

Aggregating quantitative possibilistic networks

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Abstract

The problem of merging multiple-source uncertain information is a crucial issue in many applications. This paper proposes an analysis of possibilistic merging operators where uncertain information is encoded by means of product-based (or quantitative) possibilistic networks. We first show that the product-based merging of possibilistic networks having the same DAG structures can be easily achieved in a polynomial time. We then propose solutions to merge possibilistic networks having different structures with the help of additional variables.

Introduction

The problem of combining pieces of information issued from different sources can be encountered in various fields of applications such as databases, multi-agent systems, expert opinion pooling, etc.

Several works have been recently achieved to fuse propositional or weighted logical knowledge bases issued from different sources (Baral *et al.* 1992), (Cholvy 1998), (Konieczny and Pérez 1998), (Lin 1996), (Lin and Mendelzon 1998), (Benferhat *et al.* 1997).

This paper addresses the problem of fusion of uncertain pieces of information represented by possibilistic networks.

Possibilistic networks (Fonck 1992; Borgelt *et al.* 1998; Gebhardt and Kruse 1997) are important tools proposed for an efficient representation and analysis of uncertain information. Their success is due to their simplicity and their capacity of representing and handling independence relationships which are important for an efficient management of uncertain pieces of information. Possibilistic networks are directed acyclic graphs (DAG), where each node encodes a variable and every edge represents a relationship between two variables. Uncertainties are expressed by means of conditional possibility distributions for each node in the context of its parents.

In possibility theory, there are two kinds of possibilistic causal networks depending if possibilistic conditioning is

based on the minimum or on the product operator. In the rest of this paper, we only consider product-based conditioning. In this case, possibilistic networks are called quantitative (or product-based) possibilistic networks.

The rest of this paper is organised as follows. Next section gives a brief background on possibility theory and quantitative possibilistic networks. Section 3 recalls the conjunctive combination mode on possibility distributions. Section 4 discusses the fusion of possibilistic networks having same graphical structures. Section 5 deals with fusion of possibilistic networks having different structures but the union of their DAGs is free of cycles. Section 6 proposes a general approach for merging any set of possibilistic networks. Section 7 concludes the paper.

Basics of possibility theory

Let $V = \{A_1, A_2, \dots, A_N\}$ be a set of variables. We denote by $D_A = \{a_1, \dots, a_n\}$ the domain associated with the variable A . By a we denote any instance of A . $\Omega = \times_{A_i \in V} D_{A_i}$ denotes the universe of discourse, which is the Cartesian product of all variable domains in V . Each element $\omega \in \Omega$ is called a state of Ω . In the following, we only give a brief recalling on possibility theory, for more details see (Dubois and Prade 1988).

Possibility distributions and possibility measures

A possibility distribution π is a mapping from Ω to the interval $[0, 1]$. It represents a state of knowledge about a set of possible situations distinguishing what is plausible from what is less plausible.

Given a possibility distribution π defined on the universe of discourse Ω , we can define a mapping grading the *possibility measure* of an event $\phi \subseteq \Omega$ by $\Pi(\phi) = \max_{\omega \in \phi} \pi(\omega)$. A possibility distribution π is said to be *normalized*, if $h(\pi) = \max_{\omega} \pi(\omega) = 1$.

In a possibilistic setting, conditioning consists in modifying our initial knowledge, encoded by a possibility distribution π , by the arrival of a new *sure* piece of information $\phi \subseteq \Omega$. There are different definitions of condition-

ing. In this paper, we only use the so-called quantitative (or product-based) conditioning defined by:

$$\Pi(\omega | \phi) = \begin{cases} \frac{\pi(\phi)}{\Pi(\phi)} & \omega \models \phi \\ 0 & \text{otherwise} \end{cases}$$

Quantitative possibilistic networks

This sub-section defines quantitative possibilistic graphs. A *quantitative possibilistic graph* over a set of variables V , denoted by $\mathbb{N} = (\pi_{\mathbb{N}}, G_{\mathbb{N}})$, consists of:

- a *graphical component*, denoted by $G_{\mathbb{N}}$, which is a DAG (Directed Acyclic Graph). Nodes represent variables and edges encode the link between the variables. The parent set of a node A is denoted by U_A .
- a *numerical component*, denoted by $\pi_{\mathbb{N}}$, which quantifies different links.

For every root node A ($U_A = \emptyset$), uncertainty is represented by the a priori possibility degrees $\pi_{\mathbb{N}}(a)$ of each instance $a \in D_A$, such that

$$\max_a \pi_{\mathbb{N}}(a) = 1.$$

For the rest of the nodes ($U_A \neq \emptyset$) uncertainty is represented by the conditional possibility degrees $\pi_{\mathbb{N}}(a | u_A)$ of each instances $a \in D_A$ and $u_A \in D_{U_A}$. These conditional distributions satisfy the following normalization condition:

$$\max_a \pi_{\mathbb{N}}(a | u_A) = 1, \text{ for any } u_A.$$

The set of a priori and conditional possibility degrees induces a unique joint possibility distribution defined by:

Definition 1 Let $\mathbb{N} = (\pi_{\mathbb{N}}, G_{\mathbb{N}})$ be a quantitative possibilistic network. Given the a priori and conditional possibility distribution, the joint distribution denoted by $\pi_{\mathbb{N}}$, is expressed by the following quantitative chain rule :

$$\pi_{\mathbb{N}}(A_1, \dots, A_N) = \prod_{i=1..N} \Pi(A_i | U_{A_i}) \quad (1)$$

Product-based Conjunctive merging

One of the important aims in merging uncertain information is to exploit complementarities between the sources in order to get a more complete and precise global point of view.

In possibility theory, given a set of possibility distributions π_i 's, the basic combination mode is the conjunction (i.e., the minimum) of possibility distributions. Namely (For more details on the semantic fusion of possibility distributions see (Dubois and Prade 1994)):

$$\forall \omega, \pi_{\oplus}(\omega) = \min_{i=1..n} \pi_i(\omega).$$

The conjunctive aggregation makes sense if all the sources are regarded as equally and fully reliable since all

values that are considered as impossible by one source but possible by the others are rejected.

The min-based combination mode has no reinforcement effect. Namely, if expert 1 assigns possibility $\pi_1(\omega) < 1$ to a situation ω , and expert 2 assigns possibility $\pi_2(\omega)$ to this situation then overall, in the min-based mode, $\pi_{\oplus}(\omega) = \pi_1(\omega)$ if $\pi_1(\omega) < \pi_2(\omega)$, regardless of the value of $\pi_2(\omega)$. However since both experts consider ω as rather impossible, and if these opinions are independent, it may sound reasonable to consider ω as less possible than what each of the experts claims.

More generally, if a pool of independent experts is divided into two unequal groups that disagree, we may want to favor the opinion of the biggest group. This type of combination cannot be modelled by the minimum operation. What is needed is a reinforcement effect. A reinforcement effect can be obtained using a product-based combination mode:

$$\forall \omega, \pi_{\oplus}(\omega) = \prod_{i=1..n} \pi_i(\omega).$$

Let $\mathbb{N}1$ and $\mathbb{N}2$ be two possibilistic networks. Our aim is to directly construct from $\mathbb{N}1$ and $\mathbb{N}2$ a new possibilistic network, denoted by $\mathbb{N}\oplus$. The new possibilistic network should be such that:

$$\forall \omega, \pi_{\mathbb{N}\oplus}(\omega) = \pi_{\mathbb{N}1}(\omega) * \pi_{\mathbb{N}2}(\omega).$$

We assume that the two networks are defined on the same set of variables. This is not a limitation, since any possibilistic network can be extended with additional variables, as it is shown by the following proposition:

Proposition 1 Let $\mathbb{N} = (\pi_{\mathbb{N}}, G_{\mathbb{N}})$ be a possibilistic network defined on a set variables V . Let A be a new variable. Let $\mathbb{N}1 = (\pi_{\mathbb{N}1}, G_{\mathbb{N}1})$ be a new possibilistic networks such that :

- $G_{\mathbb{N}1}$ is equal to $G_{\mathbb{N}}$ plus a root node A , and
- $\pi_{\mathbb{N}1}$ is identical to $\pi_{\mathbb{N}}$ for variables in V , and is equal to a uniform possibility distribution on the root node A (namely, $\forall a \in D_A, \pi_{\mathbb{N}1}(a)=1$).

Then, we have :

$$\forall \omega \in \times_{A_i \in V} D_{A_i}, \pi_{\mathbb{N}}(\omega) = \max_{a \in D_A} \pi_{\mathbb{N}1}(a\omega),$$

where $\pi_{\mathbb{N}}$ and $\pi_{\mathbb{N}1}$ are respectively the possibility distributions associated with \mathbb{N} and $\mathbb{N}1$ using Definition 1.

Fusion of the same-structure networks

This section presents the procedure of merging causal networks having a same DAG structures. For sake of simplicity and without loss of generality, we restrict ourselves to the case of the fusion of two causal networks.

The two possibilistic networks to merge, denoted $\mathbb{N}1$ and $\mathbb{N}2$ only differ on conditionnal possibility distributions assigned to variables.

The following definition and proposition show that the result merging of networks having same structure is immediate.

Definition 2 Let $\mathbb{N}1 = (\pi_{\mathbb{N}1}, G_{\mathbb{N}1})$ and $\mathbb{N}2 = (\pi_{\mathbb{N}2}, G_{\mathbb{N}2})$ be two possibilistic networks such that $G_{\mathbb{N}1} = G_{\mathbb{N}2}$. The result of merging $\mathbb{N}1$ and $\mathbb{N}2$ is a possibilistic network denoted by $\mathbb{N}\oplus = (\pi_{\mathbb{N}\oplus}, G_{\mathbb{N}\oplus})$, where :

- $G_{\mathbb{N}\oplus} = G_{\mathbb{N}1} = G_{\mathbb{N}2}$ and
- $\pi_{\mathbb{N}\oplus}$ are defined by:
 $\forall A, \pi_{\mathbb{N}\oplus}(A|U_A) = \pi_{\mathbb{N}1}(A|U_A) * \pi_{\mathbb{N}2}(A|U_A)$,

where A is a variable and U is the set of parents of A .

Proposition 2 Let $\mathbb{N}1 = (\pi_{\mathbb{N}1}, G_{\mathbb{N}1})$ and $\mathbb{N}2 = (\pi_{\mathbb{N}2}, G_{\mathbb{N}2})$ be two possibilistic networks having exactly the same associated DAG. Let $\mathbb{N}\oplus = (\pi_{\mathbb{N}\oplus}, G_{\mathbb{N}\oplus})$ be the result of merging $\mathbb{N}1$ and $\mathbb{N}2$ using the above definition. Then, we have :

$$\forall \omega \in \Omega, \pi_{\mathbb{N}\oplus}(\omega) = \pi_{\mathbb{N}1}(\omega) * \pi_{\mathbb{N}2}(\omega),$$

where $\pi_{\mathbb{N}\oplus}, \pi_{\mathbb{N}1}, \pi_{\mathbb{N}2}$ are respectively the possibility distributions associated with $\mathbb{N}\oplus, \mathbb{N}1, \mathbb{N}2$ using Definition 1.

Example 1 Let $\mathbb{N}1$ and $\mathbb{N}2$ be two possibilistic networks. Let $G_{\mathbb{N}}$ be the DAG associated with $\mathbb{N}1$ and $\mathbb{N}2$ and represented by figure 1.

The possibility distributions associated to $\mathbb{N}1$ and $\mathbb{N}2$ are given respectively by table 1 and table 2.

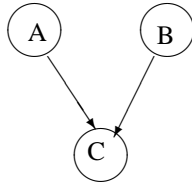


Figure 1: Example of similar networks

Then fused possibilistic networks $\mathbb{N}\oplus$ is such that its associated graph is also the DAG of figure 1 and its conditional possibility distribution is given by table 3.

It can be checked that :

$$\forall \omega \in \Omega, \pi_{\mathbb{N}\oplus}(\omega) = \pi_{\mathbb{N}1}(\omega) * \pi_{\mathbb{N}2}(\omega).$$

For instance, we have :

$$\pi_{\mathbb{N}1}(a_2 b_2 c_1) = \pi_{\mathbb{N}1}(a_2) * \pi_{\mathbb{N}1}(b_2) * \pi_{\mathbb{N}1}(c_1 | a_2 b_2) = .2 * .5 * 1 = .1$$

$$\pi_{\mathbb{N}2}(a_2 b_2 c_1) = \pi_{\mathbb{N}2}(a_2) * \pi_{\mathbb{N}2}(b_2) * \pi_{\mathbb{N}2}(c_1 | a_2 b_2) = .3 * .2 * 1 = .06$$

$$\pi_{\mathbb{N}\oplus}(a_2 b_2 c_1) = \pi_{\mathbb{N}\oplus}(a_2) * \pi_{\mathbb{N}\oplus}(b_2) * \pi_{\mathbb{N}\oplus}(c_1 | a_2 b_2) = .06 * .1 * 1 = .006.$$

Table 1: initial possibility distributions associated with $\mathbb{N}1$

a	$\pi(a)$	b	$\pi(b)$	a	b	c	$\pi(c)$	$a \wedge b$
a_1	1	b_1	1	a_1	b_1	c_1		1
a_2	.2	b_2	.5	a_1	b_1	c_2		.3
				a_1	b_2	c_1		.1
				a_1	b_2	c_2		1
				a_2	b_1	c_1		.1
				a_2	b_1	c_2		1
				a_2	b_2	c_1		1
				a_2	b_2	c_2		0

Table 2: initial possibility distributions associated with $\mathbb{N}2$

a	$\pi(a)$	b	$\pi(b)$	a	b	c	$\pi(c)$	$a \wedge b$
a_1	1	b_1	1	a_1	b_1	c_1		1
a_2	.3	b_2	.2	a_1	b_1	c_2		0
				a_1	b_2	c_1		.7
				a_1	b_2	c_2		1
				a_2	b_1	c_1		0
				a_2	b_1	c_2		1
				a_2	b_2	c_1		1
				a_2	b_2	c_2		.4

Table 3: initial possibility distributions associated with $\mathbb{N}\oplus$

a	$\pi(a)$	b	$\pi(b)$	a	b	c	$\pi(c)$	$a \wedge b$
a_1	1	b_1	.1	a_1	b_1	c_1		1
a_2	.06	b_2	.1	a_1	b_1	c_2		0
				a_1	b_2	c_1		.07
				a_1	b_2	c_2		1
				a_2	b_1	c_1		0
				a_2	b_1	c_2		1
				a_2	b_2	c_1		1
				a_2	b_2	c_2		0

Fusion of U-acyclic networks

The above section has shown that the fusion of possibilistic networks can be easily achieved if they share the same DAG structure.

This section considers the case where the networks have not the same structure. However we assume that their union does not contain a cycle.

A union of two DAGs (G_1, G_2) is a graph where :

- its set of variables is the union of variables in G_1 and G_2 and
- for each variable A , its parents are those in G_1 and G_2 .

If the union of G_1 and G_2 does not contain cycles, we say that G_1 and G_2 are U-acyclic networks. In this case the fusion can be easily obtained. We first provides a proposition which shows how to add links to a possibilistic network without changing its possibility distribution.

Proposition 3 Let $\mathbb{N} = (\pi_{\mathbb{N}}, G_{\mathbb{N}})$ be a possibilistic network. Let A be a variable, and let $Par(A)$ be parents of A in $G_{\mathbb{N}}$. Let $B \notin Par(A)$. Let $\mathbb{N}_1 = (\pi_{\mathbb{N}_1}, G_{\mathbb{N}_1})$ be a new possibilistic network obtained from $\mathbb{N} = (\pi_{\mathbb{N}}, G_{\mathbb{N}})$ by adding a link from B to A . The new conditionnal possibility associated with A is:

$$\forall a \in D_A, \forall b \in D_B, \forall u \in D_{Par(A)}, \\ \pi_{\mathbb{N}_1}(a | ub) = \pi_{\mathbb{N}}(a | u).$$

Then, we have :

$$\forall \omega, \pi_{\mathbb{N}}(\omega) = \pi_{\mathbb{N}_1}(\omega).$$

Given this proposition the fusion of two U-acyclic networks \mathbb{N}_1 and \mathbb{N}_2 is immediate. Let $G_{\mathbb{N} \oplus}$ be the union of $G_{\mathbb{N}_1}$ and $G_{\mathbb{N}_2}$. Then the fusion of \mathbb{N}_1 and \mathbb{N}_2 can be obtained using the following two steps:

Step 1 Using Proposition 3, expand \mathbb{N}_1 and \mathbb{N}_2 such that $G_{\mathbb{N}_1} = G_{\mathbb{N}_2} = G_{\mathbb{N} \oplus}$.

Step 2 Use Proposition 2 on the possibilistic networks obtained from Step 1 (since the two networks have now the same structure).

Example 2 Let us consider two causal networks, where their DAG are given by figure 2. These two DAG have a different structure.

The conditionnal possibility distributions associated with above networks are given by tables 4 and 5.

We see clearly, from figure 2, that the union of two DAGs is free of cycle. Figure 3 provides the DAG of $G_{\mathbb{N} \oplus}$ which is simply the union of the two graphs of Figure 2.

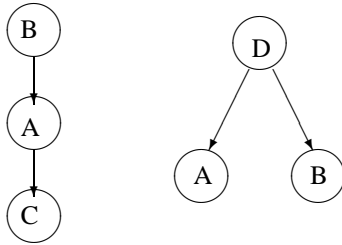


Figure 2: G_1 G_2 : Example of U-acyclic networks

Table 4: initial conditionnal possibility distributions $\pi_{\mathbb{N}_1}$

b	$\pi_1(b)$	a	b	$\pi_1(a b)$	a	c	$\pi_1(c a)$
b_1	1	a_1	b_1	.3	a_1	c_1	1
b_2	.2	a_1	b_2	1	a_1	c_2	.5
		a_2	b_1	1	a_2	c_1	0
		a_2	b_2	0	a_2	c_2	1

Now we transform both of $G_{\mathbb{N}_1}$ and $G_{\mathbb{N}_2}$ to the common graph $G_{\mathbb{N} \oplus}$ by adding the required variables and links for

Table 5: initial conditionnal possibility distributions $\pi_{\mathbb{N}_2}$

d	$\pi_2(d)$	a	d	$\pi_2(a d)$	b	d	$\pi_2(b d)$
d_1	1	a_1	d_1	1	b_1	d_1	1
d_2	0	a_1	d_2	1	b_1	d_2	.8
		a_2	d_1	1	b_2	d_1	.7
		a_2	d_2	0	b_2	d_2	1

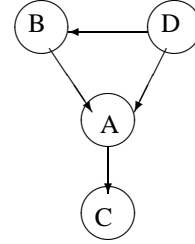


Figure 3: the DAG $G_{\mathbb{N} \oplus}$

each graph. In this case we apply the following steps:

• For $G_{\mathbb{N}_1}$ we :

– add a new variable D with a uniform conditional possibility distributions, namely:

$$\forall d \in D_D, \pi_{\mathbb{N}_1}(d) = 1,$$

– add a link from this variable D to B in the graph, and the new local conditional possibility on node B become:

$$\forall d \in D_D, \forall b \in D_B, \pi_{\mathbb{N}_1}(b | d) = \pi_{\mathbb{N}_1}(b).$$

– add a link from this variable D to A in the graph, and the new local conditional possibility on node A become:

$$\forall d \in D_D, \forall b \in D_B, \forall a \in D_A, \pi_{\mathbb{N}_1}(a | b, d) = \pi_{\mathbb{N}_1}(a | b).$$

• For $G_{\mathbb{N}_2}$ we proceed similarly, namely we:

– add a new variable C , and a link from A to C , with a uniform conditional possibility distributions, namely:

$$\forall c \in D_C, \forall a \in D_A, \pi_{\mathbb{N}_2}(c | a) = 1.$$

– add a link from B to A , and the new local conditional possibility on node A become:

$$\forall d \in D_D, \forall b \in D_B, \forall a \in D_A, \pi_{\mathbb{N}_2}(a | b, d) = \pi_{\mathbb{N}_2}(a | d).$$

Table 6 gives conditionnal possibility distributions associated with the DAG of figure 3.

Table 6: Merged conditionnal distributions $\pi_{\mathbb{N}\oplus}$

d	$\pi(d)$	a	c	$\pi(c a)$	b	d	$\pi(b d)$
d_1	1	a_1	c_1	1	b_1	d_1	1
d_2	0	a_1	c_2	.5	b_1	d_2	.8
		a_2	c_1	0	b_2	d_1	.14
		a_2	c_2	1	b_2	d_2	.2

a	b	d	$\pi(a b \wedge d)$
a_1	b_1	d_1	.3
a_1	b_1	d_2	.3
a_1	b_2	d_1	1
a_1	b_2	d_2	1
a_2	b_1	d_1	1
a_2	b_1	d_2	0
a_2	b_2	d_1	0
a_2	b_2	d_2	0

From these different tables of conditionnal distributions, we can easily show that the joint possibility of $\pi_{\mathbb{N}\oplus}$ computed by chain rule, is equal to the product of $\pi_{\mathbb{N}1}$ and $\pi_{\mathbb{N}2}$. For instance, let $\omega = a_1 b_1 c_1 d_1$ be a possible situation. Using chain rule We have:

$$\begin{aligned} \pi_{\mathbb{N}1}(a_1 b_1 c_1 d_1) &= .3. \\ \pi_{\mathbb{N}2}(a_1 b_1 c_1 d_1) &= 1. \\ \pi_{\mathbb{N}\oplus}(a_1 b_1 c_1 d_1) &= \pi_{\mathbb{N}\oplus}(c_1 | a_1) * \pi_{\mathbb{N}\oplus}(a_1 | b_1 d_1) * \pi_{\mathbb{N}\oplus}(b_1 | d_1) * \pi_{\mathbb{N}\oplus}(d_1) = .3. \end{aligned}$$

Fusion of U-cyclic networks

The previous section has proposed an approach to fuse U-cyclic networks, by expanding each network to a common network (the union of networks to fuse). This approach cannot be applied if this common structure contains cycles.

This section proposes an alternative approach which can be applied for fusing any set of possibilistic networks. This approach is based on introducing new variables. Let $\mathbb{N}1 = (\pi_{\mathbb{N}1}, G_{\mathbb{N}1})$ and $\mathbb{N}2 = (\pi_{\mathbb{N}2}, G_{\mathbb{N}2})$ be two possibilistic networks.

The following algorithm gives the construction of $G_{\mathbb{N}\oplus}$

Algorithm 1: Construction of $G_{\mathbb{N}\oplus}$

Data: $G_{\mathbb{N}1}$ and $G_{\mathbb{N}2}$

Result: $G_{\mathbb{N}\oplus}$

begin

- Initialize $G_{\mathbb{N}\oplus}$ with $G_{\mathbb{N}2}$

- Rename each variable A_i in $G_{\mathbb{N}\oplus}$ by a new variable that we denote A'_i . Each instance a_i of A_i will be renamed by a new instance denoted a'_i . We denote by V' the set of new variables.

- Add $G_{\mathbb{N}1}$ to $G_{\mathbb{N}\oplus}$

- For each variable A , add a link from A to its associated variable A'_i .

end

Namely, the fused graph $G_{\mathbb{N}\oplus}$ is obtained by first considering $G_{\mathbb{N}1}$, renaming variables of $G_{\mathbb{N}2}$ and linking variables of A_i and A'_i .

The Construction of $\pi_{\mathbb{N}\oplus}$ from $\pi_{\mathbb{N}1}$ and $\pi_{\mathbb{N}2}$ is obtained as follow:

- For each variable A , define its associated possibility distribution in $\mathbb{N}\oplus$ to be identical to the one in $\mathbb{N}1$, namely :

$$\pi_{\mathbb{N}\oplus}(A | U_A) = \pi_{\mathbb{N}1}(A | U_A)$$

- For variables A'_i , note first that parents of A'_i in $G_{\mathbb{N}\oplus}$ are those of $G_{\mathbb{N}2}$ plus the variable A_i . The conditional possibility distribution associated with each variable A' is defined as follows:

$$\pi_{\mathbb{N}\oplus}(a'_i | a_j U'_A) = \begin{cases} \pi_{\mathbb{N}2}(a_i | U_A) & \text{if } i = j \\ 0 & \text{otherwise} \end{cases} \quad (2)$$

From the construction of $\pi_{\mathbb{N}\oplus}$, we can check that : $\forall \omega \in \times_{A \in V} D_A$:

$$\Pi_{\mathbb{N}\oplus}(\omega) = \pi_{\mathbb{N}1}(\omega) * \pi_{\mathbb{N}2}(\omega).$$

Example 3 Consider the following DAG's :

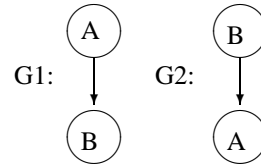


Figure 4: Example of U-cyclic networks

For lack of space, we will only illustrate the construction of the fused graph.

We remark that union of the DAG's of figure4 contains a cycle. Then the fused graph and the possibility distributions

are computed as follow:

- Move all the variables of G_{N_1} to $G_{N_{\oplus}}$.

The possibility distributions of $G_{N_{\oplus}}$ is identical to the one in π_{N_1} .

For instance:

$$\forall a \in D_A \pi_{N_{\oplus}}(a) = \pi_{N_1}(a)$$

$$\text{and } \forall b \in D_B \pi_{N_{\oplus}}(b|U_B) = \pi_{N_2}(b|U_B)$$

- Rename variables of G_{N_2} .

$$A \longrightarrow A' \text{ and } B \longrightarrow B'$$

Add the new variables of G_{N_2} to those of $G_{N_{\oplus}}$.

- Create a link from A to A' and another link from B to B'.
Compute the new conditional possibility distributions of the fused graph as defined above.
The result graph is illustrated by Figure 5.

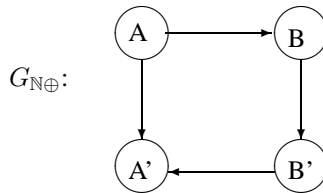


Figure 5: Example of fused graph $G_{N_{\oplus}}$

Conclusions

This paper has proposed a syntactic fusion of possibilistic networks. We first showed that when the possibilistic networks have the same structure or when the union of their DAGs is free of cycles, then the fusion can be achieved efficiently. When the union of DAGs contain cycles, then the fusion is still possible with additional variables. A future work is to analyse the problem of subnormalization that may appear when merging possibilistic networks.

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