LING 130:
Sample Quiz 1 with Answers

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1. Natural Deduction: Propositional Logic

Use **pr** for premise and **as** for assumption.

Prove the following: $p \rightarrow q, r \rightarrow s \vdash p \wedge r \rightarrow q \wedge s$

(1)	1. $p \rightarrow q$	pr
	2. $r \rightarrow s$	pr
	3. | $p \wedge r$	as
	4. | p	$\wedge$e 3
	5. | q	$\rightarrow$e1,4
	6. | r	$\wedge$e 3
	7. | s	$\rightarrow$ e 2,6
	8. | $q \wedge s$	$\wedge$i 5,7
	9. $p \wedge r \rightarrow q \wedge s$	$\rightarrow$i 3-8

2. Natural Deduction: First-order Logic

Use **pr** for premise and **as** for assumption.

Prove the following: $\forall x[P(x) \wedge Q(x)] \vdash \forall x[P(x)] \wedge \forall x[Q(x)]$

(2)	1. $\forall x[P(x) \wedge Q(x)]$	pr
	2. | a	as
	3. | $P(a) \wedge Q(a)$	$\forall$e 1
	4. | $P(a)$	$\wedge$e 3
	5. $\forall x[P(x)]$	$\forall$i 2-4
	6. | a	as
	7. | $P(a) \wedge Q(a)$	$\forall$e 1
	8. | $Q(a)$	$\wedge$e 7
	9. $\forall x[Q(x)]$	$\forall$i 6-8
	10. $\forall x[P(x) \wedge Q(x)]$	$\wedge$i 5,9

3. First-order Logic Translation

Translate into a first-order expression.
No student attended every lecture.

$\neg\exists x[Student(x) \wedge \forall y[Lecture(y) \rightarrow Attend(x, y)]]$

4. Propositional Logic Truth-table

Fill in the truth table for the following expression: $(p \vee q) \rightarrow \neg r$

p	q	r	$p \vee q$	$\neg r$	$\rightarrow$
1	1	1	1	0	0
1	1	0	1	1	1
1	0	1	1	0	0
1	0	0	1	1	1
0	1	1	1	0	0
0	1	0	1	1	1
0	0	1	0	0	1
0	0	0	0	1	1

5. Types and Type Derivations

Give the type and semantic expression for each of the following words:
a. *friend*: $e \rightarrow t$, $\lambda x[friend(x)]$
b. *admire*: $e \rightarrow (e \rightarrow t)$, $\lambda y \lambda x[admire(x, y)]$
c. *slowly*: $(e \rightarrow t) \rightarrow (e \rightarrow t)$, $\lambda P \lambda x[Slowly(P(x)]$
d. *any*: $(e \rightarrow t) \rightarrow ((e \rightarrow t) \rightarrow t)$, $\lambda P \lambda Q \forall x[P(x) \rightarrow Q(x)]$

6. Quantifiers and Entailment

Explain why the quantifier *a* allows for deductions involving $\wedge$-elimination.

While not giving the full answer, I will say that you should be able to answer this with a few sentences and an example or two. In other words, show how *a* is monotone increasing in both restrictor and scope domains.