CS112 – Friday, September 12, 2008

- Agenda
  - Natural Deduction Questions
  - FOL Translations
  - FOL Interpretations
- Natural Deduction Questions?
- First Order Logic Translations
  - Symbolization Key
    - Universe of Discourse (UD) – tells us what we can talk about
    - Predicates
    - Individual Constants
  - Translations Rules and Pointers
    - Scope
      - In order for a sentence in FOL to be well-formed (called a 'wff'), there cannot be any unbound variables.
      - All variables must fall within the scope of a quantifier.
        - The scope includes the quantifier itself (the 'x' in '∀x' is bound) and the formula that immediately follows the quantifier.
        - Use parentheses to extend the scope of a quantifier
  - Complex Translations
    - When faced with a tough translation, try breaking it down into smaller parts.
      - What is the main operator in the sentence?
    - Keep rewriting the translation when you add a quantifier or figure out a main operator until it is obvious how to fill in the pieces
    - Example
      - "Pit Bulls are dangerous, but Labradors are not."
        - Conjunction
        - <u>Both</u> pit bulls are dangerous <u>and</u> Labradors are <u>not</u> dangerous.

- <u>Both</u> <u>no matter what x I choose</u>, <u>if</u> x is a pit bull <u>then</u> x is dangerous <u>and</u> <u>no matter what y I choose</u>, <u>if</u> y is a Labrador <u>then</u> <u>it is not the case that</u> y is dangerous.
  - Note that it would have been perfectly legal to use x as the only variable in this sentence because the second time it is used (for Labradors) it is not within the same scope as the first quantifier. But it's a good practice to use different variables even if you don't have to
- $(\forall x)(Px \supset Dx)$ & $(\forall y)(Ly \supset \sim Dy)$

- When to stack quantifiers
  - To be safe, try to give your quantifiers the smallest scope possible.
  - If you must stack your quantifiers at the beginning of the sentence, use the equivalences to do so.

- English → Symbols
  - UD: people
  - Lxy: x loves y
  - Hxy: x hits y
  - Ixy: x insults y
  - Lx: x is a lawyer
  - Gx: x wears green
  - Jx: x would be a good judge
  - Cx: x is crazy
  - Jx: x is a judge
  - Fx: x is fat
  - (1) No crazy lawyer insults any fat judge.
    - $\sim\exists x(Cx \,\&\, Lx \,\&\, \exists y(Fy \,\&\, Jy \,\&\, Ixy))$
  - (2) No crazy lawyer insults every fat judge.
    - $\sim\exists x(Cx \,\&\, Lx \,\&\, \forall y((Fy \,\&\, Jy) \rightarrow Ixy))$
  - (3) No crazy lawyer loves any judge whom she insults.

        - It is not the case that there is some x such that x is a crazy lawyer and x loves some judge whom x insults.
        - ~∃x(Cx & Lx & there is some y such that y is a judge and x insults y and x loves y)
        - ~∃x(Cx & Lx & ∃y(Jy & Ixy & Lxy))
    - (4) Some crazy lawyer insults every fat judge whom she loves.
        - There is some x such that x is a crazy lawyer and x insults every fat judge that x loves.
        - ∃x(Cx & Lx & for every y, if y is a fat judge and x loves y, then x insults y)
        - ∃x(Cx & Lx & ∀y((Jy & Fy) & Lxy) → Ixy))
    - No lawyer hits any judge whom she loves unless she's crazy.
        - Ambiguous! Who does "she" refer to?
        - Case 1: "She" refers back to the lawyer
            - ~∃x(x is a lawyer and x is not crazy and x hits some judge whom x loves) == Only a crazy lawyer would hit a judge that the lawyer loves.
            - ~∃x(Lx & ~Cx & ∃y(Jy & Lxy & Hxy))
        - Case 2: "She" refers to the judge
            - ~∃x(x is a lawyer and x hits some judge whom x loves who is not crazy.)
            - ~∃x(Lx & ∃y(Jy & Lxy & ~Cy & Hxy))
- Symbols → English
    - (1) ∃x(Cx & Lx & ∀y((Fy & Jy) → Ixy))
        - Some crazy lawyer insults every fat judge.
    - (2) ∃x(Cx & Lx & ∃y(Fy & Jy & Ixy))
        - Some crazy lawyer insults some fat judge
    - (3) ∀x[(Lx & ~Cx) → ∃y(Ly & ~Jy & Ixy)]

- Every lawyer who is not crazy insults some lawyer who would not be a good judge.

- Stacking quantifiers
  - UD: positive integers
  - Lxy: x is larger than y
  - (1) For every integer, there is a greater integer.
    - ∀x∃yLyx
  - (2) There is an integer that is greater than every integer.
    - ∃x∀yLxy
  - (3) If the product of two integers is even, then at least one of them is even.
    - Pxyz: x is the product of y and z, Ex: x is even
    - ∀x∀y∀z((Pxyz ^ Ex) → (Ey v Ez))
  - (4) No product of prime numbers is prime.
    - ~(∃x)(∃y)((Px&Py)&Pxy)

- First Order Logic Interpretations
  - Review
    - In propositional logic, every line of a truth table is an interpretation.
  - FOL
    - In FOL, we need to provide an interpretation function for each predicate and individual constant.
    - Example Interpretation
      - UD: positive integers
      - Ox: {x | x is odd} == {1, 3, 5, …}
      - Ex: {x | x is even} == {2, 4, 6, …}
      - Lxy: {x y | x is larger than y} == {{2,1}, {3,2}, {3, 1}, {4,3}, …
      - a: 1, b: 2 c: 3
  - Diagrams

- Since truth tables aren't helpful for FOL semantics, using diagrams to model sentences can tell us if they are true or false for a given interpretation or model.
- (1) Any 1-place predicate is represented with a circle.
- (2) Any individual constant is represented with a dot.
- (3) 2-place predicates are represented with an arrow.

➢ When do we use these?
  - Show that a set of sentences is consistent.
    - If you can draw a diagram for a set of sentences, then we know there is at least one interpretation where all the sentences are true. == Consistency
  - Show that an argument is valid
    - If you make all of the premises of an argument true in a diagram, but you are able to make the conclusion false, then the argument is not valid.

➢ Examples – Show that the following are consistent.
  - {∀x(Fx → ∃yGyx), ∀x(Gxx → Fx), ∀x∀y(Gxy → Gxx), ∃x∃y∃z(Fx & Gyz)}
    - To show that this is consistent, provide an interpretation where all the sentences are true.
    - (1) Start with 1 dot to represent something in the domain.
    - (2) Fill in the diagram as needed to make each sentence true.
      - ♦ Existentials tell us something about the world, so they add to the diagram
      - ♦ Universals are more like tests.
    - (3) As you add to the diagram, look for triggered sentences.
    - (4) When you think you're done, check all the sentences.
    - (5) Write up the interpretation.

- {∀x∃yFxy, ∀x(Gx → ∃yFyx), ∃xGx, ∀x~Fxx}

- {∀x(Px v Qx) → ∃xRx, ∀x(Rx → Qx), ∃x(Px & ~Qx)}

➢ Examples – Show that the following arguments are valid.

- {∀xFxx v ∀x∃yGxy, ∀x∀y(Fxy → Gyx)} |= ∃xGxx
  - What we want to find is an interpretation where the premises are true and the conclusion is false (~∃xGxx)

- {∀x∀y(Fxy → ∃yFyx), ∀x∃yFxy, ∀x∃y(Fyx → Fxx)} |= ∀xFxx