Varikon Box 2×2×2 / The Minus Cube

The Varikon Box 2×2×2 puzzle is a 2×2×2 sliding cube puzzle, similar to the 3×3×3 versions Peter's Black Hole, iQube, and Magic Jack. It is a transparent box containing 7 cubes and a space. The cubes can easily be moved by tilting the box.

The puzzle pictured above has white cubes, and each side has a red or blue spot. When solved, all the blue spots are on the outside, with only the space showing three red spots. There is also a version made of red plastic with white and blue spots.

There is a tiny hole near one corner of the box, into which you can insert a small rubber pin. This prevents the puzzle from getting mixed up accidentally.

One of the earliest versions of this puzzle is the BloxBox, invented by Piet Hein around 1972.

The Minus Cube is a Russian version of this puzzle with a different colour scheme. Its 7 cubes are identical, all having three white faces (of which two are opposite each other) and three faces of another colour. Their orientations are not all the same, and the aim is to arrange it so that whichever side of the puzzle you look at you will see 4 squares that match in colour.

There are at least two versions of this puzzle. The red and white one was made in Moscow, and the blue and white version in Sverdlovsk (now Yekaterinburg). The latter is more difficult as it has fewer solutions. There also seems to be a black and white one, but I don't know what pattern it uses.
The Varikon Box was invented by Csaba Postasy, Gabor Eszes, and Miklos Zagoni. The German version of its patent, DE 3,027,556, was published on 19 June 1981.

If your browser supports it, you can click on the link below to play with a Javascript version of the Varikon Box 2×2×2.

Javascript Varikon 2×2×2

The number of positions:

There are 7 loose cubes and a space. Any even cube permutation is possible, but no odd permutations, so for any position of the space there are 7!/2 cube positions. This gives 8·7!/2 = 20,160 positions for the Varikon Box. It has only one solution.

The red Moscow Minus Cube has 3 identically oriented pieces, so it has 8·7!/3! = 6,720 positions. It has 16 solutions. The blue Sverdlovsk Minus Cube has two pairs of identically oriented pieces, so it has 8·7!/(2·2) = 10,080 positions. It has only 2 solutions.

Below is a table that shows how many positions there are at each distance from the starting position of the Varikon Box. It shows that no position needs more than 19 moves to solve.

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<th>Depth</th>
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</tbody>
</table>

Links to other useful pages:

Kvant magazine article, 1988, N10, p.66. Russian article about the Minus Cube, which unfortunately contains a number of mistakes in its analysis of the solutions.

Minus Cube Game, a nice simulation of the puzzle for Windows computers.

Solution to the Varikon Box 2×2×2:

There are three observations that will greatly simplify the solving process.

1. It seems like this puzzle has two solutions - blue on the outside, or red on the outside. This is not so, as only one of these can be solved. Trying to put the wrong color outside will leave you with a position where two cubes need to be swapped, and that is impossible. The version shown above should have blue on the outside.

2. The small cubes have different colors on opposite sides. This allows you to easily determine what the colors are on a cube's hidden sides, and therefore to determine where in the solved position that cube belongs.

3. The small hole in the outer box shows which corner will be empty in the solved position.

   a. Find the little hole in the outer box. This is the location where the space will be when the puzzle is solved.
Hold the puzzle so the hole is at the top front right corner.

b. Put the correct cube at the corner opposite the little hole, at the bottom back left corner.

c. Put the correct cube at the bottom back left corner without disturbing the previous cube.

d. Put the correct cube at the bottom front left corner without disturbing the previous two cubes.

e. If the bottom front right cube is already solved, then skip this step. Otherwise, put the correct cube at the bottom front right corner as follows:
   1. Find the cube that belongs at the bottom front right corner.
   2. Move the top layer so that that cube lies at the back.
   3. Move the bottom front left cube to the right, so that it lies at the bottom front right.
   4. Move the top layer so that the cube belonging at the bottom front right lies at the top front right, and that the space is at the top front left.
   5. Move the bottom front right cube back to the left into its correct place, and then the top front right cube down so that now the whole bottom layer is correct.

f. Move the top layer until their cubes are in their correct locations. If you can get only one correct (the other two need to be swapped) then this position is not solvable. You will have to start solving again from scratch, but with the other colour on the outside.

Solution to the Minus Cube:

The Moscow Minus Cube has 16 solutions, two for each location of the blank space. To make this puzzle more difficult, you can choose which corner to put the space before starting the solve.

Here are four solutions that are only a few moves away from each other.

If you have chosen where to put the blank space, exactly one of these four solutions is possible. You can determine which one by looking at the orientation of the three identical pieces, which is different in each solution. The same solutions can also be done in mirror image:

The other eight solutions are essentially the same but with the box turned upside down. Unless the box is marked in some way, it is not possible to distinguish these from the above solutions, but it does make it easier to solve.

The Sverdlovsk Minus Cube has 2 solutions which are mirror images of each other. You can get from one solution to the other by sliding just three cubes.
If you were to open the box and to reorient the cubes inside, there are only 38 ways to do this that have a solution (up to rotation, mirror images are considered distinct). Some of these are mirror images of each other, or are the same if their colours are swapped.