Rubik's 3x3x3 Cube

*Patent filed by Erno Rubik 1975, sold by Ideal Toys in the 1980's.*
(plastic with colored stickers, 2.2"; keychain 1.2")

The original twisty cube. Difficult, but fun to play with. One can rest at any time and pick it up later. The first puzzle of this type in a large class of puzzles in the years to follow. A number of ways to construct this puzzle have been devised over the years; here are the pieces of an original Rubik's Cube like shown above, where there is a central axis assembly and 20 pieces that interlock with it.

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Rubik 3x3x3 Six Step Solution

Notation: L (left), R (right), F (front), B (back), U (up), D (down) for 90 degree clockwise rotation of that face; a – means counterclockwise and a 2 means do it twice. Corners are named with three letters and edges are named with two letters (e.g., FR means looking at the front, it is the edge on the right).

1. Solve the down layer.

2. Solve the middle layer:
   Rotate the middle so centers are correct, and then use the edge mover sequence to move edges between the up and middle layers until the middle is solved; if an edge first needs to be flipped, move it be FU and do the edge flipper sequence of Step 3 (the edge will end up flipped at position UR, and you can rotate the top to move it back to FU):

   \[ \text{edge mover, FU -> FR: } \left( U \, R \right) \left( U- \, R- \right) , \left( U- \, F- \right) \left( U \, F \right) \]

3. Flip the up edges so they all have the correct color on top:
   If no up edges have correct top color, first do the edge flipper. Now position the cube so UL has correct top color and UF does not, and do the edge flipper at most two times.

   \[ \text{edge flipper: } F \left( R \, U \right) \left( R- \, U- \right) \] \[ F- \]

4. Move the up layer edges to their correct positions:
   As needed, re-position the cube and use the edge swapper sequence.

   \[ \text{edge swapper, UF<->UL: } \left( R \, U \right) \left( R- \, U \right) \left( R \, U^2 \right) \left( R- \, U \right) \]

5. Position the up layer corners:
   The corner cycle sequence leaves UFR alone and cycles the other three counterclockwise. Identify one corner that is correct (but may be rotated), or if there is not one, do the corner cycle. Then re-position the cube so the correct corner is UFR, and then do the corner cycle one or two times to make all corners correct.

   \[ \text{corner cycle: } \left( U \, R \right) \left( U- \, L- \right) \left( U- \, R- \right) \left( U- \, L \right) \]

6. Rotate the up layer corners:
   Position the cube so UFR is not correct and repeat these two steps until all corners correct:
   A. Repeat the corner rotator until the UFR corner is correct:

   \[ \text{corner rotator: } R- \, D- \, R \, D \]
   B. Rotate the up layer so that UFR is incorrect.

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**Remembering The Basic Rubik 3x3x3 Six Step Solution**

Each sequence has a natural rhythm, but an easy mistake is to start off wrong. The Edge Mover and Corner Cycle start with U, the Edge Flipper (after parking the F) and the Edge Swapper start with R. To avoid forgetting your place, run the sequence in your head, or simply count 1,2,3,4,... as you go; two sets of 4 for the edge mover, 4 between the F's of the edge flipper, etc.

*Edge Mover (for Step 2):*

  - *edge mover, FU -> FR:* \((U \ R) \ (U\ - \ R\ -) , \ (U\ - \ F\ -) \ (U \ F)\)

    Mixes the up layer but leaves the lower two layers unchanged except replaces FR with FU. Think of it as two pairs, the first starts with U-R, and the second with U-F.

*Edge Flipper (for Steps 2 and 3):*

  - *edge flipper:* \(F \ (R \ U) \ (R\ - \ U\ -) \ F\ -\)

    Mixes the up layer and flips UF (leaving it in the UR position). Repeat at most three times to make all up edges have correct top color; if at least one up edge is correct already, start with the cube positioned so that UL is correct. Think of this as "parking" the front with F, doing \((RU) \ (R-U)\), and then "unparking" the front with F-.

*Edge Swapper (for Step 4):*

  - *edge swapper, UF<->UL:* \((R \ U) \ (R\ - \ U) \ (R \ U^2) \ (R\ - \ U)\)

    It's R-R-R-R interleaved with U U U^2 U. Starts with R, the R's alternate + and -, and the U's keep going clockwise, where the third is 180 degrees.

*Corner Cycle (for Step 5):*

  - *corner cycle:* \((U \ R) \ (U\ - \ L\ -) \ (U \ R\ -) \ (U \ L)\)

    It's U U U L- U- interleaved with R L- R- L. Starts with U, the U's alternate + and -; remember that R and L- come first, and the next two must be R- and L.

*Corner Rotator (for Step 6):*

  - *corner rotator:* \(R\ - \ D\ - \ R \ D\)

    Exchanges UFR and DFR; repeat 6 times to restore the cube. Step 6A does it 2 or 4 times. Also exchanges DBL and DBR; cube is mixed during Step 6, but lower layers will be solved again once the up layer is. Always complete the sequence before doing Step 6B; it is easy to forget the final D when you see the correct color on top.

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Speeding Up the Basic Rubik 3x3x3 Six Step Solution

The basic six step solution is not the fastest for speed cubing, but it can be fun to do it faster:

**Step 1:** Solve the up layer and turn the cube over for the remaining steps. If needed, move the solved portion out of the way, flip a piece, put it into position, and rotate the solved portion back. To help with quickly locating pieces, pick a color for the first face that works best for you.

**Step 2:** Instead of using the edge flipper, learn the symmetric sequence that moves an edge down counterclockwise from up to middle:

\[
\text{edge cc-mover, UF -> MFL: (U- L-) (U L) , (U F) (U- F-)}
\]

**Step 3:** Before the final F-, if the right side of FR is not the top color, instead of wasting time to do F- F, repeat the (R U) (R- U-) before doing F-.

**Step 4:** Do nothing if all edges are correct, or the standard Edge Swapper if just two adjacent edges need to be swapped. Otherwise, because omitting the last move of the Edge Swapper leaves UF unchanged and cycles the other three counterclockwise, start by rotating the top to make UF correct. If UB is also correct (but UL and UR are reversed), do the first 7 Edge Swapper moves and then swap UL and UB. The other case is that UL,UB, UR need to be cycled; do the first 7 moves to cycle them counterclockwise. For clockwise, do the first 7 moves twice, or it is faster to reverse the 7 moves (easy to remember as the same R R- R R-, but interleaved with U^2 U- U- instead of U U U^2).

\[
\text{clockwise cycle UL, UB, UR: (R U^2) (R- U-) (R U-) R-}
\]

**Step 5:** If no corners are correct, learn how to tell for which orientation of the cube the corner cycle will leave things so that a counterclockwise cycle will be needed. Or, if you have identified a correct corner and a clockwise cycle of the other three is needed, instead of doing the corner cycle twice (three times returns the cube to where it was), save time by reversing the sequence:

\[
\text{reverse corner cycle: (L- U) (R U- ) (L U) (R- U-)}
\]

**Step 6:** Every iteration of the corner rotator exchanges UFR and DFR, and repeating it 6 times returns the cube to where it was. Step 6A will use the corner rotator 2 times if the top color is on the right side of the UFR corner, or 4 times if it is on the front, in which case it is faster to do the reverse sequence 2 times (easy, start with D- instead of R- and everything follows):

\[
\text{reverse corner rotator: D- R- D R}
\]

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The Corner Rotator - Why Step 6 Works

Step 6 is the same as Step 3 of the solution presented for Rubik's 2x2x2; we repeat the explanation given there for why it works:

- Step 6A affects only 4 corners by exchanging two front right corners and also exchanging the two back down corners.
- Doing Step 6A twice leaves corners in the same positions, except those four corners are rotated, and doing Step 6A six times leaves the corners the same as when you started.
- On the up layer Step 6 only modifies the front right corner.
- Since Step 6 started with the down corners correct, once three of the four up corners have been fixed, fixing the fourth up corner must leave the down layer correct. This is because when at every 6th move the two back down corners are correct, all that is left that could be incorrect are the two front right corners, but due to parity considerations, a completely solved puzzle except for two adjacent rotated corners is not possible (however, although not hard to overcome, this is not true for the Rubik 3x3x3 Void Cube).
- The only edge pieces that are affected are FR, RD, BD, which are on the lower two layers; they return back to where they were after 6 moves.

Using The Corner Rotator For Step 5

It is interesting to see that the Corner Rotator can be used for Step 5, and if you decide not to go the route of trying to solve faster, similar as how used in the alternate solution for Rubik 2x2x2, an alternate Rubik 3x3x3 solution can eliminate the need to memorize the Corner Cycle by using the Corner Rotator to do Step 5:

5 (alternate). Position the up layer corners:

Let S be the sequence of Step 6A, and let Z be the sequence S S S- S- (which does nothing) interleaved with rotating the up layer 360 degrees with U U U^2:

\[ Z = S \ U \ S \ U \ S- \ U^2 \ S- \]

Alternate corner cycle: \( Z = S \ U \ S \ U \ S- \ U^2 \ S- \)

Z does a counterclockwise cycle of UFR, UBR, UBL; repeat it until at least one up corner is correct (but may be rotated), re-position the cube so this corner is UFL, and then continue repeating it until all up corners are in their correct positions.

Although this sequence is relatively long after expanding each S to the corresponding four moves, it is relatively easy to remember as the interleaving of two do-nothing sequences. In addition, if you forget what it does, a pencil and paper can be used to draw what happens to the up layer; see the explanation and diagrams presented for the Rubik's 2x2x2 alternate solution.
Solving Rubik's 2x2x2 With 3x3x3 Sequences

A completely solved 3x3x3 cube except for two adjacent corners exchanged is not possible due to parity considerations; that is, if just two adjacent corners are interchanged, then it must be that the edges are not completely solved. However, this is possible for the Rubik's 2x2x2 cube.

If you are fast with the Step 5 corner cycle sequence, and don't want to bother remembering the sequence to exchange two corners of a 2x2x2 cube (which corrupts edges when used with the 3x3x3 cube), Steps 1, 5, and 6 will solve the 2x2x2 cube, by using the corner cycle appropriately for Step 5:

**Single Swap, UFL <-> UFR:**

The single swap is all that is needed, since it can be used 3 times for a diagonal swap and twice for a double swap. However, by using all three variations shown above, it is at most 9 moves for a single swap (8 moves for the corner cycle plus the final U) or 16 moves for the diagonal or double swap. Note each time a corner sequence or reverse corner sequence is done, the cube first needs to be repositioned so that the corner that does not move is in the UFR position. If you don't want to remember the reverse corner cycle, the second sequence of the double swap can be a standard corner cycle on D,B,A followed by a U^2; however, counting that final U^2 as two moves, it is no fewer moves than doing two single swaps.
A Corners-First Rubik's 3x3x3 Solution

Here is a different approach that starts with solving the corners, then the top and bottom edges, and finally the middle edges.

**Notation:** L (left), R (right), F (front), B (back), U (up), D (down) for 90 degree clockwise rotation of that face; a – means counterclockwise and a 2 means do it twice. Corners are named with three letters and edges are named with two letters (e.g., FR means looking at the front, it is the edge on the right).

1. Solve the corners using a solution for Rubik's 2x2x2.

2. Position up and down edges by moving to and from the middle layer:
   A. Cycle edges between the middle and up layers to get three up edges correct:
      
      \[\text{RB} \rightarrow \text{FU}, \; \text{FU} \rightarrow \text{FD}, \; \text{FD} \rightarrow \text{RB}: \; \text{F M F}^{-}\]
      
      That is, repeatedly position the cube so that the edge to be moved is RB, rotate the U layer so that where you want to move it to is FU, and cycle.
   
   B. Turn the cube over, and repeat Step A.
   
   C. Move the edge that goes to FD to the FU position; then move final edge to FU.

3. Use this to flip up and down edges:
   
   *Flip the UF edge:* \[F^{-} \; M \; (F \; M)^{2} \; F^{-}\]

4. Use rotations of the middle layer and these sequences to position middle edges:
   
   *Front back swap,* LF <> LB, RF <> RB: \[(R^{2} \; M^{2})^{2}\]
   
   *Clockwise cycle,* RF -> LB -> RB -> RF: \[(R^{2} \; M) \; (R^{2} \; M^{-})\]

5. Use this to flip middle edges (for right to left diagonal, do \(B^{2}\) before and after):
   
   *Flip RF and RB:* \[(R \; M^{-})^{3} \; R \; M^{2} \; R \; (M^{-} \; R)^{3}\]

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Rubik's 3x3x3 Cube - 25 Years Later

Plastic, stickerless, made in China, purchased from Amazon.com in 2015.
(left: Newisland, sold by YaMiYo, comes in a with a storage bag, 3.25" square;
right: DaYan, sold by Maxin, comes in a fitted box, 2.3" square)

In the early 2000's, smoother working versions of Rubik's 3x3x3 were widely available, with screws / springs for adjustable tension and smooth turning even when layers are not exactly aligned (beveled interior corners in conjunction with the spring action give a minimal degree of automatic alignment). The Newisland cube shown above was a gift from a friend who does speed cubing; it is smooth and quiet, comes with a storage bag and directions, and its literature explains PA plastic lower resistance, anti-popping, and internal construction. The less expensive Da Yan cube shown above has different but similar construction; here are photos of it apart:

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Rubik's 3x3x3 Other Versions

25th Anniversary Cube, 2.2"

Gold Cube, 2.2"

all plastic with no stickers, 2.2"
large all plastic with no stickers, 3.5"

Large Dice Cube, 3.5"
Large Alphabet Cube, 3.5"

White Maze Cube, 2.2"
Yellow Maze Cube, 2.2"

Sudoku Cube, 2.2"
Red Sudoku Cube, 2.2"
Rubik's 3x3x3 Other Versions Continued

- McDonalds, 2.2"
- Chex Cereal, 2.2"
- Jack Daniels, 2.2"
- UPS, 2.2"
- Mickey Mouse, 2.2"
- MatLab, 2.2"
- Small Cube, 1.2"
- Small Shiny Cube, 1.2"
- Dice, 2.2"
- Assembly Cube, 2.2"

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Some Other Rubik 3x3x3 Solutions (In Alphabetical Order)

Beust's Page, from: http://beust.com/rubik

Bieber's Page, from: http://www.ronaldbieber.de/Fun/Rubik


Dedmore's Page, from: http://www.helm.lu/cube/solutions/rubikscube

Dry Erase Board Page, from: http://www.thedryeraseboard.com

Fridrich's Page, from: http://ws2.binghamton.edu/fridrich/cube.html


Jasmine Page, from: http://peter.stillhq.com/jasmine/rubikscubesolution.html

Jeays' Page, from: http://jeays.net/rubiks.htm

Juergen's Page, from: http://www.mathematische-basteleien.de


McFarren's Page, from: http://www.geocities.com/abcmcfarren/math/rc/RubCub0.htm


Nerd Paradise Page, from: http://www.nerdparadise.com/puzzles/333

Olefsky Puzzle Solver Page, from: http://www.puzzlesolver.com

Ortega and Jelinek Corners First Solution Page, from: http://rubikscube.info/ortega.php

Oxford ComLab Text Solution, from: ftp.comlab.ox.ac.uk

Petrus' Page, from: http://lar5.com/cube

Rob's Rubik Repair Page, from: http://www.roobik.com/cgi-bin/rubix/rubix.cgi

Rubiks.com Solution, from: http://www.rubiks.com

Scared Cat Page, from: http://www.scaredcat.demon.co.uk/rubikscube/the_solution.html


Shon's Rubik's Place Page, from: http://www.rubiksplace.com

Still's Page, from: http://peter.stillhq.com/jasmine/rubikscubesolution.html

You Rubik Page, from: http://www.yourubik.com

Some Rubik 3x3x3 Patents


*Sugden Patent*, from: www.uspto.gov - patent no. 6,974,130


Further Reading

*God's Number is 20*, from: http://www.cube20.org

*Kociemba's Two Phase Algorithm and Cube Mathematics*, from: http://kociemba.org/cube.htm

*22 Moves*, from: http://www.springerlink.com/content/q088143tn805k124/fulltext.pdf


*Rubiks.com Page*, from: http://www.rubiks.com


*Rubiks Cube Typesetting with TeX*, from: http://www.ctan.org/pkg/rubik

*Cube Lovers Archive*, from: http://www.math.rwth-aachen.de/~Martin.Schoenert/Cube-Lovers


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