The Allies Flag Puzzle

a.k.a. The Allied Flags Puzzle

Valentine & Sons, Ltd., Dundee, British Manufacture, circa 1918.
(1 x 1 x 4.4 inch cardboard box and five paper covered 13/16” wood cubes;
labels from Henry's of Manchester are on side and bottom of box;
similar in name and theme to the Allies Flags Puzzle)

The cube sides have flags from World War I (British, French, Belgian, Japanese, Russian). Like the four block Instant Insanity puzzle, the object is to arrange the five cubes in a line so that each side has no duplicate flags. Here are the directions that came with it:

Here are two solutions and their corresponding graphs (constructed in the same way as for Instant Insanity), where in the first both cycles use the same Hamilton path, but in the second the first cycle follows a different route:

cycle 1: BR - 2 - BE - 5 - F - 1 - J - 3 - R - 4 - BR
cycle 2: BR - 3 - BE - 4 - F - 2 - J - 5 - R - 1 - BR

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