Nintendo Ten Billion Barrel

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Description:

This extremely difficult puzzle was made by Nintendo in 1980 before they limited themselves to game computers. The Ten Billion Barrel is sometimes called the Billion barrel, $10^{10}$, or the Nintendo Tumbler Puzzle. It was invented by Gumpei Yokoi. A transparent star-shaped version was made recently for members of the Nintendo Club.

The puzzle has 23 coloured balls arranged in five columns spaced evenly around the sides of the barrel. The barrel itself has six rows. Two (nonadjacent) columns have only 4 balls, which lie in the middle four rows. The other three columns have 5 balls, and the extra balls either lie all in the top row or all in the bottom row. The black central core can move up or down, and that moves those three columns along with it. The middle four rows lie in two discs, which can rotate. By rotating a disc, the balls in the two rows it contains are moved around to the other columns. The aim is to place four balls of the same colour in each of the five columns, with the three black balls filling up the top of the longer columns.

Although this puzzle looks similar to the Ivory Tower, the Whip-It and the Missing Link, those puzzles have a gap so you can move pieces individually. The Nintendo barrel only allows you to move no fewer than 10 balls at a time which makes it rather difficult.

The puzzle was invented at Nintendo by Gunpei Yokoi (who also invented/designed the Gameboy). The US patent, which also covers the Trillion puzzle, was filed on 30 December 1980 and granted on 15 March 1983 (US 4,376,537 and also design patent D267,661).

In the far east it has been produced again recently. The small bit of paper supplied with it has the following
amusing Japanglish text, quoted verbatim:

Cylindric Revolving Puzzler of good quality and fine appearance, provides educational value and also suit for decoration and giveaway in business. It holds colored beads in red, orange, yellow, green, blue, black etc, which present you 10 billion combinations.

Cylindric Revolving Puzzler original type is laid three black bead at bottom, the second layer holds 4 same color bead in a row and spread 5 different rows arranged clockwise in the roder of blue, green, yellow, orange, red. Mix the beads up, then here you start the game!

It may firstly beat your brains quite a bit. Yet you may try to arrange for a row at beginning, then two, three rows to four, four rows. You will finally grasp secret from this challenge and be in enchanted with it.

If your browser supports JavaScript, then you can play the Nintendo Ten Billion Barrel by clicking the link below:

JavaScript Nintendo Ten Billion Barrel

The number of positions:

There are 23 balls which gives a maximum of 23! positions. This limit is not reached because:

- the pieces of each colour are indistinguishable (3!·4!·5!)
- the colours are equivalent (5!)

This last factor comes about because it does not matter which colour goes in which column. This leaves 23! / (3!·4!·5!·5!) = 4,509,264,634,875 or 4.5·10^{12} positions. This far exceeds the 10 billion figure on the packaging.

If all the balls were different then, according to David Singmaster (in Cubic Circular Issue 2) all 23!/2 even permutations of the balls are achievable, even if the two discs are locked together. By turning the puzzle upside down, all odd permutations can also be achieved.

Links to other useful pages:
Steve Martin's page has a partial solution to the puzzle.
This page has a very sketchy solution.
Spanish puzzle site with solutions to the cube, magic, square 1, and Nintendo Billion Barrel here.

Notation:

The central core of the barrel will be assumed to be in its upwards position, the way it is when the puzzle is put down. The letters T and B will be used for moves which rotate one of the two discs, the Top or Bottom one. If the core is up, then Ti or Bi mean moving a disc one step to the left (clockwise when viewed from above), whereas two steps (i.e. 2/5 of full circle) will be shown as Tii or Bii. Similarly right turns are Tr, Br, Trr, Brr. If these same moves are performed while the centre core is pushed down, then small letters are used, e.g. tl, tll, trr, tr, and bl, bll, brr, br.

As said before, normally the central core will be held in the up position. The rows are counted from the bottom up, so the bottom row is called the first row, and the top row which has only three balls is the fifth row. The puzzle is always held in such a way that the two short columns are at the front left and right. One of the three long
columns is directly in front, and the two adjacent ones at the back. Lets label the columns too from A to E; the back left column is column A, the front column is C, the back right column is E. The position of each ball can then be described easily, for example C5 is the ball at the top of the front column.

Solution 1:

This is a simple solution by Pedro Luis that can be easily memorised.

This solution uses mostly just 3 very easy move sequences (and their inverses).

**9-cycle: (tr br) (Tl Bl), repeated 11 times.**

The 9 balls in columns B and C are cycled clockwise two steps, as shown in the diagram 1.

**Top 5-cycle: tr Tl, repeated 7 times.**

The 2 top balls of B and the 3 top balls of C are cycled clockwise two steps, as shown in the diagram 2.

**Bottom 5-cycle: br Bl, repeated 7 times.**

The 2 bottom balls of B and the 3 bottom balls of C are cycled clockwise two steps, as shown in the diagram 3.

**Phase 1:** Put black balls at A5 and E5.

  a. Use the move sequences above (or any other method) to bring two black balls one on top of the other in the same column (but not at A5 or E5). This can often be done intuitively, but if you can't, one sure way to do this is as follows:
      1. Put the two balls in adjacent columns, and rotate the disks so they are in columns C and D. This is always easy.
      2. Use the move sequences (e.g. the 9-cycle) to bring one black ball into position C1 (bottom of columns C).
      3. Do Tll Bll, so that one black ball is at A1, the other in column B.
      4. Use the move sequences (e.g. the 9-cycle) to put the second black ball at B1.
      5. Do Br (so the balls are at C1, D1)
      6. Do br Bl, to stack the balls up in one column.
  b. Rotate the disk(s) so that the pair of black balls lies in column D.
  c. Repeat (tr br) (Tl Bl) until the black balls are in position at A5 and E5.

**Phase 2:** Make column A of one colour.

  a. Choose a colour to solve first.
  b. If there is no ball of that colour in column B or C, then do the following:
      1. Do Tl Bl, shifting the columns left. There should now be at least one ball of your colour in column C.
      2. Do the 9-cycle until there is a ball of your colour in column B.
      3. Do Tr Br, shifting the columns back to the right.
  c. Repeatedly do the 9-cycle until B1 has a ball of the chosen colour.
  d. Put the ball in place at the bottom of column A as follows:
      1. Do Tr Br, shifting the columns right. The ball you are adding lies at C1, and any previously solved balls of that colour are at the bottom of column B.
      2. Do the 9-cycle five times (or its inverse 4 times) so that the ball has been inserted at the bottom of column B.
      3. Do Tl Bl, shifting the columns back to the left.
  e. Repeat steps b-d until column A is solved.
Phase 3: Solve two more columns.
   a. Do Tl Bl, so that E is the solved column.
   b. Do phase 2 again, solving column A with another colour.
   c. Do Tl Bl, so that D and E are the solved columns.
   d. Do phase 2 once more, solving column A with a third colour. This leaves only columns B and C.

Phase 4: Solve the last two columns.
   a. Choose which colour you want column B to become.
   b. Bring the four balls of that colour into the bottom two rows. The easiest way to do so is as follows:
      1. Do the Top 5-cycle until position C3 is of the chosen colour.
      2. Do the Bottom 5-cycle 3 times (or its inverse 4 times) to bring that ball downwards to C2.
      3. Repeat steps 1-2 until all balls of the chosen colour lie in the bottom two rows.
   c. Do the Top 5-cycle until the black ball lies at B3.
   d. Do the 9-cycle to put all the pieces in their correct positions.

Solution 2:

This solution will solve the Nintendo barrel row by row. It is based on the solution by James G. Nourse in "Simple Solutions to Cubic Puzzles".

In the first two phases the bottom two rows will be solved. There are a few useful sequences that move balls from row to row, without disturbing the bottom two rows. For example, tr Tl, moves C5 to C4, and C4 to C3. If you look carefully, you will see that you can use very similar sequences (that do not involve turning the bottom disc) to move any ball in rows 4 or 5 down to row 3.

One other useful move sequence is:
**Double Swap: bll Tll brr (Trr)**
This sequence swaps B2 and B3, and also A3 and C3. The last move in the sequence is usually not necessary because we will only use it when building the first two rows. You can use it to lift a ball from row 2 up to row 3 in column B, and vice versa, to drop a ball from row 3 to row 2. Of course, if you need to lift a ball in another column from row 2, then you can turn the bottom disc to bring it to column B, do the double swap, and return the bottom disc to its original position.

Phase 1: Solve the first (bottom) row.
   a. Consider which colours you want in each column. If you do not care which order the columns are in, and the first row already has all five different colours, then you can skip this phase completely.
   b. Turn the bottom disc so that A1 has a non-black ball. Consider that ball to be the correct colour for column A. Remember that when you have solved the puzzle, you can always move the two discs around to bring that colour to any other column anyway. Let a to e denote the colours that the columns A to E will eventually have, i.e. ball A1 already has colour a.
   c. Do Br, to bring the first ball to B1.
   d. Bring a ball of colour e to B2 as follows:
      1. Find a ball in row 3 of colour e. If there is none on row 3, then you can bring one to row 3 using the methods discussed above.
      2. Turn the top disc to bring the ball to B3.
      3. Do the double swap to bring it to B2.
   e. Do Brr, bringing the first two balls to D1 and D2.
   f. Bring a ball of colour c to B2, in the exact same way as before.
   g. Do Br bl. This puts the three balls in positions E1, D1 and B1.
   h. Bring a ball of colour b to B2, just as before.
   i. Do Bll.
   j. Bring a ball of colour d to B2, as before.
   k. Do Bll bl. The bottom row should now be correct.

Phase 2: Solve the second row.
a. Turn the bottom row until B1 and B2 have different colours.
b. Find a piece of the same colour as B1, and bring it to B2, using the exact same method as was used in the first phase.
c. Repeat steps a and b until the bottom two rows are solved.

Phase 3: Solve the third row.

a. Turn the top disc to match as many of the balls in the third row with those on the second row.
b. Find a column where the third row has the wrong colour ball. Turn both discs to bring that column to the front, to column C.
c. Find a ball that belongs at C3. There are 12 possible positions it could be in. Do the relevant sequence below:

<table>
<thead>
<tr>
<th>Column</th>
<th>Sequence</th>
</tr>
</thead>
<tbody>
<tr>
<td>A3</td>
<td>tll Til tlr tr Tr tll Ti tll</td>
</tr>
<tr>
<td>B3</td>
<td>Ti tl Til tl Ti trr Trr tll</td>
</tr>
<tr>
<td>D3</td>
<td>Tr tr Trr trr Tr tll Til tll</td>
</tr>
<tr>
<td>E3</td>
<td>trr Trr trr Trr trr Trr tr</td>
</tr>
<tr>
<td>A4</td>
<td>tr Trr trr Trr trr Trr trr</td>
</tr>
<tr>
<td>B4</td>
<td>trr Ti tl Til tr Til tll</td>
</tr>
<tr>
<td>C4</td>
<td>trr Tr trr Tr Tr tll Trr tll</td>
</tr>
<tr>
<td>D4</td>
<td>tll Tr trr TIl trr Trr tll</td>
</tr>
<tr>
<td>E4</td>
<td>tl Ti tl Til tl Til tl</td>
</tr>
<tr>
<td>A5</td>
<td>Trr tr Til tl Til Tr tr TIl</td>
</tr>
<tr>
<td>C5</td>
<td>tll Til tr Til tl Ti trr</td>
</tr>
<tr>
<td>E5</td>
<td>Til tl Trr trr Trr trr Ti tl Tr</td>
</tr>
</tbody>
</table>

d. Repeat steps b and c until the third row is correct.

Phase 4: Solve the fifth row.

Repeat the 3 steps below until all black balls are in the top row.

a. If A5 is not black, then turn both discs so that a column containing a black ball is in column D. To move the black ball from D4 to the top row, do the sequence:

\[ tll Tr tr Tr tr tll Tr tll \]

The black ball will not necessarily go to position A5, but it will go in the top row.
b. If C5 is not black, then turn both discs so that a column containing a black ball is in column B. To move the black ball from B4 to the top row, do the sequence:

\[ tll Til tr Tr tll Tr tll Tr tll Tr tll \]

The black ball will not necessarily go to position C5, but it will go in the top row.
c. If E5 is not black, then turn both discs so that a column containing a black ball is in column B. To move the black ball from B4 to the top row, do the sequence:

\[ trr Ti tlr Tr tll Ti tl Trr trr \]

The black ball will not necessarily go to position E5, but it will go in the top row.

Phase 5: Solve the fourth row.

a. Only the fourth row is left, and this can be solved quite easily by swapping the balls around. Find two balls on the fourth row which when swapped will put at least one of them in the correct column.
b. Turn both discs to bring the two columns to column C and one of A or B.
c. To swap B4 and C4, do:

\[ trr Ti tl Tr r brr Ti trr Trr bll Trr \]

To swap A4 and C4, do:

\[ tr Ti Bl bll TIl brr Til tll Br \]

Solution 3:

This solution will mostly solve the Nintendo barrel in columns. Most move sequences in this solution were found by computer.

Phase 1: Solve the black balls (the top row).
a. If A5 is not black then turn the discs to bring a black ball in column A, and repeat **Br Tr bl tl** until it is at A5.

b. If C5 is not black then turn the discs to bring a black ball in column C. Alternate the sequences **Tr tl** and **Tl tr** until it is at C5. Note that the first sequence moves A5 to E5, and the second moves it back again. If the first black ball ends up at E5 when C5 is solved, then do **tl** to prepare for the next step.

c. If E5 is not black then turn the discs to bring it to column D. Alternate the sequences **trr brr Til Bill** and **til bll Trr Brr** until it is at D4. Note that the first sequence moves A5, C5 to C5, E5 and the second moves them back again. If the black balls are now at A5, C5, D4 then do **trr** to solve them. If on the other hand they are at C5, E5, D4 then they can be solved by **Til tl**.

**Phase 2:** Solve the columns.

a. Decide what colour you want each of the columns to be. Turn the discs to bring as many balls as possible into their correct columns.

b. Find a ball that is in the wrong column. Find the column it should be, and find a ball in that column that is also wrong (preferably belonging in the first column). These two balls will be swapped.

c. Turn both discs together so that the two balls lie in column C and column A or B.

d. To swap the two balls, do the relevant sequence below:
   - **A1, C1:** Bll br Til br Trr Bl br Til br Tr
   - **A2, C1:** Til Brr brr Bl bll Brr br Bll br
   - **A3, C1:** Bl bll Bl bll Til bll Br brr Tr
   - **A4, C1:** tr Til br Bl Til br Br bl Til tl
   - **B1, C1:** Bl brr Br bll Br Tr bl brr Br Br Bl
   - **B2, C1:** Trr Bll br Ti Bl bl Tr bll Tr bl
   - **B3, C1:** Til br Til Bl br Br bl Ti
   - **B4, C1:** Tr tr Trr br Til Bl bl Br bl Ti
   - **A1, C2:** bl Trr bl Bi Til bl Trr bl
   - **A2, C2:** Br bl tl Til bll Trr tr bl Bil
   - **A3, C2:** Bl brr Tr bl TII bl
   - **A4, C2:** Bl tr br Tr br Ti tl bll Br
   - **B1, C2:** Trr brr Bl bll Bi brr Bi bl Tr
   - **B2, C2:** Bl bll Ti brr Br bll Tr brr
   - **B3, C2:** Bl bl Br Trr bl Bi Tr br
   - **B4, C2:** Bi Ti tr Til bll Trr tl Til brr Trr
   - **A1, C3:** Br Til bll Bl brr Trr bll Bl brr Bl
   - **A2, C3:** Bll Trr bll Ti br Trr bl
   - **A3, C3:** bll TII brr Trr
   - **A4, C3:** Trr ti Til brr Trr tr bll Til
   - **B1, C3:** Br Til bll Tr Br bl Bi bl Bi
   - **B2, C3:** Trr til Trr bll Til trr brr Til
   - **B3, C3:** Tr bll Trr bll Til bl Til
   - **B4, C3:** Til tl Trr bll Til tr brr Trr
   - **A1, C4:** Bll tl brr Til Br bll Trr tr brr Bl bll Brr
   - **A2, C4:** Trr Bll tl bl Ti bl Tr tr brr Bi
   - **A3, C4:** tr Trr Brr bll Til tl brr Bll
   - **A4, C4:** Ti Bll tl bll Ti tl brr Til Blil tl
   - **B1, C4:** Br tl Til bl Tr Br bl Bi tr br Bl
   - **B2, C4:** tl Til bll Trr tr Til brr Trr
   - **B3, C4:** Til Br tr brr Til bl bll Trr tl
   - **B4, C4:** Ti tll Tr ti Til tl Til tl Trr tr

Note that these sequences will mix up each column, but only the two chosen balls will move to a different column.

e. Repeat steps b-d until all the columns are solved.

**Other sequences**

1. Column swaps: (Calculated by Mike Godfrey)
Swap A,C: br Bl br Bl bll tr Tr brr Tl brr Bll bl tl
Swap B,D: Tr bll tl Trr bll Bl Tll bl Tr br b1 Ti bl tr Tl
Swap B,E: Tr br Br bl Brr bll tr Ti bl Brr Tr brr tl Brr Tl
Swap A,B: trr Bll Tl bl Ti tl Tr trr Tll brr Br bll Trr bll Tl
Swap B,C: Br bll Tr bll trr Tr tl Blr Tr bll Brr Tl br Tll trr Tll
Swap A,E: tr Bl Tr bl Ti brr Br brr tll Tl bl Tr tr Bl

Pretty patterns:

a. Spiral:
   bll Tr bll Tr bl Bl Tl br Bl
b. Zig Zag:
   Do Spiral followed by Brr.
c. Flag:
   bll Trr Brr tll brr Trr tl br Trr Bl tr br Tr Br brr
d. Topsy Turvy (is similar when turned over).
   Tr tll Tr tr Tl tl Tr tr Tll