CS112 Notes
Model Checking with Trees

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Model-Checking

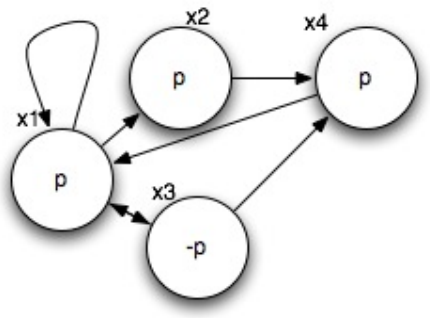
As mentioned in class last week, a valuation of an expression α at a world w_i given a model, $\mathcal{M}$, can also be viewed as a tree drawing program. Then the valuation of α can be read off the tree quite trivially.

Here is the procedure.

Given a valuation call, V_{M,w_i}, and an expression α:

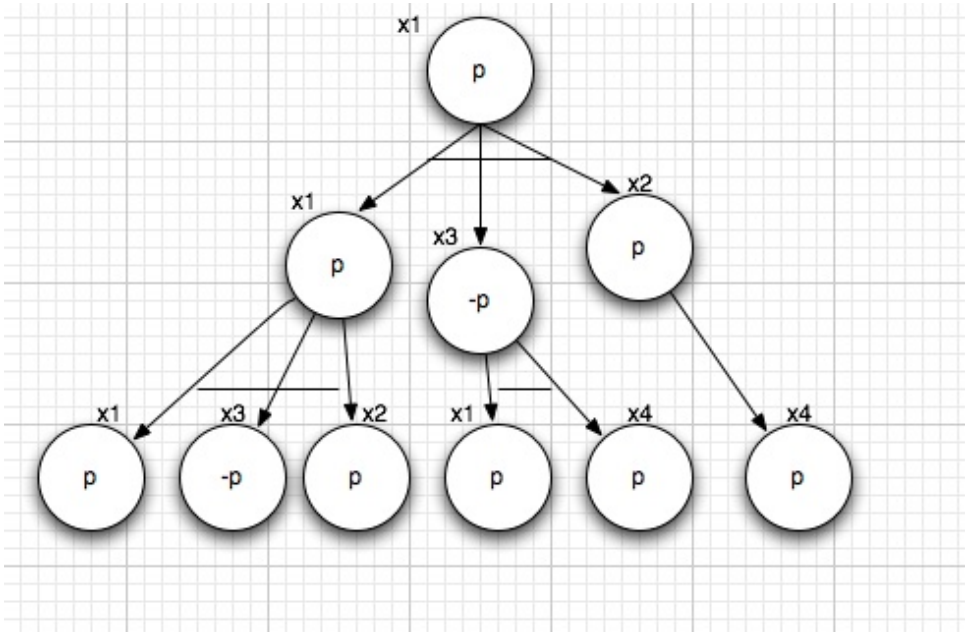
1. if α contains no modal operators, simply evaluate relative to the world w_i.
2. if α contains a leftmost $\Box$, i.e., $\alpha = \Box \ldots \phi$,
   (a) then start a tree with root w_i. $\Box$-rule: The AND-daughters to w_i are those worlds accessible from w_i; i.e., $\forall w'[w_i R w']$. Write the value of ϕ at each w' under each world node just created.
3. if α contains a leftmost $\diamond$, i.e., $\alpha = \diamond \ldots \phi$,
   (a) then start a tree with root w_i. $\diamond$-rule: The OR-daughters to w_i are those worlds accessible from w_i; i.e., $\forall w'[w_i R w']$. Write the value of ϕ at each w' under each world node just created.
4. Working inward, use the $\Box$-rule or $\diamond$-rule over the world visited at the current node, and grow the tree downward.
5. Continue till operators are exhausted.
6. Evaluate the expression according to the AND and OR conditions up the tree.

Consider the model, $\mathcal{M}$, below:



Model $\mathcal{M}$

Now consider the valuation $V_{M,w_1}(\Box\Box p)$. From walking the tree, it is easy to see that this is false.



Now consider the valuations for the following stacked modal expressions:

1. $V_{M,w_2}(\Diamond \Diamond p)$
2. $V_{M,w_3}(\Box\Box p)$
3. $V_{M,w_4}(\Diamond\Box \Diamond p)$