Qualitative Spatial Reasoning with Topological Relations in the Situation Calculus

Mehul Bhatt and **Wenny Rahayu**
Department of Computer Science
La Trobe University
Melbourne, Australia 3086.
{mbhatt, wenny}@cs.latrobe.edu.au

Gerald Sterling
Air-Operations Division, DSTO
PO Box 4331 Melbourne
Australia 3001
Gerald.Sterling@dsto.defence.gov.au

Abstract

We use a qualitative theory of spatial change and illustrate some of the key representational aspects of specifying such a theory using a formalism to reason about action & change; an effort that we regard to be essential toward a general integration of qualitative spatial reasoning with reasoning about the dynamic, causal aspects of spatial change. A topological theory of space, namely the region connection calculus, is used as the spatial metaphor in this work; the reason here primarily being that topological distinctions are inherently qualitative in nature and also because a relational approach as general as the RCC is representative of a similar class of relational techniques in the QSR domain. As such, our results can be easily generalised over a wide range of calculi, encompassing other aspects of space, that are based on similar semantics. The main aim of this paper is to illustrate first ideas on how a causal perspective to qualitative spatial reasoning may be provided using the situation calculus, which is a formalism to reason about dynamically changing domains. The minimalist notions of space and/or spatial dynamics in this paper are based on the hypothesis that it is imperative to approach the problem of the said integration at a elementary level before a higher-level abstraction involving complex actions & events is developed.

Motivation

Qualitative spatial reasoning (QSR) is a sub-field of AI concerned with developing tools and techniques for reasoning with non-metrical and incompletely specified spatial knowledge (Cohn & Hazarika 2001). QSR is important from an AI research viewpoint since it provides cognitively plausible models of human-like qualitative reasoning in a computationally feasible manner. Most research in qualitative spatial reasoning has focused on particular aspects of space such as topology, orientation, distance, size etc. However, relatively little work has explicitly addressed the need to account for the teleological or purpose-directed aspects of spatial change within a unified setup. Whereas qualitative spatial reasoning is concerned with the manner in which a set of spatial relationships[1] evolve during a certain time interval, reasoning about the teleological aspects of a system involves reasoning about action and encompasses the purpose or goal directed aspects of change. For e.g., Consider a simulated environment consisting of an agent that needs to travel from location L_1 to location L_2 via a sequence of spatial transformations. At a very simple/minimal level, there are two main closely related aspects to this problem: (a) **Spatial**: The specific sequence of spatial transformations needed in order to achieve a certain desired configuration as well as its *legality or consistency* with regard to a set of spatial dynamics. (b) **Causal**: The overall goal or the telic aspect of achieving a desired spatial configuration that dictates why does the agent want to move from L_1 to L_2, which is orthogonal to how precisely to reach location L_2. Our research is driven by the need to treat inferences about the spatial aspects in an integrated manner with inferences about the causal aspects of a system; an endeavour which we hypothesize can be achieved via relating the effects of actions with a set of spatial dynamics. One of the earliest accounts of such an attempt done within the context of the Qualitative Process Theory (QPT) can be found in (Forbus 1989). Forbus proposed *action-augmented envisionments*, which incorporate both the effects of an agent's actions and what will happen in the physical world whether or not the agent does something. Similarly, an event-based qualitative simulation system was proposed in (Gooday & Cohn 1996) by employing transition calculus, which is a high-level formalism for Reasoning about Action & Change (RAC). Although most of the important features of transition calculus involving concurrency and non-monotonic reasoning remained un-utilized, the general utility of their proposed approach cannot be taken for granted – There are many advantages of such an approach involving the use of representational tools like the transition calculus (in our case, the Situation Calculus (McCarthy & Hayes 1969)) developed in the field of reasoning about action & change – **(a)**. Rather fundamental problems (e.g., Frame, Ramification, Qualification (Shanahan 1997)) relevant to modelling systems that change have been thoroughly investigated in the context of the class of formalisms aforementioned **(b)**. More importantly for the QSR domain, these formalisms provide a rigorous account of continuous & concurrent phenomena, which manifest themselves even in the most simplest of dynamic domains. Note that such technical benefits notwithstanding, our approach is beneficial from

[1]Depending on the richness of the theory in question, these could be modelled as changing relationships relevant to various aspects of space such as topology, orientation, distance etc.

an overall QSR & RAC research viewpoint too since is promotes a more direct interaction between the two disciplines.

Approach

As general motivation, we discussed the need to integrate reasoning about the dynamic goal or purpose directed aspects of a system with a set of spatial dynamics. Such a goal can essentially be likened to a much broader objective involving the integration of various endeavours in AI. With this as our overall research context, our approach is to start at an elementary level with minimalist assumptions about the nature of space and/or the spatial dynamics and investigate the key representational issues pertaining to the intended integration. Space in this paper will be characterised by a set of topological relationships; this is because topological distinctions are not only inherently qualitative in nature, but they also represent one of the most general ways to characterise space. Consequently, spatial change in this paper will be denoted by time-varying topological relationships between objects (or their spatial extensions) in the domain. Precisely, we will use the Region Connection Calculus (RCC) (Randell, Cui, & Cohn 1992) as the spatial theory and provide a causal perspective to it using the Situation Calculus (McCarthy & Hayes 1969). Our justification behind such an approach is that it is necessary to view the problem at such a primitive level before a higher-level abstraction involving complex spatial actions or events is directly formalised using the situation (or other) calculus to perform the intended integration. The choice of the RCC-8 system is also influenced by the fact that it is representative of a general class of similar relational techniques[2] that are common-place in the QSR domain. As such, our results can be easily generalised to encompass a general class of relational systems that are based on similar semantics.

The rest of the paper is structured as follows: We begin with the Preliminaries in the next section whereupon we focus on aspects of situation calculus to be used in this paper. Note however that a general familiarity with the situation calculus is presumed. Considering the omnipresence of RCC, we refrain from discussing it at any length. For details about QSR or the RCC system, please refer to (Cohn & Hazarika 2001) and (Randell, Cui, & Cohn 1992) respectively. In the main part of the paper, we present details relevant to formalising key aspects of the RCC-8 system in the situation calculus, including an illustration of some problems encountered. Finally, we relate this paper with existing work and conclude with remarks on future outlook.

Preliminaries - Situation Calculus as a Representational Tool

The situation calculus as a representational tool for modelling dynamically changing worlds was first introduced in (McCarthy & Hayes 1969). This original formalisation viewed time as discrete and provided only an implicit account of it. Subsequently, various extensions, most notably by (Pinto 1994), (Ternovskaia 1994), (Pinto & Reiter 1995) and (Pirri & Reiter 2000), have been made so as to accommodate various features including an explicit account of continuous time and support for concurrent actions. In this paper, we refrain for committing to a precise formulation for every aspect of the calculus, especially concerning *actions*, which have originally been the only primitive means of specifying change in the calculus. Within the context of our long term research goals, we intend on making important ontological distinctions relevant to actions and events. For e.g., we intend on differentiating between various types of *Occurrences* – events in general as well as specific types of events that are actions[3]. However, in the content of this paper, we use the umbrella term '*Occurrence*' to represent changes of an arbitrary type and specifically consider only one type of occurrence, a *transition*, which merely represents a change of topological relationship between two objects.

The situation calculus formalism used in this paper is essentially a first-order language with the following 4 classes of axioms: Occurrence pre-condition axioms, successor state axioms specifying the causal laws of the domain, the initial state of the world and unique-names axioms for occurrences & fluents. Although all axioms can be intuitively followed, we make precise the most general ones to avoid any ambiguity:

1. Occurrence pre-conditions axioms are expressed using the binary predicate $Poss(e,\ s)$, denoting that the occurrence e is possible in situation s.
2. The binary functions $Result(e,\ s)$, which denotes a unique *situation* resulting from the happening of occurrence e in situation s.
3. A ternary occurrence type $trans(t,\ o_i,\ o_j)$ (read: transition) denoting the only means of change possible in within our model. The predicate expresses that o_i and o_j can transition to the state of being t.

We assume the usual unique names axioms concerning occurrences and fluents. In addition to the above, we will be using some more auxiliary predicates that will be defined in the course of the paper.

Qualitative Spatial Reasoning in the Situation Calculus

Notation

We have the following sorts in the language: Region (R), Object (O) and Situation (S) denoting *regions* of space, any arbitrary *object* and *situations* respectively. The following naming scheme will be used for these sorts: **(a)** Regions (R): $r_1,\ r_2,\ \ldots,\ r_n$, **(b)** Objects (O): $o_1,\ o_2,\ \ldots,\ o_n$ and **(c)**. Situations (S): $s_1,\ s_2,\ \ldots,\ s_n$. The constant S_0 denotes the initial situation when no occurrences have happened. By T_{rcc8}, we denote the set of RCC-8 topological relations, i.e.,

[2]For example, involving the use of a finite set of mutually exhaustive and pair-wise disjoint base relations capturing some spatial aspect, compositional inference & consistency maintenance, conceptual neighbourhood principle etc.

[3]See (McCarthy 2002) for an interesting discussion in this regard; McCarthy regards events to a much more general concept than actions.

$T_{rcc8} \equiv \{dc,\ ec,\ po,\ tpp,\ eq,\ tpp^{-1},\ ntpp,\ ntpp^{-1}\}$. In addition to the previous sorts, we also have a sort Topological Relation (T) denoted as follows: $T \equiv \{t_1,\ t_2,\ \ldots,\ t_n\}$. For representational convenience, we use the following reified and non-reified notations interchangeably: **(a)**. $(\exists t).\ Holds(t,\ r_1,\ r_2)$ and **(b)**. $(\exists t).\ t(r_1,\ r_2)$. Note however that the reified version in (a) is more advantageous both from a representational & computational viewpoint since it treats topological relations as concrete entities thereby allowing quantifications over variables of sort topological relation.

Properties that are situation dependent are termed as fluents. Currently, two functional fluents are used in the formalisation to follow: $top(r_1,\ r_2)$ and $pos(o_1)$. The former represents the topological relation between two regions of space and has a denotation of sort (T), whereas the latter represents the spatial extension of an object or the region of space occupied by it. For simplicity, we assume that the spatial extensions are regular regions of space that approximate the object in question, for e.g., using a convex hull primitive or a minimal bounding rectangle; the precise technique being irrelevant here. Also, it must be pointed out that our use of the positional fluent in this context is somewhat counter-intuitive[4]; since the theory in this paper only uses topological information, the position of an object should in actuality be denoted by its topological relationship with some other object(s). That said, in line with common-practice in the QSR domain, we continue to maintain this distinction in the rest of the paper since the issue can be easily tackled at the implementation level. Depending on context or what is more convenient, we may use it in the following ways: **(a)**. $[top(\ r_i,\ r_j,\ s)\ =\ ec]$ or **(b)**. $Holds(top(\ r_i,\ r_j),\ ec,\ s))$. Finally, a non-determinate situation is expressed in the following manner:

$$[top(\ r_i,\ r_j,\ s)\ =\ \{\ dc\ \vee\ po\ \}]\ \equiv$$
$$[Holds(top(\ r_i,\ r_j),\ dc,\ s)\ \vee\ Holds(top(\ r_i,\ r_j),\ po,\ s)]$$

We adopt the usual convention that all free variables be universally quantified from the outside. Further more, since we have sorts, the scope of all quantifications is limited to the respective sorts of the particular variable being quantified. Also, all variables with integral sub-scripts (e.g., $r_1,\ t_1$ etc) are regarded as constants.

Representational Issues - State Constraints

State constraints constitute an important representational device in our work – As will be evident later, various aspects of relational formalism such as JEPD'ness of the base relations, compositional inference etc can be modelled using state constraints. However, they also pose serious problems such as containing indirect effects in them. In the context of the situation calculus, (Lin & Reiter 1994; Lin 1995) comprehensively illustrates the need to distinguish ordinary state constraints from indirect effect yielding ones, the latter referred to as ramification constraints. This is because when ramification constraints are present, it is possible to infer new effect axioms (or simply effects) from explicitly formulated (direct) effect axioms together with the ramification constraints. Simply speaking, ramification constraints lead to what can be referred to as '*unexplained changes*' which clearly defeats the purpose of theory of change being used per se.

Impact of Indirect Effect Yielding Constraints - An Example We briefly illustrate the impact of indirect effect yielding (ramification) constraints using a simplified example. Assume that the world consist of $three$ objects and that the only (partial) information we have about them is topological in nature. Furthermore, we also add that the only change possible is that of a topological transition, represented using the ternary $trans(t,\ o,\ o')$ predicate defined previously. Additionally, we also utilize the so-called successor state axiom, which to avoid looking ahead into the paper, one may intuitively think of as explicitly specifying all possible ways in which the fluent $top(...)$ may be affected as a result of the happening of any arbitrary occurrence. Note that in the following, $dr\ \equiv\ \{dc \vee ec\}$ whereas $pp\ \equiv\ \{tpp \vee ntpp\}$. The rest of the example follows a step-by-step illustrative procedure and will be self-explanatory.

Simple world

$O = \{o_1,\ o_2,\ o_3\}$
$R = \{r_1,\ r_2,\ \ldots,\ r_n\}$

Initial state of the world

$pos(o_1,\ S) = r_1$
$pos(o_2,\ S) = r_2$
$pos(o_3,\ S) = r_3$
$\phi_1\ \equiv\ top(r_1,\ r_2,\ S)\ =\ dc\ \wedge\ top(r_2,\ r_3,\ S)\ = ec$

Ramification Constraint - RC-I

$$(\forall s).\ [dc(r_1,\ r_2,\ s)\ \wedge\ ec(r_2,\ r_3,\ s)]\ \supset$$
$$[dr(r_1,\ r_3,\ s)\ \vee\ po(r_1,\ r_3,\ s)\ \vee\ pp(r_1,\ r_3,\ s)]$$

Ramification Constraint - RC-II

$$(\forall s).\ [ec(r_1,\ r_2,\ s)\ \wedge\ ec(r_2,\ r_3,\ s)]\ \supset$$
$$[dr(r_1,\ r_3,\ s)\ \vee\ po(r_1,\ r_3,\ s)\ \vee\ tpp(r_1,\ r_3,\ s) \vee tpp^{-1}(r_1,\ r_3,\ s)]$$

Pseudo-Successor State Axiom

$$Poss(e,\ s)\ \wedge\ s'\ =\ Result(e,\ s)\ \supset$$
$$[top(r_i,\ r_j,\ s')\ = ec\ \equiv\ (\exists\, o_i,\ o_j)\,\{e\ =\ tran(ec,\ o_i,\ o_j) \wedge pos(o_i,\ s')\ =\ r_i\ \wedge\ pos(o_j,\ s')\ = r_j\}\ \vee$$
$$(\forall\, t_{rcc8})\,(\exists\, o_i,\ o_j)\,\{pos(o_i,\ s)\ =\ r_i\ \wedge\ pos(o_j,\ s)\ = r_j\ \wedge top(r_i,\ r_j,\ s)\ =\ ec\ \wedge\ e\ \neq\ tran(t_{rcc8},\ o_i,\ o_j)\}]$$

Transition Precondition

$$Poss(tran(ec,\ o_i,\ o_j),\ s) \equiv [pos(o_i,\ s)\ =\ r_i \wedge pos(o_j,\ s)\ =\ r_j]\ \wedge\ [top(\ r_i,\ r_j,\ s)\ =\ \{\ dc\ \vee\ po\ \}]$$

Given ramification constraint RC-I[5] and the initial state of the world, we can monotonically derive the following:

$$\phi_1'\ \equiv\ \phi_1\ \cup\ top(r_1,\ r_3)\ =\ \{dr\ \vee\ po\ \vee\ pp\}$$

Here, ϕ_1' is a monotonic extension of ϕ_1 in the sense that nothing previously known about the starting situation is invalidated by the state constraint; only implicit knowledge is made explicit by using the theorems

[4] This might become evident in a later section involving the specification of the dynamics of spatial change

[5] Observe that these ramification constraints are specific theorems from the RCC-8 composition table.

encoded in the RCC-8 composition table. Given the above definitions, especially the definition of the possibility of a transition and the pseudo-successor axiom or causal law governing the $top(...)$ fluent, we know that iff $s^{'} = Result(tran(ec,\ o_1,\ o_2),\ S)$, then the following will hold:

$top(pos(o_1,\ s^{'}),\ pos(o_2,\ s^{'}),\ s^{'}) = ec$

As such, in the situation $s^{'}$, the following holds:

$$\phi_2 \equiv top(pos(o_1,\ s^{'}),\ pos(o_2,\ s^{'}),\ s^{'}) = ec \wedge top(r_2,\ r_3,\ S) = ec$$

However, given Ramification Constraint II, (inspite of there being no interaction between o_1 *and* o_3) we also know that the following consistency criteria should hold:

$$top(pos(o_1,\ s^{'}),\ pos(o_3,\ s^{'}),\ s^{'}) = \{dr \vee po \vee tpp \vee tpp^{-1}\}$$

As before, let $\phi_2^{'}$ be the monotonic extension of ϕ_2 derived on the basis of RC-II:

$$\phi_2^{'} \equiv \phi_2 \cup top(pos(o_1,\ s^{'}),\ pos(o_3,\ s^{'}),\ s^{'}) = \{dr \vee po \vee tpp \vee tpp^{-1}\}$$

Obviously, according to $\phi_2^{'}$, o_1 *and* o_3 are related by $\{dr \vee po \vee tpp \vee tpp^{-1}\}$ whereas in $\phi_1^{'}$, o_1 *and* o_3 are related by $\{dr \vee po \vee pp\}$. The causal laws or the pseudo-successor axioms for our example domain *do not* lead to this inference; rather, it is a result of the ramification constraint specified in this example. For this reason, the pseudo-successor state axiom used in the example cannot be used in its current form in the presence of ramification constraints. In the sections to follow, we will illustrate how this problem has been solved, albeit at a very general level, in the context of the situation calculus (Lin 1995) and apply some of the ideas for dealing with such ramification constraints in the qualitative spatial reasoning domain.

Static Aspect - Modelling Composition Table Theorems as State Constraints

A straight-forward way to represent every theorem from the composition table is to model it as a state constraint. Using this scheme, we will need 8×8 constraints of the following form:

$$(\forall s).\ [dc(r_1,\ r_2,\ s) \wedge ec(r_2,\ r_3,\ s)] \supset [dr(r_1,\ r_3,\ s) \vee po(r_1,\ r_3,\ s) \vee pp(r_1,\ r_3,\ s)] \quad (1)$$

Using ordinary state constraints like (1), is problematic. We have already illustrated the impact of such indirect effect yielding constraints. It suffices to point out for now that instead of the ordinary constraint form shown in (1), we will use an explicit notion of causality (Lin & Reiter 1994; Lin 1995) in the form of a ternary $Caused(...)$ predicate for the specification of such ramification constraints.

$$(\forall s).\ [Holds(top(r_1,\ r_2),\ dc,\ s) \wedge Holds(top(r_2,\ r_3),\ ec,\ s)] \supset Caused(top(r_1,\ r_3),\ \gamma,\ s) \quad (2)$$
$$where\ \gamma \equiv [dr \vee po \vee pp]$$

In (2), the ternary predicate $Caused(\phi,\ \gamma,\ s)$ is used to reflect that the fluent ϕ takes on the value γ in the situation s for apparently no reason, i.e., the change in the value of this fluent is an indirect effect. Which precise occurrence caused this change is unknown (and irrelevant). Note that there is a significant level of non-determinism in the composition table theorems; reflected by the fact the γ in the above can be a disjunction of potential RCC-8 relations. For all practical purposes, additional information relevant to other aspects of space or atleast a distinction between rigid/material and non-rigid objects is necessary; after all, paradigmatic applications of such a formalism will certainly involve non-penetrable material objects [6].

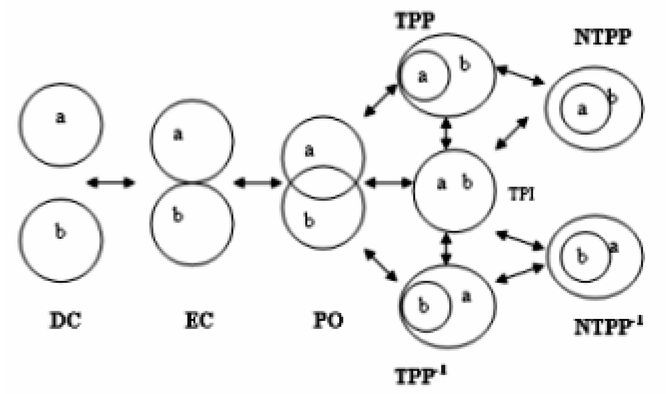

Figure 1: RCC-8 Conceptual Neighbourhood Diagram

Dynamic Aspect - Conceptual Neighbourhood Graph as Transitions

In the context of a qualitative topological theory of spatial changes, the most primitive means of change is a explicit change of topological relationship between two objects (their spatial extensions). To re-iterate, let $tran(t_{rcc},\ o_i,\ o_j)$ denote such a change; read as, o_i and o_j *transition* to a state of being t_{rcc}. In the situation calculus, the possibility of such a transition can be formally expressed in a general manner in the following way:

$$Poss(tran(t_{rcc},\ o_i,\ o_j),\ s) \equiv [Holds(pos(o_i),\ r_i,\ s) \wedge Holds(pos(o_j),\ r_j,\ s)] \wedge (\exists\, t_{rcc}^{'})\ [Holds(top(\ r_i,\ r_j),\ t_{rcc}^{'},\ s)] \wedge neighbour(t_{rcc},\ t_{rcc}^{'})] \quad (3)$$

The binary predicate $neighbour(t_{rcc},\ t_{rcc}^{'})$ in (3) is used to express the possibility of a direct continuous perturbation being consistent between two topological relations and is based on the conceptual neighbourhood principle (Freksa 1991). According to this principle, relations t and $t^{'}$ are conceptual neighbours if two objects related by t can (in line with our vocabulary) transition to the state of being $t^{'}$ and vice-versa. The conceptual neighbourhood graph for RCC-8, shown in Fig. 1, can be used to define a total of 8 axioms of the form in (3) so as to comprehensively represent the possibility criteria for every topological transition definable for the RCC-8 set of topological relations.

[6] In the context of qualitative motion, see the classification in (Galton 1993) on the basis of rigidity & non-rigidity of objects as well the relative shape & sizes of a rigid body and a region of space.

Causal Laws – The Effects of Transitions

Pseudo-Successor State Axiom (PSA) Successor state axioms specify the causal laws of the domain – what changes as a result of various occurrences in the system being modelled. Generally, the SSA is based on a *completeness assumption* which essentially means that all possible ways in which the system (the set of fluents) may change is explicitly formulated in the domain theory. In view of this assumption, the PSA in this section is based on the premise that there are no indirect effects, i.e. the *completeness assumption* is enforced. However, the topological relationship between two objects whose change this SSA is supposed to model might also change as a result of an indirect effect; this we have illustrated in an earlier section. The PSA given below assumes that there are no such ramifications or more simply, it specifies every possible way in which two objects might establish a topological relationship of being in ec.

In general, the SSA will take the form in (4) for every fluent in the domain:

$$Poss(e,\, s) \,\wedge\, s^{'} \,=\, Result(e,\, s) \,\supset\, [top(r_i,\, r_j,\, s^{'}) \,=\, t_{rcc8} \,\equiv\, (\exists\, o_i,\, o_j)\,\{e \,=\, tran(t_{rcc8},\, o_i,\, o_j) \,\wedge\, pos(o_i,\, s^{'}) \,=\, r_i \,\wedge\, pos(o_j,\, s^{'}) \,=r_j\} \,\vee\, (\forall\, t^{'}_{rcc8})\,(\exists\, o_i,\, o_j)\,\{pos(o_i,\, s) \,=\, r_i \,\wedge\, pos(o_j,\, s) \,=r_j \,\wedge\, top(r_i,\, r_j,\, s) \,=\, t_{rcc8} \,\wedge\, e \,\neq\, tran(t^{'}_{rcc8},\, o_i,\, o_j)\}] \tag{4}$$

Successor State Axiom (SSA) The successor state axioms in this section is the one that should be derived in the presence of ramification constraints. Recall our use of the ternary $Caused(\phi,\, \gamma,\, s)$ predicate in (2) toward the representation of the composition table theorems as state constraints.

The effect of including such an explicit notion of causality is that it essentially relates a particular fluent ϕ with a value γ and accords this (happening) the same ontological status as that of other occurrences in the theory. More simply, this happening, viz – ϕ assuming the value γ, is regarded as an occurrence (albeit unknown) similar in nature to $trans(t,\, o,\, o^{'})$ and is therefore no more an indirect effect of some other occurrence. Precise details notwithstanding, what remains to be done is minimizing the causal relation by circumscribing[7] it in the set of axioms introduced so far, viz – the foundational unique names axioms, the ramification constraints in (2) and the transition pre-conditions in (3), thereby resulting in the *Causation Axiom* of the form in (5). Note that this is a general form, whereas actual minimization will yield specific values.

$$Caused(top(r_i,\, r_k),\, t_{rcc8},\, s) \,\equiv\, \{t_{rcc8} \,=\, t_k \,\wedge\, (\exists r_j)[Holds(top(r_i,\, r_j),\, t_i,\, s) \,\wedge\, Holds(top(r_j,\, r_k),\, t_j,\, s)]\} \tag{5}$$

The causation axiom in (5) must to be integrated with the Pseudo-SSA to derive the following SSA:

$$Poss(e,\, s) \,\wedge\, s^{'} \,=\, Result(e,\, s) \,\supset\, [top(r_i,\, r_j,\, s^{'}) \,=ec \,\equiv\, (\exists\, o_i,\, o_j)\,\{e \,=\, tran(ec,\, o_i,\, o_j) \,\wedge\, pos(o_i,\, s^{'}) \,=\, r_i \,\wedge\, pos(o_j,\, s^{'}) \,=r_j\} \,\vee\, (\forall\, t_{rcc8})\,(\exists\, o_i,\, o_j)\,\{pos(o_i,\, s) \,=\, r_i \,\wedge\, pos(o_j,\, s) \,=r_j \,\wedge\, top(r_i,\, r_j,\, s) \,=\, ec \,\wedge\, e \,\neq\, tran(t_{rcc8},\, o_i,\, o_j)\} \,\vee\, (\exists\, r_k)\,\{Holds(top(r_i,\, r_k),\, t_i,\, s) \wedge Holds(top(r_k,\, r_j),\, t_j,\, s)\}]) \tag{6}$$

[7] Some other form of minimization with the same effect might be usable too. Note however that a step-by-step illustration of minimization using circumscription is given in (Lin 1995).

In the SSA in (6), we assume that the final disjunct is the only indirect effect, i.e., we only account for one theorem from the composition table, that too without stating it precise terms. In actuality, there is more than one entry/theorem in the composition table referring to the state of being $externally\ connected$ and a complete axiomatisation should account for each of them.

Jointly Exhaustive and Pairwise Disjoint Base Relations

The property of the RCC-8 topological relations being jointly exhaustive and mutually disjoint can be expressed using ordinary state constraints of the form in (1) in a straightforward manner. In general, we need a total of n state constraints of the form in (8) to express the joint exhaustion of a set of n base relations.

$$(\forall s).\, \neg[ec(r_1,\, r_2,\, s) \,\vee\, po(r_1,\, r_2,\, s)\, tpp \vee\, (r_1,\, r_2,\, s) \vee tpp^{-1}(r_1,\, r_2,\, s) \,\vee\, eq(r_1,\, r_2,\, s) \,\vee\, ntpp(r_1,\, r_2,\, s) \vee ntpp^{-1}(r_1,\, r_2,\, s)] \supset dc(r_1,\, r_2,\, s) \tag{8}$$

Similarly, $[n(n\,-\,1)/2)]$ ordinary constraints of the form in (9) are sufficient to express the pair-wise disjointness of n base relations.

$$(\forall s).\, \neg\, [dc(r_1,\, r_2,\, s) \,\wedge\, ec(r_1,\, r_2,\, s)] \tag{9}$$
$$(\forall s).\, \neg\, [ec(r_1,\, r_2,\, s) \,\wedge\, po(r_1,\, r_2,\, s)]$$

Additionally, as has been shown in (10), other miscellaneous properties such as the symmetry & asymmetry of the RCC-8 relations too can be expressed using state constraints.

$$(\forall s).\, dc(r_i,\, r_j,\, s) \,\supset\, dc(r_j,\, r_i,\, s) \tag{10}$$
$$(\forall s).\, tpp(r_i,\, r_j,\, s) \,\supset\, \neg\, tpp(r_j,\, r_i,\, s)$$

Symmetric relations from the RCC-8 set include $dc,\ ec,\ po,\ eq$ whereas $tpp,\ tpp^{-1},\ ntpp,\ ntpp^{-1}$ are asymmetric.

Related Work

We drew a correspondence with some of the earliest attempts that are similar in nature to our own at the start of the paper. In addition, here we include a succinct comparison with relatively recent work done in this area. In (Dylla & Moratz 2004), the line segment based Dipole calculus has been naively translated into pre-conditions and successor state axioms in the situation calculus. Although the work is similar in spirit and overall goals, there are some important distinctions: (a) Firstly, the ontological commitments with regard to space are drastically different; they deal with intrinsically oriented dipoles or line segments whereas our work follows the region abstraction of the RCC calculus. (b) Their work is primarily concerned with exploiting the conceptual neighbourhood principle in the context of orientation relations in the dipole calculus and unlike our approach, other representational aspects of a general relational calculus have not been accounted for. (c) Moreover, they directly formalise complex high-level actions on the basis of primitive orientation relations; as mentioned earlier, our aim is to approach the problem at a much more primitive level and investigate some of the finer representational issues relevant to representing dynamically changing worlds in the context of qualitative spatial reasoning. Such differences notwithstanding, there is a interesting possibility to integrate the two approaches so as to enrich the spatial theory currently in use; i.e., to use topological & orientation relations in conjunction.

Conclusion and Outlook

Albeit broadly, we presented a case for treating qualitative inferences about spatial relations in an integrated manner with the purpose or goal directed aspects of change, i.e., combining spatial change with its causal aspects. Such integrated reasoning manifests itself in wide range of practical AI applications such as intelligent control in dynamic environments, qualitative simulation for planning, prediction & diagnoses etc. Starting with minimal assumptions about the nature of space and/or spatial dynamics, we use the topological theory of the region connection calculus as our spatial metaphor and investigate the key representational aspects (& problems resulting thereof) of qualitative spatial reasoning using a general formalism to reason about change, namely the situation calculus. RCC being representative of a general class of relational formalisms in the QSR domain, our results essentially subsume other relational formalisms that are akin to RCC & encompass other aspects of space. Furthermore, we only considered one specific type of occurrence in this paper, viz – a *topological transition*. However, future endeavours involve making much finer distinctions involving deterministic *events* – events that always occur when the condition for their occurrence is satisfied and *actions* – a special type of event but which is performed by an agent and can be withhold even if all the pre-conditions are satisfied. Such an extended framework can be applied toward interesting application scenarios such as the following: (a). Spatial Planning: Given a set of spatio-temporal & other domain specific constraints, an initial state and a overall objective to be achieved, derive a sequence[8] of (spatial) actions that could possibly fulfill the desired objective. In other words, "How do we transform one spatial configuration into another?". Note that this problem, which can be considered akin to the task of arriving at a desired spatial configuration starting at a given configuration is the simplest form of spatial planning. (b) *Causal Explanation*: Given a set of spatial changes by way of temporally ordered snap-shots, the task is to extract a explanatory causal model in terms of occurrents (i.e., events & actions) from the given spatial information on the basis of a background theory that relates domain specific occurrents with spatial changes. The main objective here is to provide an explanation of what may have *caused* the system being modelled to evolve in the manner denoted by the temporal snapshots, the main objective here being to answer queries such as "*What occurrences may have caused a particular spatial configuration, or a series of temporally ordered configurations*"?. (c) *Spatial Projection & Interpolation*: Given a initial situation, predict all possible evolutions of the system or identify if a particular situation is reachable. The latter latter could be either characterised by the happening of one or more occurrences or a some spatial configuration of the objects in the domain. A related task is to interpolate missing spatial knowledge, i.e, to find a possible spatial configuration between two temporal snap-shots of a changing system. Our hypothesis is that once a sufficiently rich spatial theory encompassing various aspects of space has been formulated in this manner, complex actions & events can be composed at a much higher level of abstraction – an effort that is of special interest to our own ongoing & future work.

[8] By '*sequence*', we do not necessarily imply a linear order.

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