1 Introduction: sentence sense

Recall:

- Reference of a statement – its truth-value.
- Sense of a statement – its truth-conditions
  = situations in which the statement is true

Possible worlds: a set $W$, representing various ways things could be.
(Like Richard Montague, we also need a set of times $J$, but we’ll set that aside for now)
Think about the senses of complex sentences, in terms of the senses of their parts:

- $\neg p$
- $p \land q$
- $p \lor q$
- $p \rightarrow q$

2 Modals and possible worlds

Here is one set of expressions we cannot, as yet, represent in our semantic frameworks: modal auxiliaries and adverbs.

Interlude: modal logic and modal operators.

(1) Modals as operators on propositions:

a. Necessarily, $2+2=4$. $\approx \Box(2 + 2 = 4)$

b. Possibly, it is raining. $\approx \Diamond(\text{it's raining})$

c. John should walk in the park. $\approx \Box(\text{walk}'(j))$

d. Mary can wash the dishes. $\approx \Diamond(\text{wash}'(m, d))$

But what is the semantics of these symbols, $\Box$ and $\Diamond$?

Note that we cannot use truth-tables to define them, because a truth-value of a proposition $\phi$ does not determine the truth-value of $\Box \phi$: 
(2) a. Dogs are dogs.
   b. Dogs are pets.

We can capture what these symbols mean if we use possible world semantics.

(3) Using possible worlds:
   a. It may be raining. \( \approx \exists w [\text{it is raining in } w]\)
   b. Mary must be home right now. \( \approx \forall w [\text{Mary is home right now in } w]\)

So, we will treat modal expressions as picking out some or all possibilities in the set \( W \) of possible worlds!

### 3 INTERLUDE: Set theory

### 4 Quantifying over subsets of worlds.

(3) Using possible worlds:
   a. It may be raining. \( \approx \exists w [\text{it is raining in } w]\)
   b. Mary must be home right now. \( \approx \forall w [\text{Mary is home right now in } w]\)

Modal expressions are quantifiers over possible worlds. There are contextual restrictions which tell us which exact subset of \( W \) we’re quantifying over (4). As I indicate in parentheses, the sentence in (4a) involves quantification over worlds in which physical laws are obeyed; while (4c) quantifies over worlds in which obligations are fulfilled. This is similar to contextual restrictions for other quantifiers.

(4) a. We cannot travel back in time. (physical laws)
   b. It may be raining outside (epistemic state of conversation participants)
   c. This coffee must have sugar in it (should worlds)
   d. John should wash the dishes (epistemic state of conversation participants)
   e. You can’t do that!
      - determine whether Russell’s set \( R \in R \) - (logical laws obeyed)
      - build a perpetuum mobile - (second law of thermodynamics obeyed)
      - learn a new language in a week - (actual biological laws obeyed)
      - treat an older person with disrespect - (certain moral/social laws are obeyed)
      - use ‘they’ with singular antecedent - (prescriptive grammar obeyed)
      - do something I don’t want you to do - (my desires obeyed)
      ...

The way that these subsets of \( W \) are selected is by "accessibility relations" - relations that, for each world \( w \), select a subset of \( W \) which is "accessible" from \( W \). When we compare the sentences to the actual world, the accessibility relation will yield the subset of \( W \) which is accessible from the actual world.

**NOTE:** By itself, use of possible worlds to capture modal verbs and adverbs doesn’t mean that we have abandoned the view that sentences denote truth-values. Each particular sentence is still either true or false, since at the end, it is always compared to the actual world.