 3 8 5 2 1 4 6 7	
6-R: 5 6 1 7 3 8 2 4	**13-F:** 8 3 2 5 1 4 6 7	

Puzzle of size 10:

0: 1 2 3 4 5 6 7 8 9 0	**9-R:** 5 9 6 1 7 8 0 2 3 4	**18-R:** 1 7 4 8 9 0 2 3 5 6
1-R: 0 1 2 3 4 5 6 7 8 9	**10-R:** 4 5 9 6 1 7 8 0 2 3	**19-R:** 6 1 7 4 8 9 0 2 3 5
2-R: 9 0 1 2 3 4 5 6 7 8	**11-F:** 5 4 6 9 1 7 8 0 2 3	**20-F**: 1 6 4 7 8 9 0 2 3 5
3-R: 8 9 0 1 2 3 4 5 6 7	**12-L:** 4 6 9 1 7 8 0 2 3 5	**21-R:** 5 1 6 4 7 8 9 0 2 3
4-F: 9 8 1 0 2 3 4 5 6 7	**13-F:** 6 4 1 9 7 8 0 2 3 5	**22-F:** 1 5 4 6 7 8 9 0 2 3
5-R: 7 9 8 1 0 2 3 4 5 6	**14-L:** 4 1 9 7 8 0 2 3 5 6	**23-R:** 3 1 5 4 6 7 8 9 0 2
6-F: 9 7 1 8 0 2 3 4 5 6	**15-F:** 1 4 7 9 8 0 2 3 5 6	**24-F:** 1 3 4 5 6 7 8 9 0 2
7-R: 6 9 7 1 8 0 2 3 4 5	**16-L:** 4 7 9 8 0 2 3 5 6 1	**25-R:** 2 1 3 4 5 6 7 8 9 0
8-F: 9 6 1 7 8 0 2 3 4 5	**17-F:** 7 4 8 9 0 2 3 5 6 1	

Turnstyle Puzzles of Larger Sizes

Define **JUMP4** to advance a pair of pieces right past 4 pieces; it can be implemented in 10 moves (the discs are in bold face, **L** = shift left, **R** = shift right, **F** = flip):

1 2 3 4 5 6 7 8 9 10 11 12

L, F: 1 **3 2 5 4** 6 7 8 9 10 11 12

R, F: **3 1 5 2** 4 6 7 8 9 10 11 12

L, F: 3 **5 1 4 2** 6 7 8 9 10 11 12

L, F: 3 5 **4 1 6 2** 7 8 9 10 11 12

R, F: 3 **4 5 6 1** 2 7 8 9 10 11 12

Define **JUMP6** by using the first 8 moves of a **JUMP4** followed by 8 additional moves to advance a pair of pieces right past 6 pieces:

1 2 3 4 5 6 7 8 9 10 11 12
L, F: 1 **3 2 5 4** 6 7 8 9 10 11 12
R, F: **3 1 5 2** 4 6 7 8 9 10 11 12
L, F: 3 **5 1 4 2** 6 7 8 9 10 11 12
L, F: 3 5 **4 1 6 2** 7 8 9 10 11 12
L, F: 3 5 4 **6 1 7 2** 8 9 10 11 12
L, F: 3 5 4 6 **7 1 8 2** 9 10 11 12
R, F: 3 5 4 **7 6 8 1** 2 9 10 11 12
R, F: 3 **4 5 6 7** 8 1 2 9 10 11 12

Now consider a puzzle of size 14. A **SWAP** can be implemented by the sequence for 10 pieces presented earlier and using **JUMP4** sequences to keep the turntable in the "zone" of 10 positions. That is, call the positions initially under the turntable EF, the four positions to the left of the turntable ABCD and the positions to the right of the turntable GHIJ, and use the **SWAP** sequence for 10 pieces to **SWAP** the pieces in position EF, with the modification that whenever shift operations cause the turntable to be at positions AB and the next operation is a shift left, then first do a **JUMP4** on positions IJ, and whenever shift operations cause the turntable to be at positions IJ and the next operation is a shift right, then first do a reverse **JUMP4** on position AB.

Similarly, **SWAP** for 12 pieces combined with **JUMP4** gives **SWAP** for 16 pieces, and **SWAP** for 12 pieces combined with **JUMP6** gives **SWAP** for 18 pieces. Doing the jumps in pairs gives swaps for 20, 22, 24 pieces, in triples gives swaps for 26, 28 30 pieces, and so on, to conclude that **SWAP** is possible for any even number of pieces of 8 or more.

Of course a shortest solution for a given permutation that may be much more efficient than a sequence of **SWAPS**.

Given that **SWAP** is not possible for sizes 9 and 11 but it is for sizes 8, 10, and 12, an interesting question is whether **SWAP** is not possible for puzzles of odd size.