Problem Set 1, Part 2

cs112

Posted: Fri, September 15, 2006
Due Date: Fri, September 22, 2006 **IN CLASS**

Exercise 1 Sequent Calculus

Part 1.1 LK Rules

Discuss the following LK rules. What do they do and why are they allowed? Your discussion should also include a comparison to the appropriate ND rule.

1. $$\frac{\Gamma, A \vdash \Delta}{\Gamma \vdash \neg A, \Delta}\ (\neg R)$$

2. $$\frac{\Gamma \vdash A, \Delta}{\Gamma \vdash A \vee B, \Delta}\ (\vee R_1)$$

3. $$\frac{\Gamma \vdash A, \Delta \qquad \Sigma, B \vdash \Pi}{\Gamma, \Sigma, A \rightarrow B \vdash \Delta, \Pi}\ (\rightarrow L)$$

Part 1.2 LK Proofs

Fill in the rules that were used to create the following proofs. That is, provide the rule that was used at each derivation line to get to the next line. Then, give an explanation of what is happening in the following proofs as if you were explaining them to someone just learning sequent calculus. Your explanation should focus on why the author of the proof chose to use each rule when he did. You do not need to explain why the rule works in your explanation. For example, to explain an identity step such as $A \vdash A$, you could say, "The proof requires the proposition A to be present and this rule can introduce it to the proof." It would be incorrect, for this exercise, for your answer to discuss why you believe the rule is legal in LK.

1. $$\frac{\dfrac{\dfrac{\dfrac{A \vdash A}{A \vdash A \vee B}}{\neg(A \vee B), A \vdash}}{\neg(A \vee B) \vdash \neg A} \qquad \dfrac{\dfrac{\dfrac{B \vdash B}{B \vdash A \vee B}}{\neg(A \vee B), B \vdash}}{\neg(A \vee B) \vdash \neg B}}{\neg(A \vee B) \vdash \neg A \wedge \neg B}$$

2. $$\dfrac{\dfrac{\dfrac{A \vdash A \qquad \dfrac{\dfrac{A \vdash A \qquad B \vdash B}{A \to B, A \vdash B}}{\neg B, A \to B, A \vdash}}{A \to \neg B, A \to B, A \vdash}}{A \to \neg B, A \to B \vdash \neg A}}{A \to B \vdash (A \to \neg B) \to \neg A}$$

Exercise 2 First Order Logic

Part 2.1 Translations

Translate the following sentences into predicate logic. Give the key for the variables and predicate letters that you choose (don't forget to specify a universe of discourse). If you think that more than one translation is suitable, give the alternatives and discuss their differences. Represent as much as possible of the structure relevant to quantificational arguments.

1. **Lions are ferocious.**
2. **A lion is chasing Danny DeVito.**
3. **Ferocious lions are best avoided.**
4. **Lions and tigers are chasing Danny DeVito.**
5. **Ferocious lions and tigers are best avoided.**
6. **Danny DeVito and ferocious lions and tigers are best avoided.**
7. **If Danny DeVito is ferocious, all lions are tigers.**
8. **All professors who fail all their students deserve to be fired.**
9. **An unpopular professor either fails or gives a low grade to each of his or her students.**
10. **Professors bore all and only those of their students they are bored by.**

Part 2.2 Natural Deduction

Prove the following using FOL ND. Your proofs may use the PL rules from the last homework, the equivalences provided in the handout on the website, the new FOL rules also provided on the web, and the appendix of this homework. Make sure each step is clearly labeled with the appropriate rule.

1. $\exists x Lxx \vdash \exists x \exists y Lxy$
2. $\forall x((Px \lor Qx) \to Rx), \forall x((Rx \lor Sx) \to Tx \vdash \forall x(Px \to Tx)$
3. $\exists x \forall y Jxy, \exists y \exists z(Hzy \land \neg Py), \forall z \forall w((Jzw \land \neg Pw) \to Gz) \vdash \exists z Gz$

Appendix

Laws of Quantifier Negation

Law 1: $\neg(\forall x)\phi(x) \Leftrightarrow (\exists x)\neg\phi(x)$
Law 2: $(\forall x)\phi(x) \Leftrightarrow \neg(\exists x)\neg\phi(x)$
Law 3: $\neg(\forall x)\neg\phi(x) \Leftrightarrow (\exists x)\phi(x)$
Law 4: $(\forall x)\neg\phi(x) \Leftrightarrow \neg(\exists x)\phi(x)$

Laws of Quantifier In(Dependence)

Law 5: $(\forall x)(\forall y)\varphi(x, y) \Leftrightarrow (\forall y)(\forall x)\varphi(x, y)$
Law 6: $(\exists x)(\exists y)\varphi(x, y) \Leftrightarrow (\exists y)(\exists x)\varphi(x, y)$
Law 7: $(\exists x)(\forall y)\varphi(x, y) \Rightarrow (\forall y)(\exists x)\varphi(x, y)$

Laws of Quantifier Movement

Law 9: $(\varphi \to (\forall x)\psi(x)) \Leftrightarrow (\forall x)(\varphi \to \psi(x))$ **provided that x is not free in** φ
Law 10: $(\varphi \to (\exists x)\psi(x)) \Leftrightarrow (\exists x)(\varphi \to \psi(x)$ **provided that x is not free in** φ
Law 11: $(\forall x)\varphi(x) \to \psi \Leftrightarrow (\exists x)((\varphi)(x) \to \psi)$ **provided that x is not free in** ψ
Law 12: $(\exists x)\varphi(x) \to \psi \Leftrightarrow (\forall x)((\varphi)(x) \to \psi)$ **provided that x is not free in** ψ