Problem Set 1

cs112

Posted: Friday, September 5, 2008
Due Date: Tuesday, September 23, 2008 **IN CLASS**

Exercise 1 Propositional Logic

Part 1.1 Natural Deduction

Give a formal proof of validity for each of the following sequents. Check the website under Class Notes for a list of rules you may use and some suggested strategies. For these problems, you may only use the ND rules listed on the handout and no derived rules.

1. $p \vdash q \rightarrow (p \wedge q)$
2. $\neg(\neg p \vee q) \vdash p$
3. $\neg p, p \vee q \vdash q$
4. $p \wedge q, \neg(p \wedge r) \vdash \neg r$
5. $p \rightarrow q, \neg q \wedge r, p \vee s \vdash s$
6. $p \vee q \vdash r \rightarrow ((p \vee q) \wedge r)$
7. **EXTRA CREDIT** $(a \vee b) \wedge \neg c, \neg c \rightarrow (d \wedge \neg a), b \rightarrow (a \vee e) \vdash e \vee f$

Part 1.2 Truth Tables

Use truth tables to determine whether the following conclusions are semantically entailed from the set of premises.

1. $(c \rightarrow d) \rightarrow (d \rightarrow e), e \models c \rightarrow e$
2. $(j \vee m) \rightarrow \neg(j \wedge m), m \leftrightarrow (m \rightarrow j) \models m \rightarrow j$

Use truth tables to determine whether the following sets are truth-functionally consistent.

1. $\{a \rightarrow b, b \rightarrow c, a \rightarrow c\}$
2. $\{a, b, c\}$
3. $\{(a \wedge b) \vee (c \rightarrow b), \neg a, \neg b\}$

Exercise 2 First Order Logic

Part 2.1 Translations into FOL

Translate the following sentences into predicate logic. Give the key for the variables and predicate letters that you choose (don't forget to specify a universe of discourse). If you think that more than one translation is suitable, give the alternatives and discuss their differences. Represent as much as possible of the structure relevant to quantificational arguments.

1. **Every professor bores some of his or her students.**
2. **Some professors bore all professors.**
3. **All professors who fail all their students deserve to be fired.**
4. **An unpopular professor either fails or gives a low grade to each of his or her students.**
5. **Professors bore all and only those of their students they are bored by.**
6. **EXTRA CREDIT: Any student who doesn't listen to his or her professor doesn't understand that professor and bores that professor.**

Part 2.2 Translations from FOL

Use the following key to translate the following FOL sentences into fluent English. That is, the translations should be natural sounding. For example, use "every x" instead of "for all x".

UD: Positive Integers
Dxy: the sum of x and y is odd
Exy: the sum of x and y is even
Lxy: x is larger than y
Oxy: x times y is odd
Sxy: x plus y is even
Ex: x is even
Ox: x is odd
Px: x is prime
Pxy: x times y is prime

1. $\forall x(Ex \rightarrow \forall y Exy)$
2. $\neg\exists y(Py \wedge \forall x(Px \rightarrow Lyx))$
3. $\neg\exists x\exists y((Px \wedge Py) \wedge Pxy)$
4. $\forall x\forall y(Sxy \rightarrow ((Ex \wedge Ey) \vee (Ox \wedge Oy)))$
5. $\neg\forall x\exists y Lxy \wedge \forall x\exists y Lyx$

Part 2.3 Natural Deduction

Prove the following using FOL ND. Your proofs may use the propositional logic rules, the equivalences provided in the handout on the website, the new FOL rules also provided on the web, and the appendix of this homework. Make sure each step is clearly labeled with the appropriate rule.

1. $\exists x Lxx \vdash \exists x \exists y Lxy$
2. $\forall x((Px \vee Qx) \rightarrow Rx), \forall x((Rx \vee Sx) \rightarrow Tx \vdash \forall x(Px \rightarrow Tx)$
3. $\exists x \forall y Jxy, \exists y \exists z(Hzy \wedge \neg Py), \forall z \forall w((Jzw \wedge \neg Pw) \rightarrow Gz) \vdash \exists z Gz$
4. $\vdash (\forall x \forall y Axy \wedge \forall x(Axx \rightarrow Bi)) \rightarrow Bi$
5. $\vdash \forall x \exists y(Ax \rightarrow By) \rightarrow (\forall x Ax \rightarrow \exists y By)$
6. **EXTRA CREDIT:** $\forall y(My \rightarrow Ay), \exists x \exists y((Bx \wedge Mx) \wedge (Ry \wedge Syx)), \exists x Ax \rightarrow \forall y \forall z(Syz \rightarrow Ay) \vdash \exists x(Rx \wedge Ax)$

Exercise 3 Short Answer Questions

1. **Explain the differences between syntax and semantics in terms of natural deduction and truth tables. When and why would you use each method? What are the benefits and disadvantages of these methods?**
2. **Briefly describe the concepts of soundness and completeness in your own words. Without going into too much detail, give a sketch of the proof of each property for propositional logic. How would these proofs be modified for first order logic? (EXTRA CREDIT: Looking ahead, will the higher order logics we will discuss later this semester also have either or both of these properties? Why or why not?)**

Appendix

Laws of Quantifier Negation

Law 1: $\neg(\forall x)\phi(x) \Leftrightarrow (\exists x)\neg\phi(x)$
Law 2: $(\forall x)\phi(x) \Leftrightarrow \neg(\exists x)\neg\phi(x)$
Law 3: $\neg(\forall x)\neg\phi(x) \Leftrightarrow (\exists x)\phi(x)$
Law 4: $(\forall x)\neg\phi(x) \Leftrightarrow \neg(\exists x)\phi(x)$

Laws of Quantifier In(Dependence)

Law 5: $(\forall x)(\forall y)\varphi(x,y) \Leftrightarrow (\forall y)(\forall x)\varphi(x,y)$
Law 6: $(\exists x)(\exists y)\varphi(x,y) \Leftrightarrow (\exists y)(\exists x)\varphi(x,y)$
Law 7: $(\exists x)(\forall y)\varphi(x,y) \Rightarrow (\forall y)(\exists x)\varphi(x,y)$

Laws of Quantifier Movement

Law 9: $(\varphi \to (\forall x)\psi(x)) \Leftrightarrow (\forall x)(\varphi \to \psi(x))$ **provided that** $\boldsymbol{x}$ **is not free in** φ
Law 10: $(\varphi \to (\exists x)\psi(x)) \Leftrightarrow (\exists x)(\varphi \to \psi(x)$ **provided that** $\boldsymbol{x}$ **is not free in** φ
Law 11: $(\forall x)\varphi(x) \to \psi \Leftrightarrow (\exists x)((\varphi)(x) \to \psi)$ **provided that** $\boldsymbol{x}$ **is not free in** ψ
Law 12: $(\exists x)\varphi(x) \to \psi \Leftrightarrow (\forall x)((\varphi)(x) \to \psi)$ **provided that** $\boldsymbol{x}$ **is not free in** ψ