Efficient Bids on Task Allocation for Multi-Robot Exploration*

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Abstract

We propose a real time single item auction based task allocation method for the multi-robot exploration problem and investigate new bid evaluation strategies in this domain. In this problem, a different version of the well known NP-hard MTSP (Multiple Traveling Salesman Problem), each target must be visited by at least one robot in its open tour. Various objectives may be defined for this problem (e.g. minimization of total path length, time). In this article, we present an extensive analysis of our bid evaluation strategies for minimization of total path length objective. An integer programming (IP) approach may be used to allocate tasks to robots. However, IP approach may become impractical when the size of the mission is not small, the environment is dynamic or unknown, or the structure of the mission changes by online tasks. In real world domains, initial allocations assigned by computationally expensive methods are usually subject to change during run time. Our framework, capable of handling diverse contingencies, performs an incremental allocation method based on the up-to-date situations of the environment. Experimental results in simulations compared to both the results of the Prim Allocation method and the optima reveal efficiency of the bid evaluation heuristics combined with our framework.

Introduction

Search and rescue operations, space exploration, and reconnaissance/surveillance applications require effective multi-robot exploration. Although these tasks are similar, the overall objective for cost optimization may be different in these domains. Search and rescue operations may require time minimization, while space operations require minimization of total path length traversed by all robots, which is proportional to the total energy consumed by robots.

We propose DEMIR-CF (**D**istributed and **E**fficient **M**ult**I** **R**obot-**C**ooperation **F**ramework), a framework for heterogeneous teams of robots coordinating to complete complex missions that require diverse capabilities and collective work without a central planner/decision maker. Robots always use valid plans satisfying constraints on interdependencies among tasks. Different objective functions can be optimized by this framework when effective cost functions and constraints are supplied. The framework also includes Plan B precaution routines for handling real time contingencies and providing effective recoveries. Details of the proposed framework and related mechanisms can be found in (Sariel, Balch, & Stack 2006).

In this article, we investigate the performance of several heuristic functions integrated with our framework for multi-robot exploration tasks as a case study. Because this problem area is well studied in operations research, optimal solutions are available, so we can analyze the deviations of results generated by the framework from optima. Note, however, that optimal solutions sometimes require a very long time to compute, while our solutions can be found quickly. Although the problem domain, here in this article, consists of same types of tasks that can be executed by single robots with same type of capabilities, still it is NP-Hard due to the combinatorial structure of the problem.

The single robot multi target exploration problem (a version of the Traveling Salesman Problem) is NP-hard (Lawler *et al.* 1985). In the multi-robot case, besides the affects of route generation, allocation of targets to robots is also quite affective on the solution quality. Optimal results can be obtained using Integer Programming (IP) formulations. However these approaches may become impractical when the size of the mission is even moderate or the cost values change frequently because of the uncertain knowledge, changes in the environment (including failures) or the changing structure of the mission (e.g. online tasks). Furthermore, robots have path planning burdens for large target sets in dynamic environments. Expensive computational efforts for initial

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allocations may become redundant. Our framework eliminates these redundant efforts by means of incremental assignments based on up-to-date situations of the environment. It can also handle contingencies by precaution routines in its integrated structure. Communication failures may sometimes prevent allocations from being optimal. Our framework can also detect these situations and maintains high solution quality by dynamic task selection and task exchange scheme. Contributions of this paper are three fold: First, some important facts of multi-robot exploration problem are clearly presented for future developments. Second, effective polynomial time target allocation and route construction heuristics with near optimal solutions suitable for the situations when optimal solution cost is expensive are proposed. Third, we believe these heuristics may give inspiration for operations research methods on acquiring optimal values. As a final remark, our experiments and given sample situations reveal, we argue that target allocation and route construction should be integrated for better results in this domain.

Problem Statement

The single robot exploration problem, a variation of the Traveling Salesman Problem (TSP), is to find the minimum cost traversal of a given number of targets without considering the return cost from last target to the initial location for a single robot. The problem can be stated as finding the minimal Hamiltonian path on a given fully connected graph with all nodes to be visited. The travel costs are assumed to be symmetric. Although the TSP is NP-hard, there are many efficient k-OPT heuristic methods in the literature (Lawler *et al.* 1985). We are inspired by some of these route generation heuristics in the design of the multi-robot multi target route construction heuristics. Most of these heuristic methods assume triangle inequality principle among targets as in case of ours. The triangle inequality states that, for any triangle, the measure of a given side must be less than the sum of the other two sides but greater than the difference between the two sides.

Multi-TSP (MTSP) problem or multi-robot multi target exploration problem is a more general version of the TSP in which targets should be visited by at least one (ideally at most one) robot. This problem may be stated for different objectives such as minimizing the overall path length traversed by robots or minimizing total time of traversing all targets (similar to the makespan minimization objective). In MTSP, besides the quality of the constructed paths for robots, allocation of targets is quite affective on the overall solution quality. The Vehicle Routing Problem (VRP) from transportation and logistics research is similar to multi-robot exploration problem (Toth & Vigo 2001). Especially dynamic and stochastic multi-depot VRP problem has a similar structure with the problem presented here. In multi-depot VRP, vehicles may be in different depots, as in our case robots may be in different locations initially or during run-time.

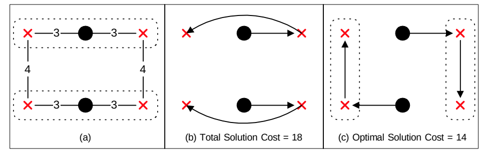

Figure 1: The optimal solution is obtained by clustering the targets

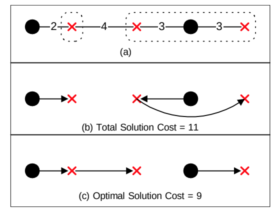

Figure 2: Clustering the targets does not necessarily result in the optimal solution

Remarks on MTSP Characteristics Separation by considering distances between robot-target pairs and assignment of the corresponding targets to robots method ignores the additional cost of returns. A sample situation is given in Figure 1. In (a) robot locations as circles, the target locations as crosses and some distance values are given. Allocation and final paths generated by separating targets based on the distances to robots results in the total cost value 18 (Figure 1 (b)). The optimal allocation cost value is 14 (Figure 1 (c)). The optimal allocation can be obtained by clustering the targets.

Clustering methods can be used to form target clusters. However clustering techniques use distance information. Therefore in some situations, clustering methods also do not produce optimal results because of ignoring additional costs as in Figure 2.

(Lagoudakis *et al.* 2005) presents an extensive analysis for multi-robot exploration problem from solution guarantees point of view. In their work, they analyze allocation approaches for both sequential tree construction and route generation, and direct allocation while constructing paths. From our point of view, generating a Minimum Spanning Forest (MSF) and constructing routes on separate MSTs is not an effective method. One reason is that there may be different MSF solutions for MTSP case in which some of the distances are the same. It may result in different allocation alternatives, and if there is not a reasonable allocation strategy other than selecting the minimum distance, the solution quality may decrease accordingly. The other reason is that the distance consideration is not an effective approach for allocating targets because of ignoring addi-

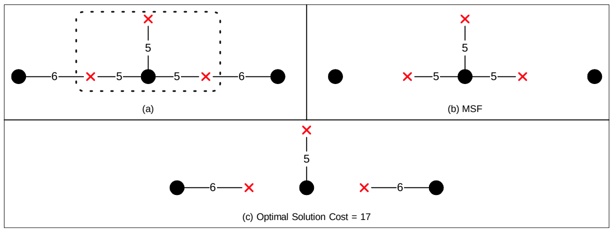

Figure 3: The optimal allocations may be completely different than the MSF allocations

tional costs added in the route construction phase. A sample situation is given in Figure 3.

Tree construction and allocations from scratch may result in sub-optimal allocations although constructions of the tree like structures are computationally efficient. An IP approach may be used for finding optimal allocations. However, for even moderate size instances, there is no guarantee on the solution time. Changes in the distance values between target and robot pairs may be frequent in dynamic environments. In this case, the solution should be reexamined again. Even very small changes may completely change the overall solution.

MTSP Solution Methods

Prim Allocation Method Prim Allocation method (Lagoudakis *et al.* 2004), based on the Prim Algorithm, generates an MSF of the targets and robots. MSF consists of separate robot trees. These trees are constructed by adding each unallocated target to the closest robot path containing the node with the minimum distance to the target, until all targets are allocated. In other words, addition of a new target is implemented by considering the distances between the target and nodes of the robot tree instead of considering the last position of the robot. Each robot offers an auction for a target and one of the targets are allocated at each round. Before robots run and visit the targets, all targets are allocated. Whenever the world knowledge is changed, allocations for the remaining unvisited targets are redone with the same algorithm. As Prim Algorithm, Prim Allocation method is bounded by 2*OPT for MTSP. Since this method is given with performance bounds and necessary details to implement, and it suggests a single item allocation method, performance of our approach is compared to this approach in simulations.

Depth-first traversal solution of a MST is bounded by 2*OPT, and the traversal and sub-tree selection does not affect the solution quality in closed version of the TSP. However for the open loop version of the TSP, selection of the sub-tree that is traversed affects the solution quality to a great extent. To improve the solutions, traversal may begin with the shortest depth sub-tree and continue with traversal on the next sub-tree. A sample situation is given in Figure 4.

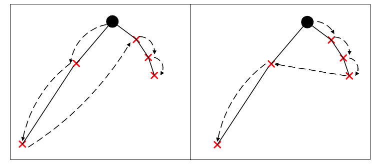

Figure 4: Effects of MST traversal strategy on the total cost for open traversal

Integer Programming Formulation Optimal results can be obtained by an efficient Integer Programming (IP) formulation. The optimal results in this paper are generated by commercial IP solver CPLEX using IP formulation given in (Lagoudakis *et al.* 2005).

Proposed Approach

Auction Based Allocation of Targets

We propose a single item auction based task allocation method. Auction based method ensures a distributed, robust and scalable allocation mechanism eliminating the complexity of the decision on allocating all targets to all robots. Each robot selects candidate targets that are suitable for it, and after a rough route plan it selects the most suitable candidate task to offer in an auction. In an auction, bidder robots send their cost values as bids for the auctioned target if the requirements are met (In MTSP definition each robot is capable of visiting each target). Since there is a tight connection between route generation and allocations, in our heuristic approach, this consideration is implemented by generating rough routes by robots. Therefore each robot (r_j) selects its most suitable target among the targets in the route target set (T_{Rj}). T_{Rj} is constructed by selecting targets, close the robot r_j, among unvisited targets (T_U) according to Eq. 1, where *dist* function returns the Euclidian distance between two points. Targets in T_{Rj} are considered as the candidate targets for the robot r_j. Therefore before selecting the most suitable target, each robot constructs these rough route sets. This heuristic does not compel an actual commitment, and targets in rough routes are not necessarily assigned to the corresponding robots in future auctions. Instead, it provides a global view to obtain close to optimal results from a local perspective.

$$\begin{aligned} reldist(r_j, t_i) &= dist(r_j, t_i) - min(dist(r_k, t_i)) \\ & \quad \{\forall k \neq j, r_k \text{ is active}\} \qquad (1) \\ T_{Rj} &= \cup t_i, \quad reldist(r_j, t_i) < 0, \ \forall t_i \in T_U \end{aligned}$$

Cost Evaluation

In our earlier work, we propose some heuristic cost functions for bid evaluation, and present preliminary results

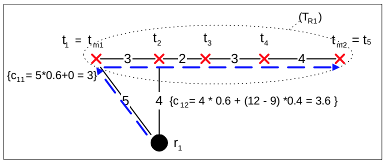

Figure 5: Target selection strategy by FAC Heuristic function is illustrated. Although t_2 is closer to r_1 than t_1, t_1 cost value is smaller by the FAC heuristic evaluation. The robot path is shown by the dashed arrows.

of these functions (Sariel & Balch 2005). We extensively analyze two heuristic cost functions (FAC and CC) combined within our framework in this work. These cost values are evaluated for the targets in T_R for each robot. CC (Closest Cost) heuristic cost value for robot r_j and target t_i is evaluated by the Eq. 2. This heuristic cost function only considers the distance between targets in T_{Rj} and the robot r_j.

$$c_{ji} = dist(r_j, t_i)\ t_i \in T_{Rj} \tag{2}$$

FAC (Farthest Addition Cost) heuristic function considers costs as if there is a route for T_{Rj} as in Eq. 3. t_{m1} and t_{m2} are the target pairs in T_{Rj} with the maximum distance value. FAC forwards robots to these targets in T_{Rj} to some degree ($\alpha = 0.6$). The main idea behind this approach is that the open loop traversal should contain both t_{m1} and t_{m2}s. If the robot heads towards one of these targets, if profitable (α), this maximum distance can be traversed by traversing other targets on the path. A sample illustration of this cost function is given in Figure 5.

$$\begin{aligned} c_{ji} &= \alpha * dist(r_j, t_i) + (1-\alpha) * [dist(t_{m1}, t_{m2}) \\ &\quad -max(dist(t_i, t_{m1}), dist(t_i, t_{m2})] \\ &\quad \{dist(t_{m1}, t_{m2}) = max(dist(t_k, t_l))\ t_{i,k,l} \in T_{Rj}\} \end{aligned} \tag{3}$$

The algorithm for target allocation in auctions is given in Algorithm 1. Each robot implements the same algorithm until the mission ends in a distributed fashion. The given algorithm may be used to allocate all targets from scratch by considering the latest locations of robots. However an incremental approach eliminates redundant allocations for dynamic environments.

Limitations for the MTSP

Original implementation of our framework allows multiple winners in different auctions at a time step. However for the given objective of total distance minimization, ending one auction at a time results in better performance when there is relation between targets. This is the reason why some auctions are cancelled when there are related multiple auctions at the same time.

Precaution Routines

For dealing uncertainties because of the message losses, each robot keeps track of the models of known tasks (targets) and other robots in their world knowledge. When robots get information from others they update their world knowledge accordingly. Whenever communication becomes unreliable, world knowledge of each robot may be inconsistent. The proposed framework ensures an update mechanism when conflicts are detected to reduce inconsistency. When robots receive inconsistent messages, they either warn others or correct themselves. These inconsistencies occur when robots are not informed about the tasks that are completed, under execution or under auction. It is assumed that robots are trusted and benevolent. Complete set of precaution routines designed for handling several contingencies can be found in (Sariel, Balch, & Stack 2006).

Algorithm 1 Target Allocation Algorithm for robot r_j

> **while** T_U is not empty **do**
>   **if** there is no auction/execution in progress **then**
>   form T_{Rj} (Eq. 1)
>   **if** $\|T_{Rj}\| > 0$ **then**
>   select $argmin_{t_i}(c_{ji})$, $t_i \in T_{Rj}(*)$
>   offer auction for t_i
>   **end if**
>   **end if**
> **end while**
> **if** an auction message for t_k from r_l is received **then**
>   **if** (auction/execution is in progress) & ($c_{ji} > c_{jk}$) $||$ (($c_{ji} > c_{lk}$) & (t_k or r_l is close to the T_{Rj})) **then**
>   cancel auction for t_i
>   **end if**
>   send bid value for t_k
> **end if**
> **if** auction negotiation deadline is reached **then**
>   end auction; award the best bidder/begin execution
> **end if**
> **if** an award message is received & $\|T_{Rj}\| = 0$ **then**
>   begin execution of the task
> **end if**
> **if** world knowledge is changed affecting T_{Rj} **then**
>   cancel auctions or executions
> **end if**
> $*$ c_{ji} is evaluated either by Eq.2 (CC) or Eq.3 (FAC)

Analysis of Approaches

Prim Allocation algorithm performance is proved to be bounded by 2*OPT (Lagoudakis *et al.* 2004). Difference of Prim allocation and CC heuristic approach is the robot location taken into consideration. CC heuristic approach forwards robots into one of the sub-trees of the MST and either leave the other sub-tree to be traversed by another robot or itself. If the first robot is traversing the sub-tree the solution cost is the same as the Prim Allocation solution cost. Otherwise the generated solution is better than depth-first traversal of the tree because the other sub-tree is left to another robot resulting in a cheaper cost value. CC heuristic can be listed as one of the BidSumPath heuristics (Lagoudakis *et al.* 2005) and shown that the solution is bounded by 2*OPT. FAC heuristic forwards robots to one of the border sub-trees. In the worst case scenario, the next selection phase forwards robot to the next sub-tree in the MST before completion of traversal of a sub-tree (usually this results in elimination long connections among sub-trees and better results). However by triangle inequality returning back to the previous sub-tree cannot be greater than two times traversal of the corresponding MST edges in this worst case.

Experimental Setup

In the first set of experiments, FAC heuristic function for a single robot exploration problem (TSP) is analyzed for the known TSP instances (TSPLIB). In the second set of experiments, heuristic functions combined with the proposed auction based approach and Prim Allocation method are evaluated on randomly generated test sets for different robot and target numbers. Basically, algorithms are run on distance matrices in these set of experiments. Environment size is taken as 100*100, numbers of robots change in the range [1-50] and the number of targets in [10-50]. The optimal results are generated by the LP solver CPLEX for the IP formulation given in (Lagoudakis *et al.* 2004). One observation from our experiments: the constraints (3^{rd}) cannot be fed into the IP formulation for the instances with 18 targets and higher. As explained in (Lagoudakis *et al.* 2004) a cutting plane method is used to solve the integer program. This method slows down the IP approach for obtaining the optimal result. Time comparison results are taken for the centralized implementation of the auction based methods. Distance calculations among targets and robots are excluded from run time period while IP model generation is included in the time period since it is part of the solution. All approaches are assumed to be running on the distance matrices. Results are presented as averages of 100 independent runs for randomly generated test instances. Running time results are averaged over 30 runs.

Experimental Results

First set of experimental results are given in Table 1. These results reveal near-optimal performance of FAC heuristic function with at most 15.24% deviation from the optimum (for a large TSP instance). Note that the results are for the open loop TSP. In Figure 6, open loop routes of the FAC heuristic function and the Optimum is given for the ATT48 TSP instance (targets represent geographic locations of capitals of US cities).

Running time comparison of the MTSP solution approaches is given in Figure 7. Large standard deviation values for the IP approach present the dependency of the solution time on the problem instance structure. IP approach performance is close to the worst case performance for some instances, not given in this statistics. Depending on the application and the instance size or the frequency of the changes in the distance values, IP approach may be impractical without guarantees on the solution time. Allocation by using a heuristic approach can be implemented in a very short time as expected and given in Figure 7. This graph presents running time results of the approaches for the single robot case. With decreasing amount of target densities, IP approach solution time decreases accordingly.

Overall performance results are given in Figure 8. PRIM-ORG values represent results of the Prim Allocation method without considering sub-tree sizes on the traversal while PRIM-SD values represent the results with shortest sub-tree selection improvement. FAC heuristic approach results are promising even for single robot instances. With increasing number of robots, the solution quality is also affected by the target allocation. Therefore the CC and FAC heuristic results

Table 1: FAC Heuristic Function Performance Result for known TSP Instances

TSP Instance	FAC	Optimum	Deviation from the Optimum
ATT48	33537.83	31470.4	6.5
EIL51	444.01	413.51	7.37
BERLIN52	8104.99	7305.38	10.94
EIL101	725.31	629.38	15.24

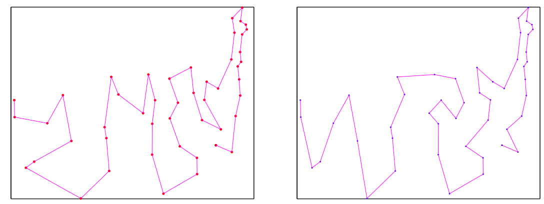

Figure 6: Open-loop routes of the FAC Heuristic function and the Optimum for the ATT48 TSP instance

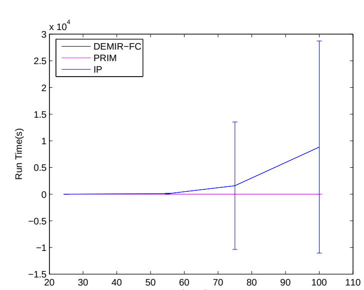

Figure 7: Runtime comparison of the approaches for single robot route generation

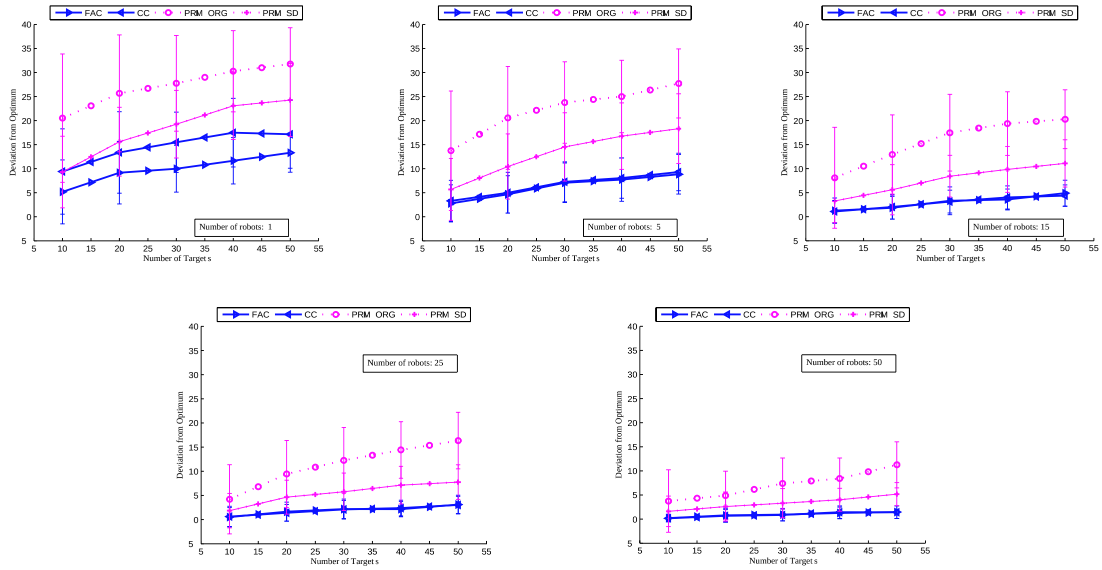

Figure 8: Performance results of the heuristic approaches given as deviations from the optima with standard deviation, averaged over 100 runs

become closer with a very small value of deviation from the optima. Decrease in target/robot proportion results in a decrease in the deviations of the results from the optima. However Prim Allocation results still deviate from the optima because of the allocation method. This is prevented in our approach by dynamic selection of T_Rs. Note that, our results can be further improved by using 2-OR exchange (Toth & Vigo 2001) type improvements if the communication is reliable.

Conclusion

We present results of our multi-robot coordination approach with two heuristic bid evaluation functions in the multi robot exploration domain. The results are promising in the sense of deviation from the optima. It should be noted that optimal results can be obtained by IP approach for a given configuration of the robot and target locations on small instances. However this approach may become impractical for increasing number of targets, or when the distance values change frequently because of the uncertain knowledge, dynamism of the environment or the changing structure of the mission. Therefore this method may be too expensive added to the path planning burdens for large target sets. Finally as results in this paper reveal, allocating all targets from scratch and generating routes of robots may result in suboptimal solutions. Therefore, target allocation and route construction should be integrated for near optimal results as in our approach. Integration of route construction and allocation in an incremental assignment approach is also useful for eliminating redundant efforts especially in highly dynamic or unknown environments. Porting our framework to the real robots will shape our future work to analyze the performance of our framework on real dynamic environments.

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