

THE ULTIMATE SOLUTION TO RUBIK'S CUBE

A unique solution using only two series

Requires only 65 moves to solve a scrambled Rubik's Cube

This site also contains solutions to

2x2x2 cubes

3x3x3 cubes with images

4x4x4 cubes

5x5x5 cubes

dodecahedron

The Ultimate Solution to Rubik's Cube

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If you have mastered The Ultimate Solution to Rubik's Cube and also solved scrambled versions of the other cubes given above, then perhaps you are ready for a still more challenging puzzle. In theory one can design a true four dimensional analog of Rubik's Cube (3x3x3x3). While one cannot make a working model of this puzzle in the real world, one can project its image into three dimensions which can then be displayed, manipulated and solved using standard 3D computer graphics. You can find [MAGIC CUBE 4D](#) on the web. It was designed and constructed over a period of several years by Daniel Green and Don Hatch with later help from E. Jay Birkenbilt.

Check it out!

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THE EDGE PIECE SERIES

In this approach to solving a scrambled Rubik's Cube all 12 edge pieces are placed first. The first four are placed in Step One which is straightforward. Some of the others are moved into place with a simple replacement process. The remainder utilize a symmetric four turn series which moves three edge pieces around a corner of the cube.

The red face of the cube in Fig. 1a is front. The red/yellow edge piece at front/top belongs at front/right. We cannot simply rotate the front face clockwise by 90° because we have already placed four blue edge pieces on the bottom face and would not want to move any one of them out of place. But an Edge Piece Series will move the red/yellow, orange/yellow and red/green edge pieces around the front/right/top corner of the cube without moving other edge pieces. (These edge pieces have been numbered #1, #2 and #3.) According to standard notation this series is F R' F' R.

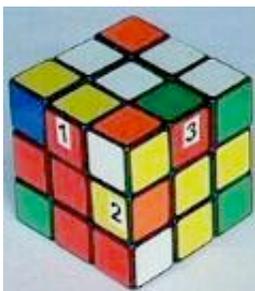


Fig. 1a

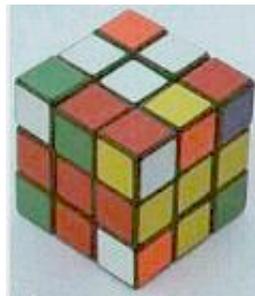


Fig. 1b

Applying this series gives the cube shown in Fig. 1b. We find that edge piece #1 is now in the position originally occupied by edge piece #2. That is, #1 replaced #2. Further, #2 replaced #3 and #3 replaced #1. The three pieces moved about the front/top/right corner of the cube in a counterclockwise direction. The blue edge pieces on the bottom of the cube are undisturbed. Some corner pieces have also moved but that is immaterial. At this time we are interested only in the movement of edge pieces.

We can also describe this series in terms of the movement of edge pieces #1, #2 and #3.

Turn one	Front clockwise	#1 replaces #2
Turn two	Right counterclockwise	#3 replaces #1
Turn three	Front counterclockwise	reverse turn one
Turn four	Right clockwise	reverse turn two

Now let us consider what has happened to these edge pieces. If the front face in Fig. 1b is turned 90° counterclockwise we find that edge piece #1 (red/yellow) has the same

orientation as before. The same is true of edge piece #2 (turn the right face 90° counterclockwise). But edge piece #3 is different. If we turn the top face 90° counterclockwise we find that edge piece #3 has been inverted.

This is always true of these edge pieces. Edge pieces #1 and #2 are not inverted while edge piece #3 is inverted. You are free to number the three edge pieces in any way you want to. Then, provided you always follow the turn sequence given above, you will find that #1 replaces #2, #2 replaces #3 and #3 replaces #1. Edge pieces #1 and #2 will not invert while #3 inverts.

Look at the cube in Fig. 2a. This is the same cube as in Fig 1a except that the top face has been turned 90° in a counterclockwise direction. We still want the red/yellow edge piece to move into the front/right position. But to be properly placed it must invert. Hence it must be edge piece #3. Since #3 replaces #1 then orange/yellow must be edge piece #1. Green/white then is edge piece #2.

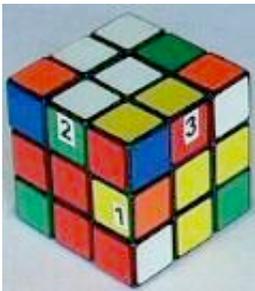


Fig. 2a

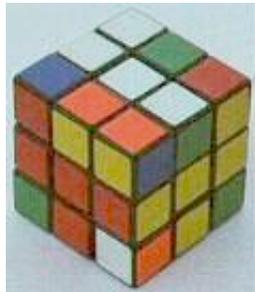


Fig. 2b

We will apply the series as before.

turn one	#1 replaces #2
turn two	#3 replaces #1
turn three	reverse turn one
turn four	reverse turn two

Now examine the result, the cube in Fig. 2b. Again we have accomplished our main purpose, the movement of the red/yellow edge piece into its proper position and orientation at front/right. The orange/yellow piece is involved in both instances but it moves to a different location. However, red/green is involved in the first case while the green/white piece has replaced it in the second series. In the second example, the three pieces have moved in a clockwise direction about the front/right/top corner of the cube.

The red/yellow edge piece may be moved into its correct position/orientation from a position above either the red or the yellow center pieces by an Edge Piece Series. Which version of the series you will want to use will depend on what else you are trying to accomplish. We will visit this situation again in Step Two of the Ultimate

Solution to Rubik's Cube.

Go to the [Corner Piece Series](#)

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THE CORNER PIECE SERIES

The corner pieces are put into proper position/orientation following the placement of the 12 edge pieces. After the edge pieces are placed we will find some corner pieces may already be correctly placed while some are properly positioned but not properly oriented. The remainder will be in the wrong position. The incorrectly placed corner pieces will be moved into proper position/orientation using the Corner Piece Series.

The Corner Piece Series has eight 90° turns and is perfectly symmetrical. The first turn is always a turn of the top face. Such a turn is indicated by a curved arrow on the top face of the cube with the point of the arrow ending on the back part of this face. The first figure in the set of eight below indicates that the top face is rotated clockwise. We will say that this face is turned to the right because the arrow at the back of the top face points to the right. This is simply a convention but it will enable you to more easily recall this series. The symbols below indicate the eight turns in one example of the series. You are looking down on the top face of the cube.



The first move of the top face may be either to the right or to the left, but the rules governing the succeeding moves are the same in either case. The second symbol shows that the top surface of the right face moves away from you.

1. The first move is a turn of the top face, either to the right or to the left.
2. The second move is a turn of the side face toward which the top was turned. It is turned away from you.
3. The top is then turned in the direction opposite to the first turn.
4. The side toward which the top was just turned is then turned away from you.
5. The top is then turned in the same direction as the first turn.
6. The side toward which the top was just turned is then turned back toward you.
7. The top is turned in the opposite direction.
8. The side toward which the top was just turned is turned back toward you.

The top is turned every other turn, alternating directions. Each side face is first turned away and then back toward you. Note that each of the initial turns is later reversed.

This series will cause three corner pieces on the top face of the cube to move about a triangle. If the first turn of the top face is to the right then the corner pieces will move

in a counterclockwise direction about the triangle. If the first turn is to the left then three pieces (two are the same but one is different) will move in a clockwise direction about a triangle which is a mirror image of the first triangle.

Let us apply a Corner Piece Series (with the first move being a turn of the top face to the right) to the cube in Fig. 3a. In this case the orange face is the front of the cube. The result is the cube in Fig. 3b. We can see that the left/front/top, the left/back/top and the right/back/top pieces have been moved. We get another view of the changes by rotating the cube 180° giving us Fig. 3c.

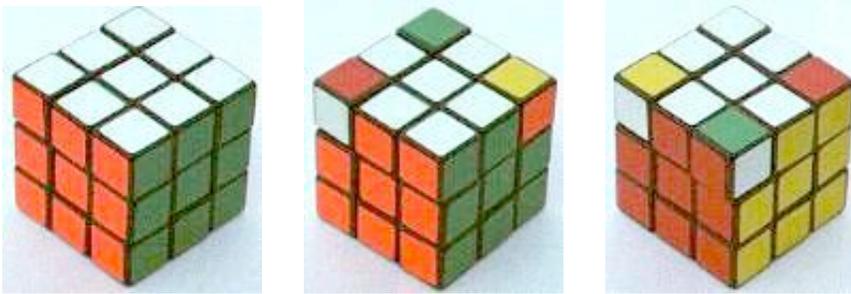


Fig. 3a

Fig. 3b

Fig. 3c

The red/green/white corner piece moved along the red/white edge and into the red/yellow/white corner. In the process it "rolled over" so that the white face is now on the side of the cube and in the yellow face. The red/yellow/white corner piece moved along the yellow/white edge and also "rolled over". The yellow/orange/white piece moved across the diagonal of the top face. In the process it's yellow face, which had been on the left (yellow) side, came to the top.

Note that the two corner pieces on the back of the top face and the corner piece at left/top/front are involved in this triangle. The pieces moved counterclockwise. All three "rolled over".

If the first turn of the top face had been to the left the two pieces at the rear would still be involved but the third piece would be the one in the front/right/top corner. This piece would move across the diagonal to the left/back/top corner. The pieces would move in a clockwise direction about the triangle.

Obviously the two pieces at the back of the top face are always involved along with one of the pieces at the front. How can you tell which one? You should note that the first turn immediately moves the front corner piece involved to the back of the top face. It is this piece which will eventually move across the diagonal of the top face.

If the three out-of-position corner pieces were to move in the opposite direction about the same triangle they would move back into position/orientation. These three pieces illustrate what you will be looking for as you try to solve a scrambled cube.

The red/white/green piece may be made to move along the red/white edge of the cube. As it does so it will roll over into its proper position/orientation. The yellow/white/red corner piece will do the same thing as it moves along the yellow/white edge of the

cube.

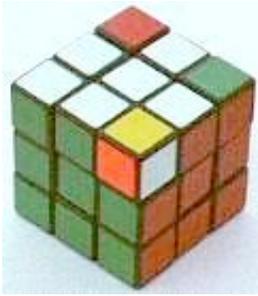


Fig. 3d

The cube has been turned another 90° in Fig. 3d. Now you see an example of a corner piece which can be moved across the face diagonal to its proper position and orientation. For this corner piece, the color of the top face (white) is on the right side. The orange and yellow colors do not match any side which is visible. But you should know that orange is opposite red and that yellow is opposite green. Hence this corner piece has the colors of the opposite corner.

The Corner Piece Series which begins with a turn of the top face to the left will move the orange/white/yellow corner piece across the diagonal with the white face coming to the top. The other two pieces move along cube edges and roll over into their proper position and orientation. The cube is again complete.

Of course you will not often find three corner pieces arranged so that all move into correct position/orientation in a single series. Most of the time you will find one which can move into place along a cube edge. Less often it will be a corner piece which can move across a face diagonal and into its proper position and orientation. At times there will be two pieces which can be moved into place in a single series and rarely three. But if you should find three in such an arrangement you certainly would want to take advantage of that fact.

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STEP ONE: THE CROSS

This solution to Rubik's Cube begins, as many solutions do, with the formation of a cross on one face of the cube. You can choose any color but it is best to choose a face which already has an edge piece matching the center color of that face. The scrambled cube shown in Fig. 4a has three faces for which this is true. I have chosen the blue face but any one of the three could have been chosen. (On my cube, red is opposite orange, yellow is opposite green and white is opposite blue.)

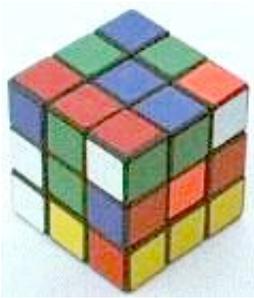


Fig. 4a

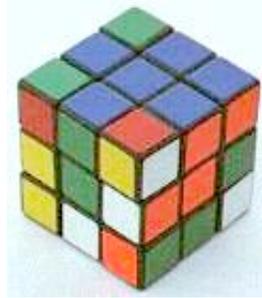


Fig. 4b

Naturally it would be better if two edge pieces had their blue color on top but only if these two pieces are in the proper position relative to each other. More often than not this is not true and one edge piece would have to be changed. In two trials out of three, two will not be better than one.

What is the best way to transform the cube to the one in Fig. 4b? Many solutions would tell you to begin by turning the top (blue) face one turn (90°) clockwise. The blue/green piece would move into its proper place on the cube. These solutions emphasize the bottom face. Target edge pieces are taken to the bottom of the cube, the bottom is turned and the target piece brought to the top in proper position and orientation. That approach would require eight moves to form the cross on the blue face of this cube.

But it is better to emphasize the top face. Edge pieces are moved to the top face so as to be in the proper relationship to top edge piece(s) already on the top face. First, examine the cube. The blue/green edge piece is already on the top face; the blue/red piece is at front/right; the blue/yellow piece is at bottom/front; and the blue/orange piece is at back/bottom.

The blue/yellow piece must be opposite the blue/green piece. In addition, the blue/red piece belongs where the green/orange piece is now. If the blue face is rotated 90° counterclockwise then a 90° clockwise rotation of the orange face will move the blue/red edge piece into place relative to the blue/green piece and a 180° rotation of the green face moves the blue/yellow piece into place, also relative to the blue/green piece.

Three edge pieces are in place on the blue face. We can now rotate the blue face by

180° and then the yellow face by 90° clockwise. Unfortunately this moves the blue/yellow edge piece out of position. After the blue/orange piece is moved into position/orientation by a 90° counterclockwise rotation of the orange face, the blue/yellow edge piece is moved back into place by a 90° counterclockwise rotation of the yellow face. This approach required seven moves.

But it is still better to anticipate. The first move should have been a 90° counterclockwise rotation of the yellow face. This moves the blue/orange edge piece into position so that, following placement of the blue/red and blue/yellow edge pieces, a clockwise rotation of the red face moves the blue/orange piece into its proper position/orientation relative to the other three. Finally, a 180° rotation of the blue face moves all blue edge pieces into place. This approach requires six moves. In standard notation the moves are (back is B):

B' T' R F² L T²

In actual practice, of course, how you proceed to form the cross is up to you. Just make sure that you can do it somehow. With sufficient practice you should be able to reduce the average number of moves required to about seven.

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STEP TWO: CENTER SECTION EDGE PIECES

In Step Two of The Ultimate Solution to Rubik's Cube you will have two objectives which are to be met simultaneously. You will use the Edge Piece Series to move a central section edge piece from the top face into its proper position and orientation in the center section. You will repeat this process until three of the center section edge pieces are in place.

As you carry out this process some of the pieces you will be moving will be top section edge pieces. Make sure you carry out the Edge Piece Series so that, if possible, the top edge pieces wind up with their top color on the top face of the cube. This step is complete when:

- a. three edge pieces are in place in the center section,
- b. three top edge pieces have their top color on the top face of the cube,
- c. one top edge piece is in the center section and
- d. the fourth center section edge piece is in the top section of the cube.

In the beginning it is possible that one or more center section edge pieces are already in place. Fine. But it is also possible that one or more are in the proper position but with the wrong orientation. Tough. You will have to remove it (them), also with an Edge Piece Series.

Remember that a center section edge piece may be moved into proper position and orientation with that edge piece starting above either of the faces with one of the edge piece colors. The orange/green edge piece in Fig. 5a (at top/right) could be moved into place at front/right if it is treated as Edge Piece #1. But this would be a bad idea because the white/red edge piece would be inverted in the process and we would have reduced the number of top edge pieces with their top color on top of the cube from one to none.

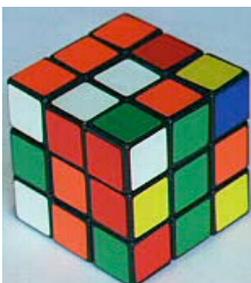


Fig. 5a

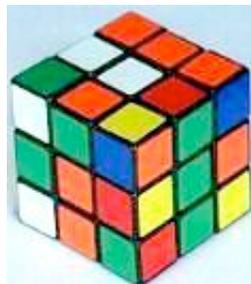


Fig. 5b

On the other hand we could treat the orange/green edge piece as Edge Piece #2 in Fig. 5a (red/white would be #1) or rotate the top face 90° clockwise (giving Fig. 5b) and treat orange/green as Edge Piece #3. The red/yellow edge piece would be #1. In either

case the orange/green edge piece moves into the proper position and orientation and the red/white edge piece will continue to have its white color on the top face of the cube.

But we need to get more white edge piece faces on the top face of the cube. In Fig. 5c the red/green edge piece belongs at front/right. If we treat the red/green edge piece as #1 an Edge Piece Series will cause the orange/white edge piece to come to the top of the cube with its orange face on top, not white. On the other hand if we treat the yellow/red edge piece as #1 then red/green is #2 and orange/white would be #3. Orange/white would come to the top of the cube with its white face on top (since edge piece #3 always inverts).

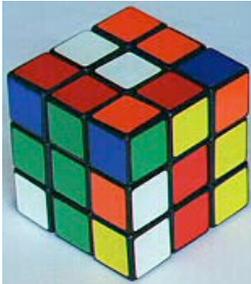


Fig. 5c



Fig. 5d

Or we could rotate the top face 90° counterclockwise (giving Fig. 5d) and apply an Edge Piece Series with the red/green edge piece as #3. The orange/white edge piece will be #1 and the red/white edge piece will be #2. The orange/white edge piece will come to the top of the cube with its white face on top and the white face of the red/white edge piece will remain on top. We will now have two white faces on the top face of the cube.

Continue this process until three center section edge pieces are in place, three top edge pieces have their top color on top and the fourth top edge piece is in the center section. The fourth center section edge piece will be on the top of the cube.

If you should, by chance, properly place three center section edge pieces but have only two top edge pieces with their top colors on top, then apply one more Edge Piece Series in which all of the pieces involved are top edge pieces. The top edge piece in the center section comes to the top with its top color on top and the top edge piece which does not have its top color on top moves into the center section.

You should note that you do not always have to complete the fourth turn of the Edge Piece Series. Whenever the fourth turn is a turn of the top face it does not need to be made because a turn of the top face can have no effect on the bottom face. Remember, the Edge Piece Series was devised not only to put center section edge pieces into proper position/orientation but also to prevent the movement of edge pieces on the bottom face.

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STEP THREE: THE TOP EDGE PIECES

In this section we will place the fourth center section edge piece and the four top edge pieces in their proper positions and orientation.

At the conclusion of Step Two, three top edge pieces were to have their top color on the top face of the cube. Those three edge pieces could be in any one of the following possible arrangements:

1. All three could be out of order.
2. Two adjacent pieces could be in the proper order relative to each other with the third out of order.
3. Two opposite pieces could be in the proper order relative to each other with the third out of order.
4. All three could be in the correct order relative to each other.

The three white edge pieces on the top of the cube in Fig. 6a are all out of order. Note that the top edge piece face you cannot see on the back of the cube must be red. If the top face of the cube is turned 90° clockwise then the yellow face of white/yellow would be adjacent to the yellow face center and white/yellow would be in the proper position/orientation.

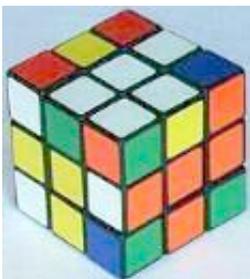


Fig. 6a

But neither of the other two white edge pieces would be in the correct position. The green color would be above the red face center and the red color would be above the orange face center. No matter which of the three top edge pieces is put into place the other two will be out of place.

We can correct this by applying an Edge Piece Series to the cube. However, the series must involve three top (white) edge pieces. Rotate the top face until three white edge pieces are adjacent to the top/front/right corner as they are in Fig. 6a. The cube obtained by rotating the top face 90° clockwise would also be suitable.

Always be sure that three white pieces are in the series and that it concludes with three white faces on top of the cube. In all cases the white edge piece in the center

section will be edge piece #1. In Fig. 6a white/orange will be #1 and white/yellow will be #2. Any other choice would not leave three white edge pieces with their white faces on top of the cube at the end of the series. This series will change the top edge pieces to one of the other arrangements.

In the case of the cube in Fig. 6a the series converts it to the cube in Fig. 6b. Here two adjacent edge pieces are in the proper order (white/yellow and white/orange) and one is out of order (white/red where red is above green). These pieces may be put in the proper order by turning the top face by 180° and applying an Edge Piece Series to the top edge piece in the center section (white/green), the top edge piece which is out of order on the top face (the white/red piece) and the fourth center section edge piece (yellow/orange). The latter is moved into proper position/orientation and all four top edge pieces will be in their proper order. A 90° counterclockwise turn of the top face will put all top edge pieces in their proper place. Step Three is complete.

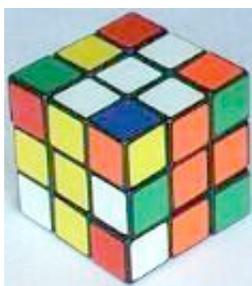


Fig. 6b

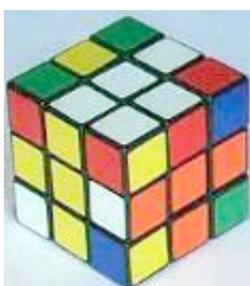


Fig. 6c

In the third case two opposite edge pieces are in position relative to each other and the third is out of place as shown in Fig. 6c. The white/yellow top edge piece is in place. By the process of elimination the color you cannot see on the white edge piece is green. Since the green face on my cube is opposite the yellow face the white/green piece must also be in its proper place. But the white/red piece is out of place. It belongs on the other side of the top face.

The white/orange piece belongs where the white/red is now. The former can be put there by a simple 90° rotation (clockwise) of the orange face. Turn the top face by 180° and a 90° counterclockwise turn of the orange face will move the white/red edge piece back to the top face and into its proper position relative to the other three. Finally another 180° turn of the top face moves all white edge pieces into place. Step Three is complete.

In the fourth case all three top edge pieces on the top face are in their proper position relative to each other. This is shown in Fig. 6d. The unseen color of the white edge piece is red and it clearly is in the red face of the cube. This arrangement is converted to one in which all four white edge pieces are in the proper order on the top face by a series of replacements.

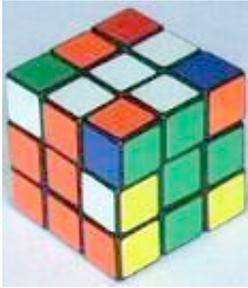


Fig. 6d

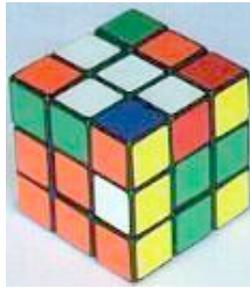


Fig. 6e

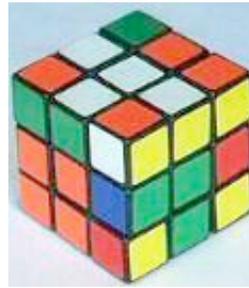


Fig. 6f

We begin by replacing a white edge piece on either end of the three on the top face with the white edge piece in the center section. For example, rotate the top face in Fig. 6d by 90° clockwise giving Fig. 6e. Rotate the green face 90° clockwise giving Fig. 6f. The white/red piece is replaced by the white/yellow piece and the former moves to the center section.

Now turn the top face 90° counterclockwise (Fig. 6g) and replace the white/green piece with the white/red piece (i.e. rotate the green face by 90° counterclockwise) to give the cube shown in Fig. 6h. Continue in this fashion until, in the fourth turn of the green face the last white edge piece (in this case the white/orange piece) returns to the top face of the cube and the fourth center section edge piece moves into place at front/right.

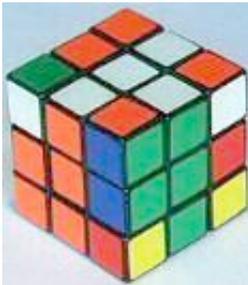


Fig. 6g

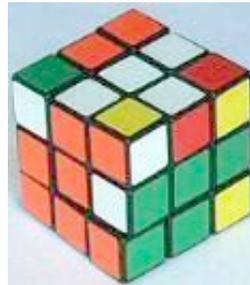


Fig. 6h

You might ask how one knows that the fourth center section edge piece has the proper orientation and is not inverted. While you made sure that the top edge pieces had their white color on top you did nothing to ensure that the center section edge piece had the proper orientation. This is true for each of these examples. But you don't need to worry about the 12th edge piece. If 11 edge pieces are in the proper position and orientation then the 12th must be in the proper position and have the proper orientation as well. It cannot be otherwise.

Turn the top face so that all top edge pieces are in position (in this case 90° clockwise). Step Three is complete and all edge pieces have been properly positioned/oriented.

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STEP FOUR: FIVE CORNER PIECES

Once the edge pieces have been correctly placed you will look for corner pieces which can be moved into position/orientation by a Corner Piece Series. In the discussion of that series we learned that one or more corner pieces will, almost inevitably, be situated so that this series will move that corner piece along a cube edge and into position/orientation. Others will be in a position, such that the series will move them across a face diagonal and into the correct position and orientation.

Figure 7a shows the first kind of corner piece in the front/top/right corner of the cube. The proper series will move the green/red/white corner piece along the green/red edge of the cube and into its correct position and orientation at the back/top/right corner of the cube. (That is, in the green/red/white corner.) In orienting these cubes we will always use the same convention. We will always show the front, right and top of the cube. In this case that means blue is the front, red is the right side and green is the top.

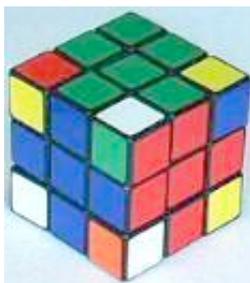


Fig. 7a

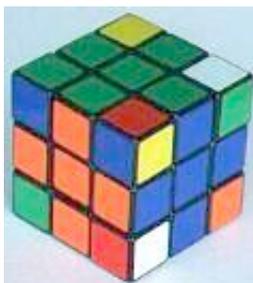


Fig. 7b

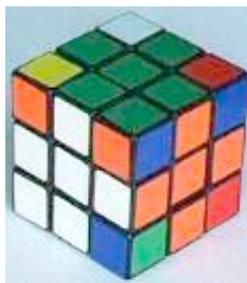


Fig. 7c

If the entire cube is turned 90° counterclockwise we obtain the cube in Fig. 7b. Now orange is front and blue is right. A standard Corner Piece Series which begins with a turn of the top face to the right will move our target corner piece to the back/top/left corner. It will be correctly placed in the green/red/white corner of the cube.

Turning the cube another 90° counterclockwise gives the cube in Fig. 7c. The Corner Piece Series is the same (it begins with a turn of the top face to the right) but the target corner piece now moves along the left/top edge rather than the back/top edge. You can use either orientation of the cube and still move the green/red/white corner piece into place. The difference is that the blue/green/orange corner piece is involved in moving about the first triangle and the blue/red/yellow corner piece has replaced it in the second case.

The red/yellow/blue corner piece is at front/top/right in Fig. 7d. This corner piece belongs somewhere on the top face because of its blue color. But the red and yellow colors do not match the other two sides (front and right) which we can see. Yellow is opposite green and red is opposite orange so this red/yellow/blue corner piece belongs in the back/top/left corner of the cube.

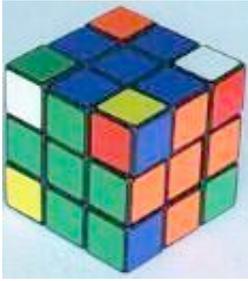


Fig. 7d

A Corner Piece Series which begins with a turn of the top face to the left will cause the red/yellow/blue piece to move across the diagonal of the top face and into place in the back/top/left corner. In this case there is only one way this can happen.

If possible, one would like to place two corner pieces at the same time. You may find, by chance, that a piece goes along an edge while another goes across the face diagonal at the same time. Or perhaps two may move along adjacent cube edges. But most of the time you will have to arrange for these things to happen.

While other solutions will use a relatively large number of series in solving a scrambled cube this method is able to solve a cube while using only two series because of the use of preliminary face turns which change the corner pieces involved in a Corner Piece Series.

Turning the orange face in Fig. 7c by 180° gives Fig. 7e. We apply the same Corner Piece Series as before (start with a turn of the top face to the right) but the blue/red/green piece has become a part of the triangle. This is the piece which belongs at back/top/left and it is not only moved to the correct position it is also properly oriented. Two corner pieces have moved into place at the same time. Following this series the orange face is again turned 180° returning it to its original position.

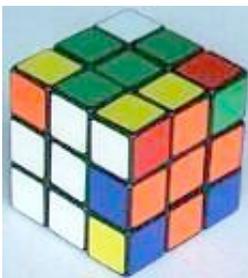


Fig. 7e

The problem with this approach is that you will have to search for the new piece without any clue as to where it will be found. Rather than trying to find the piece which will replace your target piece, it is better to work with the piece which the target piece is going to replace. This is true because a glance at the latter piece will immediately tell you where it must go.

The target piece in Fig. 7c will replace the orange/yellow/blue corner piece at front/top/left. The Corner Piece Series (first turn is to the right) will move this piece

across the diagonal with its blue face coming to the top. But where does the orange/yellow/blue corner piece belong? Since none of its colors match any of the faces it touches it must belong in the opposite corner of the cube. And more specifically its blue color must be in the blue face of the cube.

The front of the cube is white so the back is blue. Turn the orange face (in Fig. 7c) 90° counterclockwise and you get Fig. 7f. Apply the Corner Piece Series and the blue color of the orange/blue/yellow corner piece moves next to the blue color of the blue/orange edge piece. Turn the orange face 90° clockwise and two corner pieces have been moved into place.

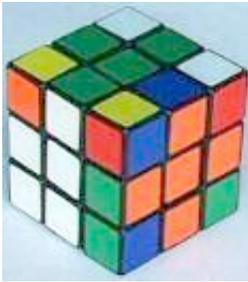


Fig. 7f

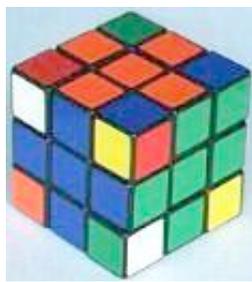


Fig. 8a

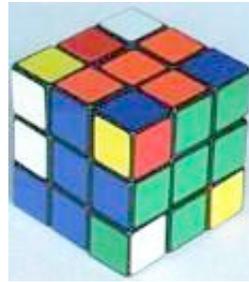


Fig. 8b

Fig. 8a shows another example. The orange/blue/green corner piece at back/top/right will move along the top/right edge of the cube and "roll over" into proper position/orientation in the front/top/right corner of the cube. It replaces the yellow/blue/red corner piece. What needs to be done to cause the latter piece to move into position/orientation at the same time?

According to its colors the yellow/blue/red piece belongs in the corner diagonally across the blue face of the cube. Rotate the yellow (left) face of the cube 180° giving Fig. 8b. Apply the Corner Piece Series (first turn of the top face is to the left) and the yellow/blue/red piece moves into place with its red color next to the red edge piece color on the left side of the top face. A second 180° turn of the left face and two corner pieces have moved into place during the same sequence of moves.

Fig. 9a shows a potential problem. A Corner Piece Series (first turn of the top face is to the left) moves the red/yellow/blue piece across the blue (top) face of the cube to its correct position and orientation. But it would also move the yellow/blue/orange out of place. To avoid this, turn the orange face 90° clockwise (giving the cube in Fig. 9b) and apply a Corner Piece Series which begins with a turn of the top face to the right. Reverse the preliminary turn of the orange face. The red/yellow/blue piece winds up in the correct place but the yellow/blue/orange piece is not disturbed.

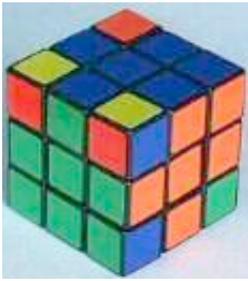


Fig. 9a

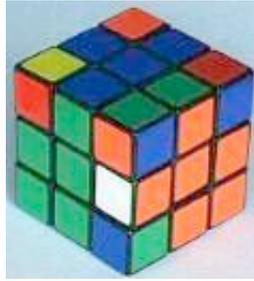


Fig. 9b

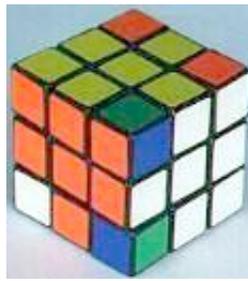


Fig. 10a

We could move three corner pieces on the top face of the cube in Fig. 10a about the triangle: front/top/right; back/top/left; back/top/right. The yellow/orange/white piece at back/top/right moves along the top/right edge and rolls over into place at front/top/right. The blue color of orange/green/blue comes to the top in the back/top/left corner. This piece belongs in the front/bottom/left corner. The blue color of the orange/green/blue piece obviously must go in the blue (left) face of the cube. It must replace the red color of the piece which is now there.

Turn the bottom face 90° counterclockwise and then turn the left face 90° clockwise. This gives the cube in Fig. 10b. A Corner Piece Series which begins with a turn of the top face to the left causes the blue color of the orange/green/blue piece to replace the red color we see in the back/top/left corner of the cube. Then reverse the two preliminary turns (left face 90° counterclockwise, bottom face 90° clockwise). Again we have placed two corner pieces during the same sequence.

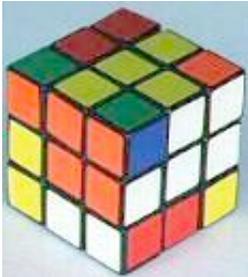


Fig. 10b

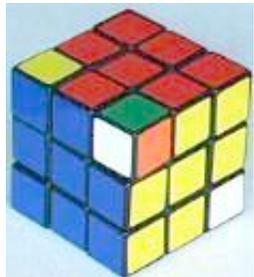


Fig. 11

The blue/yellow/red corner piece at front/top/left in Fig. 11 could move along the front/top edge and roll over into place at front/top/right. But we do not have a triangle of three out-of-place corner pieces on the top face. No problem. Rotate the white face 90° clockwise and the out-of-place piece at back/bottom/right comes to the top face making a temporary triangle. Apply the Corner Piece Series and then turn the white face 90° counterclockwise.

The yellow/red/blue piece at front/top/left in Fig. 12a will move along the front/top edge and into place at front/top/right. The green/red/blue piece has the most difficult situation you will find in trying to place two corner pieces at the same time. This is indicated by the checkerboard appearance it makes with the red/blue edge piece next to it. This corner piece belongs at back/top/right and its green face must be in the green face of the cube.

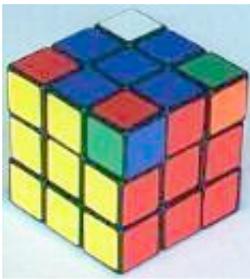


Fig. 12a

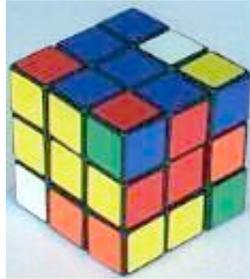


Fig. 12b

It will replace the blue/green/orange corner piece which now has its blue face in the green face of the cube. This means we must find (create) a Corner Piece Series in which the green face of the green/red/blue piece replaces the blue face of the blue/orange/green piece. Turn the green face 90° counterclockwise; turn the white (bottom) face 90° clockwise; and turn the green face 90° counterclockwise. We have the cube in Fig. 12b. As required, the blue face we are looking for is on the top face at back/top/left.

Apply a Corner Piece Series moving corner pieces about the triangle: front/top/left; front/top/right; back/top/left. Now reverse the preliminary turns (green face 90° clockwise; white face 90° counterclockwise; green face 90° clockwise). Again, two corner pieces have been moved into place at the same time.

The cube in Fig. 13 shows an unusual arrangement. The pieces making a checkerboard pattern on opposite sides of the red/yellow edge piece need to be exchanged as do the pieces across the diagonal on the white face. We could put one of the first pair in place using the preliminary three turn sequence described in the previous paragraph. Or we could put one of the latter pair in place using a two turn sequence. We will do the latter simply because it is shorter.

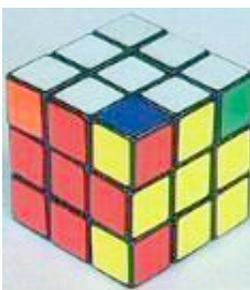


Fig. 13

Move the green/red/white corner piece so that it is across the cube diagonal (not a face diagonal) from its proper position (turn the orange face 90° counterclockwise). Next turn the blue (bottom) face 90° counterclockwise. Note that this turn is at right angles to the first turn. Now the green/red/white corner piece is diagonally across the red face from its proper position and can be moved into position and orientation with the proper Corner Piece Series.

If the second turn in the paragraph above had been 180° instead of 90° then the green/red/white piece could have been moved into place along a cube edge (the green/red edge) with the proper Corner Piece Series.

When five corner pieces have been placed Step Four will be completed. If you can place two corner pieces at a time obviously the total number of applications of the Corner Piece Series will be reduced. However, anytime you find a piece in place but with the wrong orientation you will have to remove it. You can do this at the same time that you move another corner piece into place but you can't place two pieces while removing a third. And anytime you find two corner pieces are each located in the other's proper place you will be able to place only one corner piece while moving the other out of the way.

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STEP FIVE: THE END GAME

With only three corner pieces remaining out of position all must be involved in the final Corner Piece Series. Further, each must move simultaneously into its correct position and with the right orientation.

It is possible that the three target corner pieces are on the same face and will move into place via a Corner Piece Series without any preliminary moves. But that would be rare indeed. We won't even think of it. Rather, the first thing for you to do is to look for a corner piece which can move along a cube edge into its correct position while rolling over into its correct orientation. Surprising as it may seem, in about half of all instances you will find that such a piece exists. About one-fourth of the time you will even find two such pieces.

The corner piece at back/top/right in Fig. 14a is just such a piece. (Its color on the back side of the cube is blue.) The proper Corner Piece Series will cause it to move along the top/right edge to the front/top/right corner of the cube where it will be in the correct position/orientation. There are two different cube faces which may be used as the front face in the Corner Piece Series and two different triangles about which the corner pieces may move.

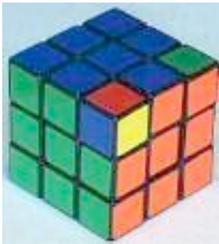


Fig. 14a

In one such case, the blue/red/yellow piece at front/top/right will move to the back/top/left corner (i.e. the yellow/blue/red corner) with its yellow face on top of the cube. In that series, the green face is front and the first move is a rotation of the blue (top) face to the left.

In the other case, the front face is red (the left side in Fig. 14a) and the first turn is again a rotation of the blue (top) face to the left. In this case the blue/red/yellow piece will move to the corner now occupied by the blue/green/red corner piece. Again the yellow face of the blue/red/yellow corner piece will come to the top of the cube.

You should note that the first two corner pieces will be on the same face and will be adjacent to each other. The first will go into position/orientation replacing the second. The second will go to either of the two remaining corners on the top face with the color which is on the right side of the cube at the start of the Corner Piece Series coming to the top. We will refer to the third and fourth corners as the "target positions".

Naturally the second corner piece must replace the third corner piece with the third

replacing the first. But where is the third corner piece? A little thought should convince you that, in this case, the third piece is somewhere in the yellow face of the cube. Fig. 14b (the entire cube in Fig. 14a has been rotated 90° clockwise on an axis from top to bottom) shows that the third corner piece is in the yellow/blue/red corner. It should be clear that if the yellow face of the second corner piece replaces the orange face of the third corner piece (the face which is in the yellow face of the cube) the second piece would be in the right position and have the right orientation.

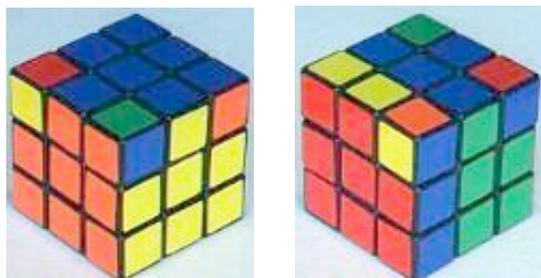


Fig. 14b

Fig. 14c

How can we make that happen? We must bring that orange face to the top (blue) face of the cube in one of the target positions. This can be done with a 90° clockwise rotation of the red face as shown in Fig. 14c. Then, with red as the front face, a Corner Piece Series beginning with a turn of the blue face to the left will cause these three corner pieces to move clockwise about the front/top/right; back/top/left; back/top/right triangle.

The yellow face of the yellow/red/blue corner piece replaces the orange face of the orange/blue/yellow corner piece and when the red face is rotated 90° counterclockwise (the preliminary turn is reversed) each of the corner pieces will be in its correct position and orientation.

But, you might say,..."You made sure that each of the first two pieces went to the right place but you did nothing about the third". You don't have to worry about the third corner piece for, when all other pieces on the cube have been properly placed, the last piece must be in position and must be properly oriented as well. It is physically impossible for it to be otherwise. Thus you must ensure that the first two pieces are correctly placed and the third one will take care of itself.

As indicated above, when you reach the End Game, half the time at least one of the corner pieces will be in position such that a Corner Piece Series will cause it to move along a cube edge and into position/orientation. If you don't find such a corner piece you must create one. About 70% of the time you will do so with a single 90° rotation of a cube face. And about 30% of the time you will do so with a 180° rotation of a cube face.

The cube in Fig. 15a does not have a corner piece which can be moved along a cube edge and into place. But the red/white/yellow corner piece is diagonally across the yellow face from the corner where it belongs. It could be made part of a triangle which

would send it across the diagonal to its proper place. A corner piece in such a position may be moved to an adjacent corner so as to create a corner piece which can be moved along a cube edge and into place. You can do this with a 90° rotation of either vertical face in which the corner piece is located. For example, the orange face could be rotated 90° clockwise. Fig. 15b shows the resulting cube with white as the front face.

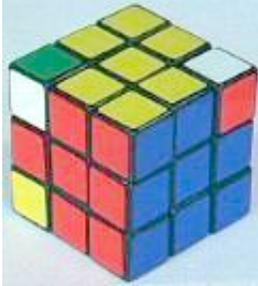


Fig. 15a

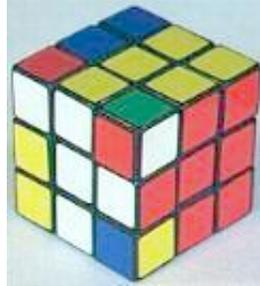


Fig. 15b



Fig. 16

The cube in Fig. 16 does not have an appropriate corner piece adjacent to its correct position. Neither does it have a corner piece which could be moved across a face diagonal to its proper place. But it does have a corner piece which can be rotated 180° and be in a position from which it can move along a cube face and into proper position/orientation. That corner piece in Fig. 16 is the green/red/white piece in the back/top/left corner of the cube (all you can see is the green face).

You can turn either of the faces, which (of the three remaining out-of-position corner pieces) contain only the green/red/white corner piece, by 180° and that piece will be in position such that the proper Corner Piece Series will move it along a cube edge and into place. Turning the third face of the cube (yellow in this case) will not work.

With this understanding we can construct the rules for successfully completing the End Game:

1. Find a corner piece which may be moved along a cube edge and into place. If none can be found create one.
2. Note the face color (of the second corner piece) which will come to the top face of the cube in a target position during a Corner Piece Series.
3. Locate the third corner piece in the cube face of this same color and note the color of the third corner piece face which is in that cube face.
4. Bring the third corner piece to the top of the cube, by various face turns, such that that color is in the top face of the cube.
5. Perform the Corner Piece Series so that the target color of the second corner piece replaces this color (named in 4 above).
6. Reverse the preliminary face turns indicated in 4 above.

This process is much easier to carry out than it is to describe. Several examples will be given below.

The white/yellow/orange corner piece at front/right/top of the cube in Fig. 17a may be moved along the white/yellow edge and into place. It will replace the red/green/blue piece. The latter will move to either of the target positions with its blue face coming to the top. We examine the blue face of the cube and find the yellow face of the white/red/yellow corner piece in that blue face. Hence we need to get the yellow face of the third corner piece into one of the two target positions. This is done by a 180° rotation of the green face giving the cube in Fig. 17b (the entire cube has been rotated 180° so you can see all three faces of the third corner piece).

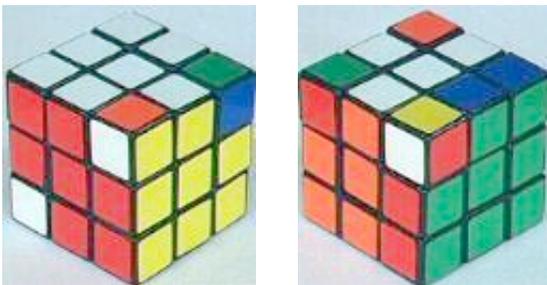


Fig. 17a

Fig. 17b

Now hold the cube with the green face in front and begin the Corner Piece Series with a turn of the top (white) face to the right. The third corner piece goes across the diagonal of the white face (with its white face coming to the top) and into its proper position and orientation.

You do not always have to examine the face of the cube which has the target color. When the white/yellow/red piece in Fig. 18a moves along the white/yellow edge of the cube the green face of the second corner piece comes to the top in either of the target positions. We need to move the face of the piece at the lower left of the cube which is now in the green face (we see only its white face) into one of the target positions.

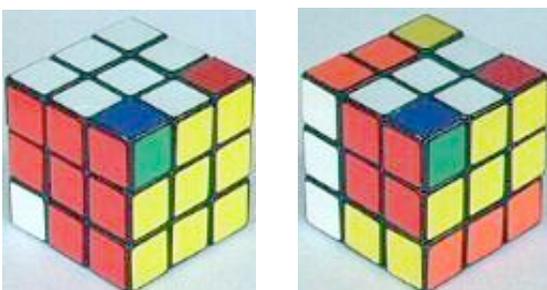


Fig. 18a

Fig. 18b

Rotate the bottom face by 90° counterclockwise and then the left face 90° clockwise. This gives Fig. 18b. The face we were looking for is yellow (we would have seen that if

we had turned the cube to reveal the green face). The first turn of the top face is a rotation to the top left. When the series is completed reverse the original two turns and the cube is complete.

The corner piece at back/top/left in Fig. 19a will move along the top/left edge of the cube and into place. The green face of the white/orange/green corner piece will come to the top in a target position. The yellow face of the third corner piece is in the green face of the cube. We can bring that yellow face to a target position by the three turn sequence we learned in Step Four. It is: green counterclockwise; blue (bottom) clockwise; and green counterclockwise.

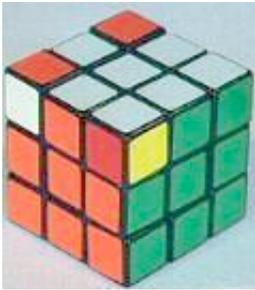


Fig. 19a

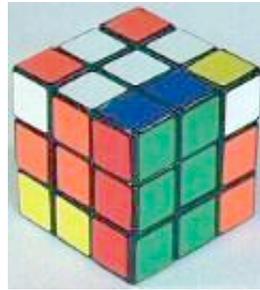


Fig. 19b

There is the yellow color on the top face in the back/right corner of the cube (Fig. 19b). Apply the Corner Piece Series and reverse the three preliminary turns. The cube is restored.

The cube in Fig. 20a has two corner pieces which can move along a cube edge and into position. We could use either one of them as the first corner piece. Note that all three of the final corner pieces have an orange face and, in each case it is in the orange face of the cube.

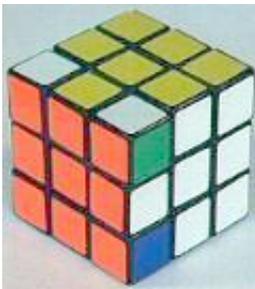


Fig. 20a

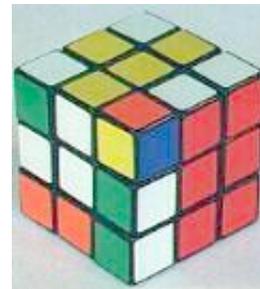


Fig. 20b

We will use the yellow/white/orange piece as the first corner piece. Let us turn the entire cube 90° clockwise on an axis from top to bottom. Now turn the bottom (green) face 90° clockwise and then the red face also 90° clockwise. This gives the cube in Fig. 20b. (Again we chose this view so you could see all three faces of the third corner piece.) With red as front, turn the top face to the right and complete the Corner Piece Series. Reverse the two preliminary turns and the cube will be done.

In Fig. 21a none of the pieces is adjacent to any other. As a general rule this should be

avoided but it is not much of a problem in this case because any one of the three corner pieces could be turned 90° and thereby create a corner piece which could be moved along a cube edge and into its proper place. For example, turn the orange face 90° counterclockwise (Fig. 21b shows the result except that the entire cube is turned clockwise in order to show all three faces of the green/white/red corner piece) or turn the red face 90° clockwise.

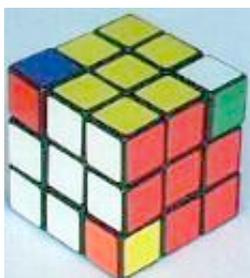


Fig. 21a

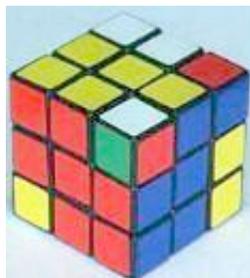


Fig. 21b

But if the orientation of each were to be changed so as to put a corner piece face into the cube face of the same color then things would change drastically.

(For example, the yellow color of the red/blue/yellow piece is moved to the yellow face of the cube at front/top/left, the red face of red/white/green is moved to the red face of the cube and the white face of orange/white/yellow is moved to the white face of the cube.) Although this arrangement is rare, it is the most difficult you will encounter.

I will leave this for you to solve for yourself. But I'll give you a hint. You will have to move one of the corner pieces in two different directions.

If you have one of those cubes with symbols on four of the faces then go to [Rubik's 4th Dimension Cube](#).

If you are having trouble following this description you may be interested in a little help. A videotape is available showing a 53 minute description of this solution. Or you may simply wish to give the videotape to someone else.

[video](#)

[Home](#)

THE ULTIMATE SOLUTION TO RUBIK'S POCKET CUBE

The Ultimate Solution to Rubik's Pocket Cube utilizes the methods given in [The Ultimate Solution to Rubik's Cube](#). Obviously the Edge Piece Series is not used here but you must first master the [Corner Piece Series](#) and [The End Game](#) before attempting this method of solving Rubik's Pocket Cube.

Solving a Rubik's Pocket Cube (2x2x2) is not the same thing as solving a Rubik's Cube (3x3x3) after all of the edge pieces have been properly placed. Rather, it is simpler and, at the same time, more complex.

It is simpler because the corner pieces of the first face can be placed with one or two moves of cube faces or, at the most, a three turn series. In fact, since there are no center or edge pieces to match, the first corner piece is in place even before you begin!

But it is more complex because of a fundamental difference between the 3x3x3 and 2x2x2 cubes. In the former cube when only three corner pieces remain you will find them either all in position or all out of position. You cannot, for example, find that all pieces are in place except two corner pieces which are out of position. But with the Pocket Cube you can find exactly that, six corner pieces are in position and properly aligned and two are out of position. That creates a new problem.

In addition, the lack of center and edge pieces means that you have no aids to remind you of the proper orientation of the cube, of the turns you have made or the turn you must make next. Concentration is of critical importance, particularly in a complex End Game.

THE FIRST FACE

Since there are no center or edge pieces with which the corner pieces must be aligned, the first corner piece is in place simply because you say it is. Choose a color, any color. This is the color of the first (top) face. Choose the piece which you will proclaim to be in place with its top face color on top of the cube. Although it is possible that a second corner piece will be in place relative to the first corner piece, we will assume that it is not.

As was true in The Ultimate Solution to Rubik's Cube, all pocket cube images shown here will depict the front, top and right sides of a cube. If an entire cube is rotated by, say, 180° the new image will be described in terms of its new front, top and right faces.

The cube in Fig. 1a shows the yellow/orange/blue corner piece in place at right/front/top. The easiest corner piece to deal with at this point is one with its top color (blue in this case) on the bottom of the cube. If we rotate our cube by 180° we get the cube in Fig. 1b. In this figure the corner piece at right/front/bottom has its blue face on the bottom. A simple rotation of the right face by 180° (either direction) will move this corner piece into place at right/back/top (Fig. 1c). Its orange face will be next to the orange face of the first corner piece.



Fig. 1a



Fig. 1b



Fig. 1c

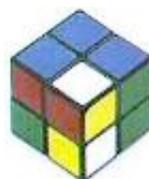


Fig. 1d

It is preferable that the second corner piece goes into place in one of the positions adjacent to the first

corner piece (as we have just done). This will leave a vertical cube face which may be rotated freely (in this case the front face). If, instead, a corner piece is moved into place in the opposite corner from the first corner piece it would then be impossible to move a third corner piece into place with a single move (since that would move one of the first two pieces out of place).

Another blue corner piece may be seen at right/front/top in Fig. 1c but the red/yellow/blue piece is at left/back/bottom and has its blue color on the bottom surface of the cube. A 180° rotation of the bottom face and then a 180° rotation of the front face moves the third corner piece into place at left/front/top (Fig. 1d).

In trying to place the second or third corner piece you may find that the blue color of the target corner piece is on a vertical face of the cube. You may be able to see how a face turn will move the corner piece into place. Or a rotation of the bottom face may move your target corner piece into such a position. But such moves are frequently blocked (by corner pieces already in place). If such a move is not obvious then rotate a face so as to move the blue color of a corner piece to the bottom of the cube and proceed as above.

Once you have placed three pieces by this method you must use a three turn series to place the fourth (or the third and fourth if the first two are diagonal to each other). The target corner piece must be in the bottom half of the cube and have its blue color on a vertical face. The bottom half of the cube is rotated so that the target corner piece is below its appropriate place on the cube as in Fig. 2a. Rotate the front face one turn clockwise (Fig. 2b); rotate the bottom face one turn clockwise (Fig. 2c); rotate the front face one turn counterclockwise (Fig. 2d).

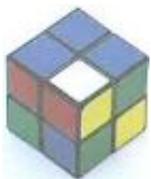


Fig. 2a



Fig. 2b



Fig. 2c



Fig. 2d

If the blue color is in the right face rather than the front face (Fig. 3a) then the series is: right face one turn counterclockwise (Fig. 3b); bottom face one turn counterclockwise (Fig. 3c); right face one turn clockwise (Fig. 3d) completing the top face. This series is seen to be a mirror image of the first series.



Fig. 3a



Fig. 3b



Fig. 3c



Fig. 3d

The target corner piece may be in position but not properly oriented. In this case replace it with another corner piece using one of the two series just described. Which one? The one which brings the target corner piece down to the lower half of the cube with its blue color on a vertical face.

You might complete the placement of the first three corner pieces and find that the fourth has its blue color on the bottom of the cube. In this case rotate the bottom face of the cube until the target piece is below the place where it belongs. Rotate the right face counterclockwise, the bottom face clockwise and the right face clockwise. This is similar to the first series. It differs in that the second move is a clockwise rotation of the bottom face rather than a counterclockwise rotation. It moves the target corner piece to the left/back/bottom corner of the cube and puts the blue face on a vertical face of the cube. Now a 180° rotation of the bottom face moves the target corner piece to the right/front/bottom corner

and it is ready for the series which will place it properly on the top of the cube.

THE SECOND FACE

When the first face is complete turn the cube over. White (in this example) will now be the top color. There are many possible arrangements of the pieces on this face. The pieces could be in different positions relative to each other and each one could have any of three different colors on top of the cube.

What you want to find (or create) is the situation in which the top face is ready for the End Game (the same one you learned in the solution to Rubik's Cube). In this case one piece will be in place (proper position and alignment) and the other three will all be out of position.

Suppose that, when you turn the cube over you find two pieces with their white color on top of the cube. Select one of them and rotate the top face until this piece is in position. Now, let us assume that when you examine the cube you find two adjacent pieces in proper position and two which are out of position. One of the two pieces out of position has its white color on top of the cube (Fig. 4a). Rotate the top face one turn to place this piece in proper position and alignment (Fig. 4b). Surprise! You now have the End Game arrangement.



Fig. 4a



Fig. 4b

If we consider only the order of the pieces on the top face, and not the possible color combinations, we find there are only three possibilities. The number of possible arrangements increases when we consider the colors of the corner pieces on the top surface. However, it develops that the only time color is significant is when we ask whether or not a given piece has its top color on top of the cube.

The following are the significant arrangements:

- 1) All four pieces are in the correct order. Color is immaterial.
- 2) Two pieces opposite each other are in the correct order. The other two pieces are also in the correct order relative to each other but are out of order relative to the first two pieces. Color is immaterial.
- 3) Two pieces adjacent to each other are in order and the other two are out of order. None of the out-of-order pieces has its top color on top of the cube.
- 4) Two pieces adjacent to each other are in order and the other two are out of order. At least one of the out-of-order pieces has its top color on top.

Note that there are only four different combinations of the top corner pieces, including colors, which are significant. One of the four (arrangement 4) may be converted to the End Game by a single turn of the top face. Each of the other three may be converted to arrangement 4 by a single Corner Piece Series.

Although it is not necessary, many will find it easier to identify the arrangement of pieces when at least one of the pieces is in position. If, when you turn the cube over after completing the first face, at least one corner piece has its common color on top, turn the top face to place that piece in proper

position/orientation. If none of the pieces has its common color on top then choose any piece and put it in position (if it is not already there).

1. You might have all pieces in position. This is the first arrangement. Apply a Corner Piece Series to any three pieces such that the piece which moves across the diagonal of the top face has its common color come to the top of the cube. Move the piece which moved across the diagonal into place (rotate the top face by two turns) and you will have the End Game arrangement.
2. Suppose you find two opposite pieces in position. The other two are out of position (relative to the first two). This is the second arrangement. Apply the Corner Piece Series to any three pieces. Again you must make sure that the common color of one of the pieces comes to the top. However, this time it must be one of the pieces which moves along an edge of the cube. Either one will do. No matter which three pieces you choose, nor which piece (moving along an edge) has its common color on top at the end of the series, you will always be one move away from the End Game arrangement.
3. You may have the third arrangement: two pieces adjacent to each other are in place and neither of the other pieces has its top color on the top face. Again you will convert this arrangement to the End Game by applying a single Corner Piece Series. One of the out-of-order pieces must move across the diagonal and its common color must come to the top. This piece is then moved into place giving the End Game arrangement of corner pieces. However, this time you do not have complete freedom as to which corner pieces to include. You may include only one of the two adjacent corner pieces which are in position relative to each other.
- 4) If you have the 4th arrangement, two adjacent corner pieces in place relative to each other and at least one of the out-of-order corner pieces with its common color on top, this is, in fact, the End Game. You may find it as the End Game itself or you may find it with two pieces in place and two out of place. This will become the End Game upon a 90° turn.

The third arrangement is shown in Fig. 5a. The white/red/green piece (at right/front/top) and the yellow/white/red piece (at right/back/top) are both in position. The other two top corner pieces are out of position. None of these pieces has the top color on the top face of the cube. Only one of the pieces in position (either one) may be involved in the required Corner Piece Series. One of the out-of-position pieces must move across the diagonal and have its white color come to the top. We will send the white/yellow/orange corner piece (at left/front/top) across the diagonal with its white color replacing red. The result of this series is shown in Fig. 5b.



Fig. 5a



Fig. 5b



Fig. 5c

Two of the corner pieces in Fig. 5b are in place and adjacent to each other. The other two are out of place. If we rotate the top counterclockwise one turn we will obtain Fig. 5c. The orange/yellow/white corner piece (at left/back/top) is in position and aligned. The other three top pieces are out of place and ready for the End Game.

In any of these situations you may have a problem getting the common color of the appropriate corner piece to come to the top during a Corner Piece Series. The cube in Fig. 6a has two opposite pieces in position. How can you move the green/red/white corner piece (at left/front/top) along a cube edge to left/back/top and have its white face on the top face of the cube.



Fig. 6a



Fig. 6b

You have seen this kind of problem before in the solution to Rubik's Cube. If you rotate the front face by 90° clockwise you will get the cube in Fig. 6b. Now the white/red/green piece can move across the diagonal of this cube and its white face will be on the top face of the cube. Following the Corner Piece Series turn the front face back and you are ready for the End Game.

The End Game is exactly the same as it was with Rubik's Cube. However, the lack of center and edge pieces creates an orientation problem. You must be careful to keep the orientation of the cube in mind and to keep track of the turns you have made. Remind yourself repeatedly of the turn you will make when the Corner Piece Series is completed. The process is not too difficult but you cannot let your mind wander.

Rubik's Pocket Cube is not well suited to racing. It is far less solidly built than Rubik's Cube and, in particular, you cannot spin faces rapidly or the cube may come apart. While it may not be broken this will certainly spoil the race.

Once you have learned the solution you may want to try to reduce the number of moves required to restore a scrambled cube. The best I have ever done with a Pocket Cube is six moves and I have done this more than once. It should be clear that, with a number this low, when the first face was done the second face was found to be complete as well. The average of 100 trials was 22 moves, less than half of the average number required for any other solution I have seen.

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RUBIK'S 4th DIMENSION CUBE

If you have a Rubik's 4th Dimension Cube you will need to learn how to correctly align the symbols which are found on four of the six faces. If you have solved the colors on all of the faces first then you will need to learn several more complex series of face turns in order to rotate the four symbols so that all are upright.

But that would be a waste of time and effort for, if you align the symbols first, the process is simplicity itself provided you use the method given on these pages to solve the cube. The cube must be held so that the symbols are on the vertical faces while placing edge pieces but orientation is immaterial while placing corner pieces. If a vertical face is rotated in a given direction early in an Edge Piece Series then it is rotated in the opposite direction later. At the conclusion of every series the four symbols will each have the same alignment as they did in the beginning. Therefore, if you properly align all four symbols before you begin to apply The Ultimate Solution then the symbols will be properly aligned when you have completed the solution.

Hold the cube so that either the orange or the red face is on the top (that is, either of the faces without one of the symbols on it). Put three of the top edge pieces in place relative to each other. Make sure that the fourth top edge piece is on the bottom of the cube with its top face color on the bottom face of the cube.

The position where the fourth top edge piece belongs is above one of the four vertical face symbols. It makes no difference which one. The fourth top edge piece must not be below this symbol. If necessary, rotate the bottom face so that this fourth edge piece is in one of the other positions. Now rotate the vertical face (if necessary) so that the symbol is upright.

Rotate the top face 90° and repeat this process for the second symbol. Again rotate the top face 90° and carry out this process for the third symbol.

Rotate the top face one more time and the fourth symbol will be below the slot where the fourth top edge piece belongs. This time rotate the vertical face so that the symbol is upside down. Now rotate the bottom face so that the fourth top edge piece is below the last symbol. Rotate this vertical face by 180° . The fourth symbol is properly aligned (upright) and the fourth top edge piece is in place relative to the other three. Rotate the top face until the top edge pieces are properly aligned with the vertical face colors.

All four symbols are properly aligned as are the top edge pieces. Complete the remaining steps of The Ultimate Solution and the Rubik's 4th Dimension Cube will be restored to its original condition. Now wasn't that easy?

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THE ULTIMATE SOLUTION TO RUBIK'S REVENGE

A scrambled Rubik's Revenge does indeed look formidable. And it is. Prof. Rubik did more than just increase the number of pieces on an edge from three to four or the number on a face from nine to 16. The center pieces did not simply increase from one to four on each side. Rather, these center pieces can now be moved relative to each other. Instead of 20 movable pieces of two kinds we have 56 movable pieces of three kinds.

But take heart. Things are not as bad as they seem. Rubik's Revenge can be solved using the same two series described in [The Ultimate Solution to Rubik's Cube](#), namely [The Edge Piece Series](#) and [The Corner Piece Series](#). Once the center pieces and the edge pieces are put together the cube can be assembled using the five steps described for Rubik's Cube.

Can this method be applied by the neophyte? Certainly. Recently (7-19-99) I received an e-mail message from a world-travelling businessman who claims never to have solved a Rubik puzzle of any kind. While in Tokyo he picked up a Rubik's Revenge at a toy store with the intent of spending the 12 hours returning to the states in an effort to solve it. He did try but, not surprisingly, he didn't succeed.

The puzzle was set aside. Periodically, over the next seven years, he would try again. The most recent time was just this month (July of 1999). Again he was unsuccessful but this time he thought to try the internet for help. The first site he found was this one. Although this method required that he first learn how to solve Rubik's Cube, he persevered and the restored Rubik's Revenge sits on his desk.

THE FACE CENTERS

If done first, the center pieces can be placed very easily. You must learn a simple process but there are no new series of turns to memorize. For example, consider the cube in Fig. 1a. We may arbitrarily set the color of the first face and, from the looks of things on these three faces, it appears that a good choice would be to make the right face orange.

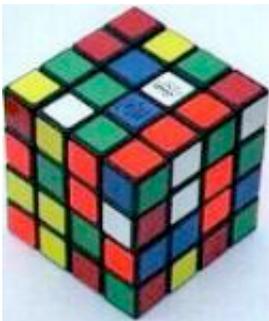


Fig. 1a



Fig. 1b

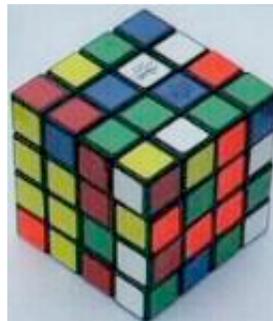


Fig. 1c

During the first part of the solution, constructing cube faces, we may freely turn certain cube faces. We will turn the right face 90° clockwise. This gives the cube in Fig. 1b. Now when the top half of the cube is rotated 90° counterclockwise (giving Fig. 1c) the orange center piece from the front face moves to join the two orange pieces in the right face. If we had not made the first turn then one orange piece would have moved out of the right face as the third moved in.

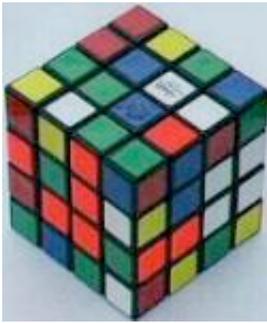


Fig. 1d

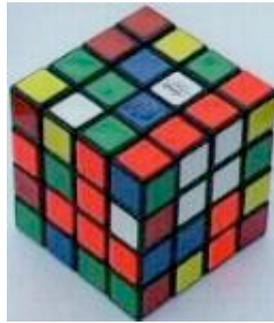


Fig. 1e

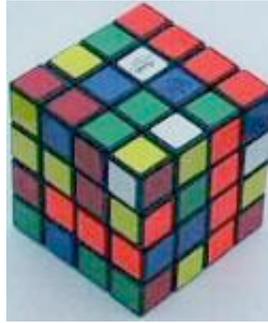


Fig. 1f

If we rotate the entire cube 90° clockwise (Fig. 1d) we see that the fourth orange center piece is in the next face (now the right face). How do we get it to join the other three in the front face? Note that if we turn the top half 90° counterclockwise the two orange pieces in the right face would not line up but would be diagonal to each other. In order to get the two pieces to move together they must be adjacent to each other. To prepare for the turn of the top half of the cube we first turn the right face 90° counterclockwise giving the cube in Fig. 1e. Now when we turn the top half of the cube 90° counterclockwise (Fig. 1f) the two orange pieces in the right face will be adjacent to each other. Turn the right face 90° counterclockwise (Fig. 1g) and then turn the top half 90° clockwise and the orange face is complete.



Fig. 1g

Which face shall we do next? We know that red is opposite orange. We know that this is true on our Rubik's Cubes. It is also true for both the Rubik's Pocket Cube and Rubik's Revenge. It is true for all the cubes I own including the 5x5x5 "Professor's Cube".

If we choose to place the red face center pieces at this time some of our moves will break up the orange center and we will have to take precautions to restore it. On the other hand, if we choose one of the vertical faces in Fig. 2a we can turn the top and bottom halves of the cube freely. We can solve a vertical face so long as none of its pieces is now on the bottom of the cube. White appears to be a good choice.

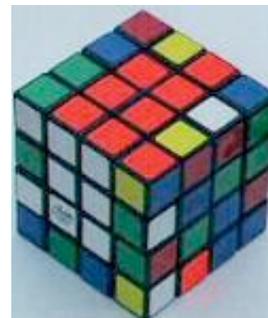
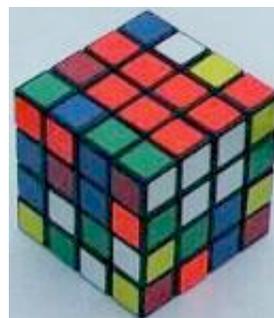
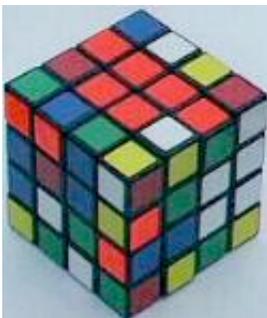


Fig. 2a

Fig. 2b

Fig. 2c

The right face in Fig. 2a is turned 90° counterclockwise giving Fig. 2b. The top half of the cube is now turned 90° clockwise and we have three of the four white pieces in place. We need to "go and get" the fourth white piece from the back side of the cube and the white face will be done (Fig. 2c). Now we have no choice but to work on a face which will cause faces which have already been completed to be temporarily broken up. Blue is opposite White. We will do it next.

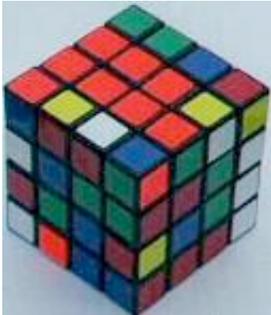


Fig. 3a

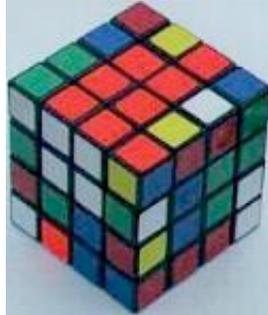


Fig. 3b

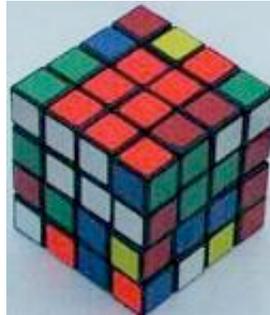


Fig. 3c

The entire cube has been turned 90° clockwise (about an axis from top to bottom) in Fig. 3a. White is on the left side so we must assemble the blue face center pieces on the right side. The top half of the cube is turned 90° counterclockwise. Two blue face center pieces are adjacent to each other on the right side (Fig. 3b). Note that the top half of the white face center now appears on the front side of this cube. Turn the right side 90° counterclockwise (Fig. 3c) and the two blue pieces are in the bottom half of the face center. They will be left behind in the right face when the top half of the cube is turned 90° clockwise (Fig. 3d).

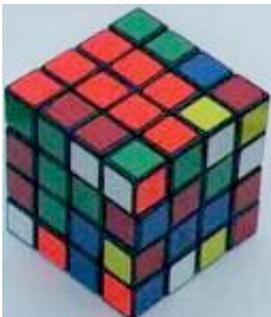


Fig. 3d

The most recent turn restored the white face. It will be alternately broken up and then restored as we move other face center pieces into position. For example, how do we move the blue face center piece in the front face of Fig. 3d into the right face where it belongs? Answer: rotate the top half of the cube in Fig. 3d by 90° clockwise; rotate the front face 90° counterclockwise; rotate the top half 90° counterclockwise. The white face was broken up and then restored as the third blue piece was moved into place. When we get the fourth blue piece into place the blue center face will be complete.

The blue face of the cube in Fig. 4 is complete and the front face already has three green face center pieces. It should be fairly easy to complete that face by retrieving the fourth green face center piece. But that would be an error. It was permissible to arbitrarily locate the white face as we did. Then the blue face had to be opposite the white face. But the front face in Fig. 4 could be either green or yellow (which are opposite each other on my cubes). How do we know which?

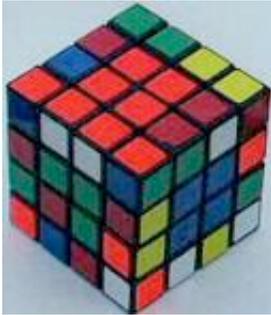


Fig. 4

We can either memorize the order of the appearance of colors as we rotate a cube or we can rely on very clear indicators. Look at the corner piece at right/front/top in Fig. 4. It shows that, when orange is on the top, the green face is to the right of the blue face. Hence, green must be on the right side of the blue face, not on the left. This means that the front face must be yellow.

The green, yellow and red face centers are completed using the method outlined above. Note that the last two faces are completed simultaneously. When it comes to the final parts of the solution of Rubik's Revenge these face centers will act as a single piece and will not be moved relative to each other.

THE EDGE PIECES

As opposed to Rubik's Cube, the edge pieces are in two parts. Most of the edge pieces have been split in two and are scattered about the cube. Now we must pair up these edge pieces so the edge pieces as well as the center pieces may be considered as single units in the final parts of the solution.

In the illustration given here the two halves of the orange/blue edge piece are brought together to make the "whole". One of the orange/blue edge piece halves is shown on the right/front edge of the cube in Fig. 5a. It is in the third horizontal layer of the cube numbering from bottom to top. The other orange/blue edge piece is on the right/back edge of the cube and is in the second layer. The bottom half of the cube is rotated 90° counterclockwise giving the cube in Fig. 5b.

We have joined the orange/blue edge pieces to make the "whole". Note that each of the white, yellow, blue and green faces has been split. You cannot turn front or right halves of the cube without making a mess. But you can turn cube faces providing you restore any vertical face before again turning the bottom half of the cube. We want to preserve the orange/blue edge piece which means that we must remove it from the vertical edge where it now resides.

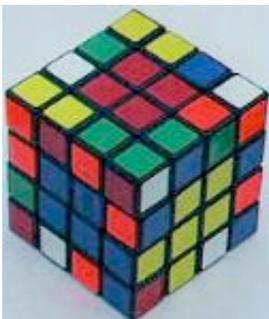


Fig. 5a



Fig. 5b

This is done with an Edge Piece Series. One can use either the full four turns or only the first three turns omitting a final turn of the top face. In this case only three turns were used. They were:
one right face clockwise

two top face clockwise
three right face counterclockwise

Following the Edge Piece Series the orange/blue piece is on the front/top edge (Fig. 5c) and the turn of the bottom half of the cube does not affect it (Fig. 5d).



Fig. 5c

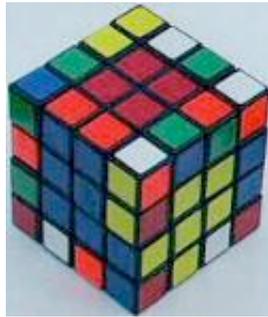


Fig. 5d

If the two edge piece halves you are trying to recombine are across a cube face and on the same layer of the cube how can you bring them together? Rotate the face containing one of the edge pieces by 180° . The two pieces are now on different layers and may be brought together by a 180° rotation of half of the cube.

One thing you do not have to worry about is getting two edge piece halves together but not aligned. This can never happen. Although it has other problems the mechanics of Rubik's Revenge are such that, whenever the two halves of a given edge piece are brought together they are always in proper alignment.

If you carry out this rejoining of edge pieces as described above you will have to do it eight or nine times as you prepare for the final recombination. However, you should note that you introduced two edge piece halves just prior to the turn which restored the cube face centers. If you selected properly at least one of them would be in proper color alignment with the edge piece half which it joins. In that case you would be creating two "whole" edge pieces (and possibly three) each time you carry out this process.



Fig. 6a



Fig. 6b

How do you complete the process? You do it with three unaligned units remaining. The cube in Fig. 6a shows unmatched pairs at right/front, right/back and right/top. The two colors you cannot see at right/back are both white. A 90° counterclockwise rotation of the bottom half of the cube brings the two yellow/white halves together on the right/front edge (Fig. 6b). The yellow/white edge piece is replaced by the unmatched edge piece pair at right/top. It is done in such a way that orange/white is in the second layer (Fig. 6c). Now when the opening turn of the bottom half of the cube is reversed the two orange/white halves are together and in proper alignment (Fig. 6d). So are the orange/yellow halves at right/front but you don't have to worry about them. They have no other choice.



Fig. 6c

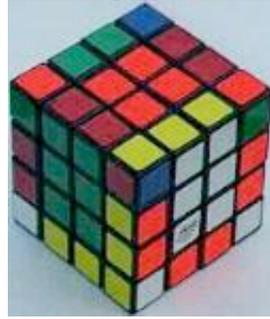


Fig. 6d

You might make a mistake and overshoot the mark leaving only two unmatched edge piece pairs. How do you treat this situation? Arrange these unmatched pairs so that they are on facing cube edges with the identical edge piece halves on the same layer of the cube. If the two identical halves are on different layers then rotate a face containing one of the unmatched pairs by 180° . The identical halves are now in the same layer. Rotate a cube half to bring two edge piece halves together. Remove this unmatched pair with an Edge Piece Series. Then replace this pair but invert it in the process. Now when the original turn of half a cube is reversed all edge pieces will be properly paired.

FINISHING THE CUBE

With the face centers in place and all edge pieces matched, Rubik's Revenge may be treated as a Rubik's Cube ($3 \times 3 \times 3$). This means that you turn the faces of the cube, never half a cube. Even though there are two parts to each edge piece they will respond to the Edge Piece Series in exactly the same way as the edge pieces in Rubik's Cube. And even though there are two rows of cubes between corner pieces (rather than one) the corners react to the Corner Piece Series in the same way.

We may find a problem when all of the edge pieces have been placed. One of the edge pieces may be inverted. If we were dealing with Rubik's Cube we would be tempted to believe that our cube had been sabotaged. But with Rubik's Revenge this problem developed when we placed the center pieces. There appears to be nothing we can do to prevent it. We will get it 50% of the time. We just need to learn how to deal with it.

According to Glenn Rhoads (recent e-mail) the inverted edge piece will show up whenever, following the completion of the face centers, the number of permutations of edge pieces required to put all in place is odd. Since the edge piece permutations occur in pairs, if you try to correct the inverted edge piece you will invert some other edge piece.

To eliminate this situation you must change the arrangement of some of the center face pieces. This change must be such as to give an odd number of permutations to the edge pieces. Andy Olsen (e-mail on 7/29/99) suggests that I should use one turn of half the cube. For example:

turn the bottom half of the cube 90° clockwise

If orange is on the top and green in front then white will be on the right side (look at the cube in Fig. 6d). The 90° turn of the bottom half of the cube will disrupt the green, white, yellow and blue faces leaving the orange and red faces intact. We must restore the faces one at a time using the "go and get" method described above. (We cannot simply turn the bottom half 90° counterclockwise.) We are looking for a process which has an even number of turns of half a cube. When these are added to the first turn we just made we will always get an odd number of turns of a cube half. This will invert the problem edge piece. For example, the green face is restored by:

turn the right face 180°
 turn the bottom half 90° clockwise
 turn the right face 180°
 turn the bottom half 90° counterclockwise

Always remember the face color which is to be restored next. It might be wise, whenever you carry out the edge piece inversion, to hold the cube in the same way and match up the same color first. In this case orange is on top and green is restored first. The next color restored will then be white. Now restore the yellow face (this step also restores the blue face). When you have completed this process you will find that only two edge pieces have been broken up. These are restored simultaneously using the method given above for two remaining unmatched edge pieces.

Most cubers will say that either a 90° rotation or a 180° rotation of half a cube (or of a cube face) is a single move. But that does not mean that they are identical when considering permutations or parity. With a Rubik's Revenge a 90° rotation of half a cube will invert an edge piece. A 180° rotation of half a cube will not. The latter is the same as two 90° rotations which means that the first inversion is followed by a second inversion and you are right back where you started from. If you would like to see a further discussion of permutations and parity as applied to various Rubik puzzles refer to Alan Hensel's site, [How to Solve Almost Any Rubik-like Puzzle](#).

There is another potential problem. The 4x4x4 cube, like the 2x2x2 cube, can have six corners in place and two out of place. Or perhaps five are in place while two of the remaining pieces are out of position with the third in position but not properly oriented. But unlike the 2x2x2 cube you cannot get out of this dilemma by simply turning the top face. We will have to be more innovative. Again someone has sabotaged your cube. Not really. That's just the way things are.

To correct this situation rotate the top half of the cube by 180° and remove one of the unmatched edge pieces in the middle section of the cube with an Edge Piece Series. Now return that pair to the same position but inverted by another Edge Piece Series. Do the same thing with the unmatched pair on the opposite side of the cube (which looks just like the first pair). Rotate the top half by 180° .

None of the face centers or edge pieces have been broken. And two corner pieces and half of the edge pieces are still in place. Complete the cube in the standard manner for a Rubik's Cube.

Each time the center pieces are moved into place you will have a 50-50 chance of completing the edge pieces without having one of them inverted. And each time you complete the matching of the edge pieces you will have a 50-50 chance of placing all corner pieces in proper position and alignment. Therefore, 25% of the time you will be able to complete the cube without incident. For 25% of your attempts you will have to change only the center pieces and for another 25% you will have to change only the edge piece pairs. Unfortunately, for another 25% you will have to go through both processes.

How does this method compare to others? In general other methods avoid the two problems described above by completing the edges before the centers and the corners before the edges. But these methods are extremely complex. One needs to learn 50 to 60 new series, (for example, Minh Thai's Rubik's Revenge (Dell, 1982) and Denny's [Rubik's Revenge Solution](#)), and the total number of moves far exceed the number needed here.

Thai's method required an average of about 400 moves. Denny's required 476 moves the first time I tried it and 500 the second. The method described here requires an average of about 140 moves provided that neither of the problems described above occur. The correction for the inverted edge piece requires about 50 moves and that for the misplaced corner piece about 45 moves. Hence, even if you have to make both corrections the average is about 235 moves, far below either of the other methods. And you don't have to memorize any new series!

Rubik's Revenge is also not well suited for racing. Although it is more substantial than Rubik's Pocket Cube, some of the pieces are attached to the turning mechanism by thin plastic pins. If you do not have the various layers properly lined up one or more of these pieces may become stuck and a pin may snap if you try to force it.

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THE ULTIMATE SOLUTION TO THE PROFESSOR CUBE

The Professor Cube (5x5x5) has some of the characteristics of Rubik's Cube (3x3x3) and some of those of Rubik's Revenge (4x4x4). For example, the piece in the center of each face is fixed relative to the other five faces. Hence, as with Rubik's Cube, the face colors are predetermined. You will not have to figure out what the color of any one of the faces should be. It is the color of the piece in the center of the face.

But the edge pieces are like the edge pieces of Rubik's Revenge. They may be inverted. However, the inversion is also different from that in Rubik's Revenge. The edge piece is made up of three small pieces (or cubies according to some). While the first and third will always have their colors aligned the piece in the middle may have its colors inverted from the colors on either side.

Naturally you would not deliberately move the three parts into place with the center one turned the wrong way. However, you can control only the first 11 edge pieces. It's the twelfth edge piece which might have its center section inverted. If you encounter this inversion you will have a parity problem and you will correct it in the same manner as with the inverted edge piece in Rubik's Revenge.

The corner pieces are like those in Rubik's Cube. If six are in place then the last two are in place as well. Unlike Rubik's Pocket Cube and Rubik's Revenge (the even numbered cubes) the odd numbered cubes (Rubik's Cube and the Professor Cube) cannot have six corners in place and two out of place.

Even though you will have, at most, one correction to make, the solution for this cube will still require more moves than Rubik's Revenge. After all, there are now 48 center section pieces which must be moved into place along with 36 edge piece parts. .

THE FACE CENTERS

The first step, as with Rubik's Revenge, is to complete one of the center sections, that is, one of the 3x3 sections in the center of each face. (These 3x3 sections will be treated as a single face center piece in the final part of the solution.) Note that there are three different kinds of "center pieces" in a 3x3 section. The first is the one in the center which can only rotate. The second is the piece with an edge adjacent to an edge of the center piece. These have been called "cross pieces" by [Davenport/Monroe](#) because, when all are in place on one face, the center piece and these four pieces form a cross. The third type touches the center piece only at a corner and therefore we will call them "center/corner pieces". These three types are not interchangeable.

A 3x3 center section is completed by making strings of three pieces each. The first string contains the center piece (first type) and two cross pieces in a straight line. In Fig. 1a two blue pieces, including a center piece, are already together on the top face. A cross piece is missing. It belongs where the yellow piece is now.

Look at the front (red) face. There are two blue faces in the 3x3 center section. Both are cross pieces. We will use the one in the second vertical slice from the left.



Fig. 1a

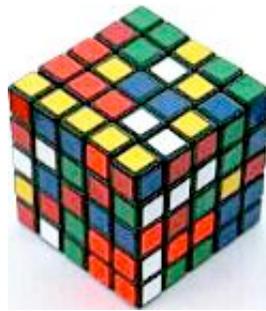


Fig. 1b



Fig. 1c

As a general rule, if we wish to turn an inner (second) slice of the cube we will turn the adjacent face as well simply because it is easier and because there is no reason not to do so. If the two left slices are rotated 90° counterclockwise the blue piece from the front face of the cube will move to be adjacent to the blue center piece on the top face. However our three blue pieces will not be in a straight line. We must prepare the top face prior to the turn of the two left slices. Therefore, we rotate the top face 90° clockwise first (Fig. 1b) and then rotate the two left vertical slices 90° counterclockwise. This gives Fig. 1c. We have our first string of three center section pieces.

The next two strings will both contain one cross piece flanked by two center/corner pieces. For the second string we will use the cross piece in the front face, second vertical slice from the right, of the cube in Fig. 1c. We need to put center/corner pieces both above and below this piece. One such piece is found on the back side of our cube and a 180° rotation of the two bottom slices puts it in place as shown in Fig. 2a.



Fig. 2a

There is another blue center piece of the third kind on the front face. It is at the intersection of the second horizontal slice from the top and the second vertical slice from the left. It needs to replace that red piece above our two blue pieces. This can be done quite easily provided we first move that horizontal row out of the front face. Rotate the top two horizontal slices 90° counterclockwise (Fig. 2b) and then rotate the right face 90° clockwise giving the cube in Fig. 2c. The blue piece has replaced the red piece. Now rotate the top two slices 90° clockwise and the second string is completed

(Fig. 2d).



Fig. 2b

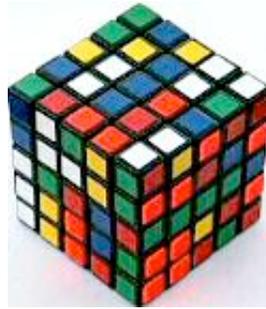


Fig. 2c

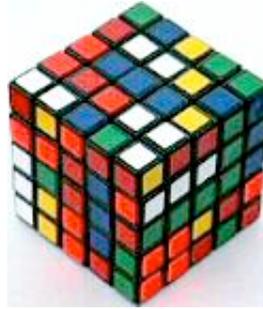


Fig. 2d

This string is put in place by rotating the top face 90° clockwise (Fig. 3a) and then rotating the two right slices 90° clockwise (Fig. 3b). Now we need to complete the third string of blue center pieces (note that two pieces are already together on the front face in Fig. 3b) and move that string into place completing the blue face of the cube (Fig. 3c).

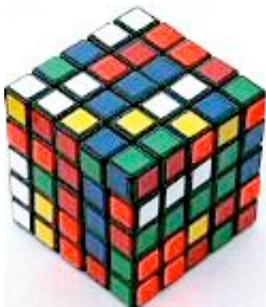


Fig. 3a

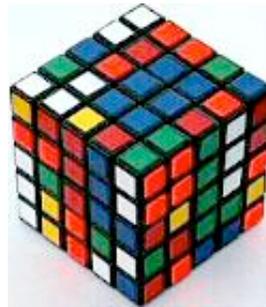


Fig. 3b

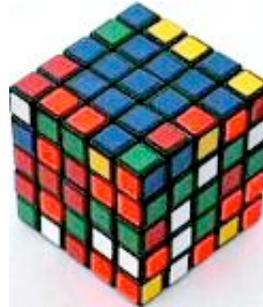


Fig. 3c

The first face was completed without worrying about disrupting faces which had already been completed. But usually with the second face, and certainly with the third, some moves will move pieces out of place in a face which has already been completed. In such cases it is essential that you use a process which may be described as "go and get" the piece which you want to put in place.

For example, consider the cube in Fig. 4a. The blue face (on the left side of the cube) has already been completed. We are working on the green face and have completed the first string of three green face pieces. The second string has two pieces in place in the second vertical slice from the left on the front face. We want to replace the orange face below those two green faces with the green center piece now in the right face (in the second vertical slice from the back). We could rotate the right face 90° clockwise and then rotate the two bottom slices 90° counterclockwise. The string of three green center pieces would now be complete but we would have moved a string of blue center pieces out of place on the left side of the cube.



Fig. 4a



Fig. 4b

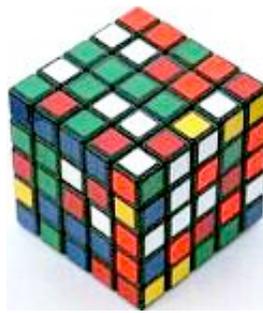


Fig. 4c

To solve this problem we will send the orange piece to the right face to get the green piece. First we will have to move the green piece out of the second horizontal slice. Rotate the right face 90° counterclockwise (Fig. 4b) and then rotate the bottom two slices 90° clockwise giving Fig. 4c. Note that the bottom string of three face pieces from the blue side now appears on the front face. Rotate the right face 180° (Fig. 4d) and the two bottom slices 90° counterclockwise (Fig. 4e). The string of three green pieces is complete. Rotate the top face 90° clockwise and the two left slices 90° counterclockwise and two strings will be in place on the green face.



Fig. 4d

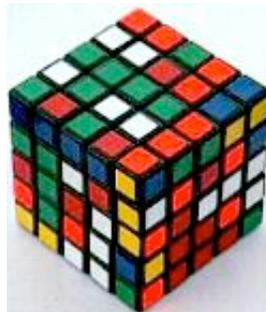


Fig. 4e

The last turn of two slices will not disrupt any faces which have already been completed. Hence the "go and get" process is not needed in this case. If either the yellow or orange faces had been completed then a "go and get" process would have been required to bring the second string of center face pieces into the green face.

We can use the "go and get" process either to bring a center/corner piece from another face into place next to our chosen cross piece or we can move a center/corner piece from the same face into a second face, reorient it, and then bring it back into place next to the chosen cross piece. This is what we did in completing the second string of blue center pieces. The "go and get" process is also used to place cross pieces and to move a string of three center pieces into place on the third or fourth faces.

The first four faces should be completed in such a way that the final two faces are adjacent to each other. This is not an absolute requirement but it helps to visualize the final moves. Let's assume that the final two faces are green and white. Use the "go and get" process to maximize the number of green center pieces in the green face. That is,

send two or three green pieces from the white face (by a quarter turn of the two vertical front or back slices) to the green face, rotate the green face and bring down as many white pieces as possible. You should turn the green and white faces so that white pieces brought from the white face line up with those already in the green face making the white strings in the green face as long as possible.

The aim is either to reduce the number of white pieces on the green face to one center/corner piece or to complete both green and white faces directly. (The former will also reduce the number of green pieces on the white face to one of the same type.) To assist you, here are some typical examples of cube faces which you will encounter.

In Fig. 5a, rotate the two front slices 90° counterclockwise. Rotate the top face 90° clockwise. Rotate the two front slices 90° clockwise. The number of white pieces in the green face has been reduced from four to three. This creates a cube similar to that in Fig. 5c.

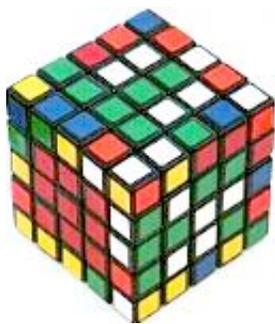


Fig. 5a



Fig. 5b



Fig. 5c

The cube in Fig. 5b is a straightforward example of a "go and get" process in which two strings of three center pieces each are exchanged and all faces are completed.

In Fig. 5c, rotate the two slices on the back of the cube 90° clockwise. Rotate the top face 180° . Rotate the two slices on the back 90° counterclockwise.

In Fig. 5d, rotate the two slices on the front of the cube 90° counterclockwise. Rotate the top face 90° clockwise. Rotate the two slices on the front of the cube 90° clockwise.



Fig. 5d



Fig. 5e



Fig. 5f

In Fig. 5e, rotate the front two slices 90° counterclockwise. Rotate the top face 90° counterclockwise. Rotate the front two slices 90° clockwise.

In Fig. 5f, rotate the front two slices 90° counterclockwise. Rotate the top face 90° clockwise. Rotate the front two slices 90° clockwise.

The moves described for the cubes in 5c, 5d and 5e leave you with one center/corner piece of the opposite color in each of the green and white faces. On the other hand the example in Fig. 5f completes the cube faces.

Most of the time you will wind up with the situation shown in Fig. 6a. In this case the final two faces are completed by the following process: rotate the two back slices 90° clockwise; rotate the top face 90° clockwise; rotate the two back slices 90° counterclockwise. You now have a string of three white pieces in the green face and a string of three green faces in the white face. Use the "go and get" process to exchange these two strings.



Fig. 6a



Fig. 6b

On occasion you might obtain the cube shown in Fig. 6b. In this case the single foreign piece in each of the white and green faces is a cross piece. Rotate the two back slices 90° clockwise. Rotate the top face 90° clockwise. Rotate the two back slices 90° counterclockwise. The result is the same as Fig. 5e, except that both the green and white faces have been turned. Turn the faces so as to duplicate that figure and carry out the process described there.

THE EDGE PIECES

In Rubik's Revenge you had to pair up two parts for each "edge piece". You had no trouble getting the colors to match. In the Professor Cube there are now three pieces which must be brought together. You will have no trouble with the alignment of the two outside pieces. If the yellow color of the blue/yellow piece at the top faces to the right then so will the one at the bottom. You won't be able to put it into its proper space with any other alignment. But the piece in the middle is different. It can go into place facing in either direction and you have to make sure you get it right.

In Fig. 7a two parts of the yellow/white edge piece are already in place so that a 180° horizontal rotation of the top two slices would bring them together. The white face of a yellow/white piece shows at the top of the front/left edge. You can see a yellow face in the middle of the right/back edge. The color on the back side is white. However, the third yellow/white edge piece part is not in place. Rather, you can see it on the right/top edge of the cube. It could be moved into place along side the other yellow/white pieces if it were at the bottom of the right/front edge of this cube, that is, where the orange/yellow piece is Figure 7a. In order that it face in the correct direction its white face must be in the blue face of the cube.



Fig. 7a



Fig. 7b

The following Edge Piece Series will put this yellow/white piece where we want it.

right face 90° counterclockwise

front face 90° clockwise

right face 90° clockwise

front face 90° counterclockwise

The yellow/white piece is now at the lower part of the right/front edge as shown in Fig. 7b.

The bottom two slices are held stationary. The middle horizontal slice is rotated 90° clockwise (looking down from the top) and the top two slices are rotated 90° counterclockwise. This gives the cube in Fig. 7c.

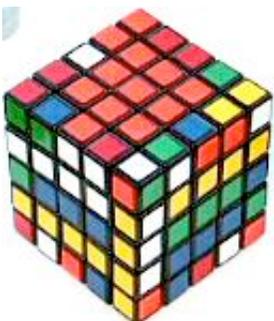


Fig. 7c



Fig. 7d

The yellow/white edge piece has been correctly joined but we must remove it from its

current position in order to preserve it. Again this is done with an Edge Piece Series.

right face 90° clockwise

top face 90° counterclockwise

right face 90° counterclockwise

top face 90° clockwise

This gives the cube shown in Fig. 7d. Reversing the original turns of the horizontal slices restores the four vertical faces (yellow, blue, green and white). You have put one edge piece together. You must now repeat this process for nine other edge pieces.

Sometimes you will find two parts of an edge piece side by side with the proper orientation relative to each other. Good. Find the third part and put it in place relative to the other two using the method described above.

When you have reduced the number of scrambled edge pieces to four or three you must remember to place one of the remaining scrambled edge pieces on either the top or bottom of the cube. This follows because you must use a scrambled edge piece to replace (and thus preserve) the next edge piece you put together.



Fig. 8a

When you get down to two scrambled edge pieces you must change your approach. In Fig. 8a the edge pieces at right/front and right/back are scrambled. The top and middle parts of the edge piece at right/back both have blue on the back side. The bottom part has green on the back side. If you turn the two bottom slices clockwise by 90° you would find that the orange/blue edge piece would be complete. On the other hand, if you turned the two bottom slices 90° counterclockwise you would find that the three blue/green edge piece parts would be together but would have an inverted middle section.

You might think that this would mean that our cube has a parity problem. In fact it does not. You would have a parity problem only if the process described above showed either that both edge pieces have an inverted middle section or that neither has an inverted middle section.

But how do we get these edge piece parts together? Rotate the front face by 180° giving the cube in Fig. 8b. The effect of this turn is to invert all three parts of one of

the scrambled edge pieces. Now instead of having one of its parts in each of the second, third and fourth horizontal slices (as in Fig. 8a) each of the two remaining edge pieces has two of its parts in the same horizontal slice. There are two orange/blue parts in the second horizontal slice from the top of Fig. 8b and two blue/green parts in the fourth horizontal slice from the top.

Now rotate the two bottom slices so that one part of the blue/green pair which do not match is replaced by the second part in the fourth slice from the top (giving the cube in Fig. 8c). (The entire cube in this figure has been rotated 90° counterclockwise about an axis from top to bottom in order to get a complete view of the edge piece in question.) The two adjacent parts of the blue/green edge piece now have matching colors.



Fig. 8b



Fig. 8c



Fig. 8d

Use an Edge Piece Series to remove the scrambled edge piece from the right/front edge. Now use another Edge Piece Series to return this edge piece to the right/front edge but in an inverted position (Fig. 8d). Each of the parts in this edge piece has been turned around. When the two bottom slices are rotated 180° the blue/orange part will return to match the other two blue/orange parts on the left/back of our cube and the blue/green part will come to the right/front and will match the blue/green parts already there.

If you should actually obtain a cube in which the final edge piece is inverted (Fig. 9) then you can change it by using the same method used for Rubik's Revenge. Turn the two bottom slices 90°. This disrupts four faces around the cube as well as four edge pieces (including the faulty edge piece). Restore each face in order around the cube using the "go and get" process. (Do not simply turn the bottom two slices back 90°.) Now restore the four edge pieces. The edge piece inversion will have been removed.



Fig. 9**FINISHING THE CUBE**

Once the faces and the edges have been restored the Professor Cube will behave just like a standard 3x3x3 Rubik's Cube. Hence the cube is completed using the procedures of the [Ultimate Solution to Rubik's Cube](#).

This method requires about 245 moves (nearest multiple of 5) to solve a scrambled Professor Cube provided we do not encounter a parity problem. But an inverted edge piece adds only about 25 moves because we can recognize it even before all edge pieces have been restored. On the other hand, with Rubik's Revenge you must restore all edge pieces and then put them in place before you will know that the last one is inverted. Oddly enough, the number of moves required to solve a Professor Cube, including the correction (about 270 moves), is not much more than that required for Rubik's Revenge with its corrections (about 235 moves).

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THE ULTIMATE SOLUTION TO MEGAMINX

Recently I received an e-mail from Jerzy Wieczorek and Mark Snyder. It seems that these gentlemen have discovered that the methodology of The Ultimate Solution to Rubik's Cube could be applied to Ewe Meffert's Megaminx. While I have had a Megaminx for some time I had never tried to solve it. Behold, a little research showed that they were right.

The Megaminx, a dodecahedron, looks quite formidable. Each of the 12 faces is a pentagon. Each face has five edge pieces and five corner pieces. If one had trouble with Rubik's Cube or Rubik's Revenge, one would surely have a lot of trouble with this fiendish device. After all, the number of corners has gone from eight to 20 and the number of edges from 12 to 30. In addition, while the number of colors is the same as on Rubik's Cube (six) each color is found on two separate areas (the opposite faces each have the same color).

As with Rubik's Cube there are three kinds of pieces on this puzzle. The face center cannot move relative to the other face centers but may rotate. Each edge piece has two different colors and may be moved freely about the Megaminx. Each corner piece has three colors and may also be moved about the entire puzzle.

But there are major differences. For each color combination (say green and orange) there are two identical edge pieces. Either one may be placed between a particular pair of green and orange face centers. This can present a problem as will be discussed later. There are also two different corner pieces with the same set of colors, for example, the green, orange and blue corners. However, these corner pieces are not identical. Rather, they are mirror images of each other. You might be confused by the colors but you will certainly not put the wrong corner piece into any given corner. (Or rather, perhaps, you will not leave it there.)

The steps in solving the Megaminx are similar to those for Rubik's Cube. But there are also significant differences. While the Edge Piece Series still has four turns each turn is $1/5$ of a full rotation or 72° . The series still involves two adjacent faces and causes three edge pieces to move about a given corner piece. In the first turn, edge piece #1 replaces #2 and in the second turn #3 replaces #1. Turn three is the reverse of turn one and turn four is the reverse of turn two. Hence, this is virtually identical to the Edge Piece Series for Rubik's Cube.

Step One: The Star

Hold the Megaminx with any one of the faces on top. I will use a blue face. That is, there is a blue face center on the top face. The other pieces may or may not have a blue color on the top face. You will find that there are five faces, with five different colors in the center, each with an edge in contact with one of the five edges of the top

blue face (see Fig. 1a).



Fig. 1a



Fig. 1b

Bring five edge pieces with a blue color on one face and one of each of the other five colors (yellow, green, red, orange and tan) on the other face, into the upper half of the dodecahedron. By rotating the various faces place these edge pieces on the top face of the cube so that a blue star is formed on the top and the colors of the faces hanging down from the blue face match the colors of the face centers with which they are in contact.

In Fig. 1a the blue/green edge piece is already in place. But you cannot put the blue/red piece in place by a simple counterclockwise rotation of the red face. The green and red colors would not have the proper relationship to each other. By looking at the red and green faces you can see that the red face must be to the left of the green face. Rotate the top face 2/5ths of a turn counterclockwise (Fig. 1b) and now a 1/5th rotation of the red face puts the blue/red and blue/green edge pieces in proper relationship to each other.

You should note that this order of red and green is not universally true. If you had chosen the other blue face as your top face this order would have been reversed. Now place the other three points of the star in the proper order.

Step Two: Second Level Edge Pieces

Turn the puzzle over so that the blue star is on the bottom. You will see positions for five edge pieces (placed horizontally) just above the bottom face of the puzzle. Bring an edge piece of the appropriate colors into one of the positions in the third (middle) level of the puzzle either by rotating one of the faces in the top half of the puzzle or by applying an Edge Piece Series to the appropriate edge pieces. Now apply an Edge Piece Series to move this edge piece into its appropriate position and orientation. Repeat this process for the other four edge pieces of the second level.

In Fig. 2 the yellow/orange edge piece belongs where the tan/green piece is now. It is put into place by an Edge Piece Series which begins with a clockwise rotation of the yellow face. The next turn is a counterclockwise rotation of the orange face.



Fig. 2

Step Three: Third Level Edge Pieces

There are ten edge pieces in the third or middle level. They are inclined at a slight angle to the vertical. Some of them may be moved into place by an Edge Piece Series using edge pieces in the third and fourth levels.

In some cases edge pieces already in place in the third level will prevent the use of an Edge Piece Series to place another edge piece adjacent to them. But you will always be able to put an edge piece into place in the middle level by making a simple edge piece exchange in the top level.



Fig. 3a



Fig. 3b

The yellow/green edge piece in Fig. 3a is already in place. The tan/red edge piece to the right of the yellow/green piece needs to be replaced by a yellow/red edge piece. If you look carefully you will see a yellow/red edge piece on the left side of the top face. A $2/5$ th counterclockwise rotation of the yellow face places the tan/red piece in the top face (Fig. 3b). Now a $1/5$ th rotation of the top face counterclockwise will replace the tan/red edge piece with the yellow/red piece (Fig. 3c). Finally a $2/5$ th clockwise rotation of the yellow face returns the yellow face to its starting position with both the yellow/red and yellow/green edge pieces in their correct positions (Fig. 3d).



Fig. 3c



Fig. 3d

Of course the edge piece you are looking for will not always be on the top face with the proper color hanging down. But you can always apply an Edge Piece Series to move a piece from the fourth level to the top with the proper orientation or to invert a piece already in the top level.

You should also remember that there are two edge pieces for each color pair you are looking for. For example, if you turn the puzzle in Fig. 3b around you will find another space for a yellow/red edge piece on the back side. In this case the red side will be up and the yellow side will be down. In other words, if you still have two yellow/red spots to fill, a yellow/red edge piece in the top layer will fit one or the other of those spots without being inverted.

Continue placing middle edge pieces until all ten middle edge pieces are in position.

Step Four: Fourth Level Edge Pieces

Use the Edge Piece Series to move fourth level edge pieces from the top level into the fourth level where they belong. If you find a fourth level edge piece in its proper place but with the wrong orientation you will have to remove it and put it back in place but correctly oriented. As you carry out this process try to ensure that the top level edge pieces have their blue color on top.

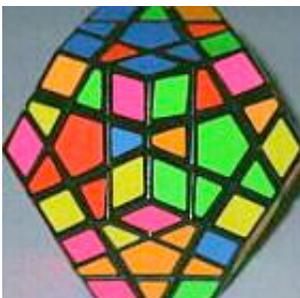


Fig. 4

In Fig. 4 the red/green edge piece must be inverted as it moves into place in the fourth level. Therefore it must be edge piece #3. The blue/tan piece is #1 and blue/red is #2. The green face is rotated clockwise and then the blue face is rotated counterclockwise. Reverse turn one and then reverse turn two. Both the blue/tan and

blue/red pieces wind up with their blue color on top.

Continue until there is one top level edge piece in the fourth level and one fourth level edge piece in the top level. You must have three top edge pieces in order and one out of order.

Step Five: Top Level Edge Pieces

Look at the top level edge pieces. First we need to find if three of them are in order. If not, choose one piece. It is #1. Piece #2 is the one with the color just to the left of color #1 on the faces below the top blue face. Color #3 is the color just to the left of color #2 on these same faces. Use the Edge Piece Series to put the blue edge piece with color #2 in the top face just to the left of color #1. Repeat this process with the blue piece which has color #3. It goes just to the left of the blue edge piece with color #2.

Hold the puzzle with the fourth level target position toward you and turn the top face so that the three top edge pieces in the proper order are in the back. One of the remaining top edge pieces should be in the fourth level edge piece position and the final top edge piece and the final fourth edge piece are in the top level. Apply an Edge Piece Series so that the final fourth level edge piece goes into place with the proper orientation and the top edge pieces are on top with their top color on top.

All thirty edge pieces have now been placed and we are ready for the moment of truth. You should be able to turn the top face so that all top edge pieces line up with the colors of the five faces below. But you have only a 50-50 chance of accomplishing this. Half of the time the last two top edge pieces will be reversed as in Fig. 5.



Fig. 5

This comes about because of the 15 pairs of identical edge pieces. We can assume that in the original puzzle the edge pieces were correctly placed. Every time we exchanged one of the pairs we brought about a reversal. If we did this an even number of times we would be right back where we started. But if we did it an odd number of times we would wind up with a pair of edge pieces reversed.

How do we get out of this parity problem? One way is to exchange two of these identical edge pieces. Suppose we take the blue/yellow edge piece from the top face

and move it down to the second level by means of three successive Edge Piece Series. Then rotate the bottom blue face so that a rotation of the proper lower face will replace the bottom blue/yellow piece with the top blue/yellow piece. Bring the displaced bottom blue/yellow piece to the top face with appropriate Edge Piece Series. Choose these series so as to minimize the disruption to the puzzle. Repair the edge piece portion of the puzzle beginning with the second level and working your way up to the top level.

Now when three top edge pieces are put in place (in the proper order) and an Edge Piece Series is applied to the final two top edge pieces and the fifth fourth level edge piece you will find that all five top level edge pieces are in order. The edge piece reversal has been eliminated.

Step Six: Seventeen Corner Pieces

You will apply the same Corner Piece Series to put corner pieces into place as you did with Rubik's Cube, Rubik's Revenge, etc.. However, it won't look quite the same because you won't have a square on the top of your puzzle. As you looked down on top of Rubik's Cube and started the Corner Piece Series with a turn of the top face to the left you then made the left side turn away from you. Turn the top to the right and then turn the right side away from you.

Hold Megaminx so that it "points" toward you as in Fig. 6. The "right" (tan) and "left" (red) sides of the puzzle are not parallel to each other. Rather, they angle in toward each other at the back side of the puzzle. With the front "prow" of the pentagon pointing straight toward you the right side angles-in by 18° at the back of the pentagon as does the left side as well.



Fig. 6

When the first turn of the Corner Piece Series is to the left the corner piece at the right-front moves to the left-rear. The piece at the left-rear moves to the right-rear and the one at the right-rear moves to the right-front. In this example the tan/green/yellow piece at the left-rear rolls over and goes into proper position and orientation at the right-rear of the top face of the puzzle. The corner piece at the middle-front and the corner piece at the left-front do not move. Hence the major difference is that two top corner pieces do not move when the Corner Piece Series is applied to Megaminx while only one does not move in the case of Rubik's Cube.

Sometimes you will have to temporarily turn faces in order to form a complete triangle about which three corners can move as in Fig. 7. Here the green side may be rotated 1/5th turn clockwise and the blue-orange-yellow corner piece moves to the right-rear of the top face. That piece moves to the left-rear and the left-rear piece (red-blue-yellow) moves into position and orientation at left-front. The piece at left-front moves to right-rear. At the completion of the Corner Piece Series the green face is rotated 1/5th turn counterclockwise.



Fig. 7

As you work to move corner pieces into place you will note that you can seldom move more than one corner piece into place at the same time. This is because there are so many pieces to choose from, because the pieces are far apart on the puzzle and because it is difficult to visualize the proper movement of pieces to get them into position for multiple placement.

The fact that each corner piece has a mirror image simply further confuses the issue. Each piece you look at could go in either of two distinctly different areas on the puzzle. As a general rule you will just have to plod away to get 17 pieces into place.

Step Seven: The End Game

The methodology for the End Game is exactly the same in Megaminx as in Rubik's Cube. But most of the time, in the case of Rubik's Cube, you will find two of the final three pieces side by side. In the case of Megaminx, most of the time you will not find two pieces together as in Fig. 8a. This is going to make The End Game more difficult. The appropriate turns to make a successful End Game may be quite complex. Sometimes it may be helpful to solve only one corner piece while moving one which was already in place if this makes the final arrangement easier to visualize.

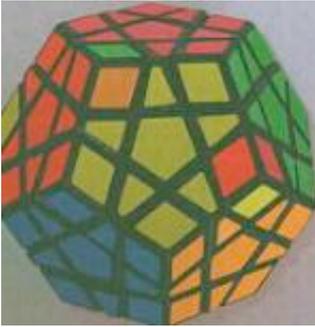


Fig. 8a

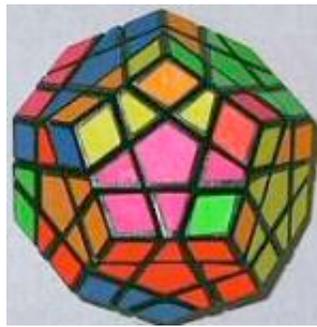


Fig. 8b

In Fig. 8a the pieces to be placed are red-orange-yellow, tan-orange-green and yellow-green-tan (you can see only the yellow color). The puzzle at the conclusion of the preliminary moves is shown in Fig. 8b. What were those moves?

According to Wieczorek and Snyder the use of The Ultimate Solution "made the supposedly near-impossible Megaminx easily solvable on my first try. It is still a long process as pieces can be located at their antipodes, requiring many rotations to place them appropriately, yet it is as simple as the regular Ultimate Solution method."

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OTHER PUZZLES

PYRAMINX

As shown in the section on the Megaminx, a puzzle does not have to be a cube in order to be solved using the methods of The Ultimate Solution to Rubik's Cube. All that is required is that it has corners and edges. Recently I received an e-mail from Patrick Fincannon pointing out that the Ultimate Solution approach makes the solution to Ewe Meffert's Pyraminx and Tetraminx extremely simple. This is indeed true. We do not even need the Corner Piece Series. In the case of the Pyraminx, following a few preliminary moves, edge pieces are placed using the Edge Piece Series. Then the solution is completed by placing the three final edge pieces using a single Edge Piece Series and the principles of the End Game.

The Pyraminx sits on a triangular base and has three other triangular sides. It has three kinds of pieces. There is a three sided tip at each apex. There are four tips. Each of the three faces of a given tip is of a single color. This tip is like the face center of a Rubik's Cube in that it may be rotated but it cannot be moved to another part of the Pyraminx.

The tip is attached to a face piece. The face piece has three sides each of a different color. Like the tip it may be rotated but it also cannot be moved to another part of the Pyraminx. There are four face pieces.

Finally, there are six edge pieces. Each has two faces and each face has a different color. These edge pieces are the only pieces which may actually be moved from one part of the Pyraminx to another.

When solved each face of the Pyraminx will show a single solid color.

Let us assume you are presented with a scrambled Pyraminx. Its colors are orange, green, blue and yellow. The first step is to determine the target color of a single face. Look at one tip. It has three colors. The one I am looking at it is blue, green and yellow. Therefore the color of the opposite face is the color missing from this tip or orange.

Rotate each tip so that its colors match the colors of the face piece to which each is attached. Now rotate one of the apexes in the orange target face so that its orange color comes to that face. This means that you are to turn a pyramid which is two little triangles high. Repeat this maneuver for the other two apexes containing the orange color. Six of the nine pieces in the orange face are now properly placed.

Now look at the fourth apex, the one without an orange face. Rotate that apex so that its colors match the colors in the blue, green and yellow faces. A total of eight pieces are now in place. Six edge pieces remain although it is possible that one or more of them were accidentally moved into the proper position and orientation by these preliminary moves.

Look at the "orange" face of the Pyraminx. An Edge Piece Series (to be described) will cause the three edge pieces you see here to move about the orange face. As you look at this face one edge piece may be described as being on the right. Call it #1. One edge piece is on the bottom (#2) and one is on the left (#3).

Rotate the right hand apex counterclockwise so that edge piece #1 replaces #2. Now rotate the left hand apex clockwise so that #3 replaces #1. Reverse turn one and then reverse turn two.

Edge piece #1 has replaced #2, edge piece #2 has replaced #3 and #3 has replaced #1. Edge pieces #1 and #2 simply roll over into place. That is, #1 will wind up looking just as it did at the end of the first turn. Edge piece #2 will roll over in a similar manner. But #3 does not roll over. Rather, the color of that piece which was in the orange face at the beginning of the series is still in the orange face at the end of the series (although it has moved from the left side to the right side).

If you want the edge pieces to move the opposite way then rotate the left hand apex first and the right hand apex second. If you want a different edge piece to be the one which does not roll over then change the apex which points up, etc..

You may find that two pieces you want to use are in a given face but the third position is occupied by the correct piece with the correct orientation. No problem. Simply rotate a base or bases so as to move another incorrectly placed edge piece into the target face. Apply the Edge Piece Series and rotate the base (bases) back where they came from.

When only three edge pieces remain you are at the End Game. However, now you will use an Edge Piece Series rather than a Corner Piece Series as with the cubic puzzles. Two of the remaining edge pieces will be on the same face. If the third is not then you must bring it to that face with appropriate base turns so that a given Edge Piece Series will move all edge pieces into the correct position and with the correct orientation.

Don't overshoot the mark. You may find that the base turns to get the third edge piece in the proper face are the inverse of the final turns of the Edge Piece Series and you obtain a solved Pyraminx before you expect it.

TETRAMINX

Despite any appearances to the contrary, you will find that the Tetraminx and the Pyraminx are basically the same puzzle. Just take the four tips off of the Pyraminx. This will leave a space the size of one of the small triangles. Paint these four spaces with the four colors of the original Pyraminx and, behold, you have a Tetraminx. Omitting all of the references to the apex tips, the remainder of the directions given above may be applied to solve a scrambled Tetraminx.

There is one point where you may find an alternative approach to be more convenient and faster (it has fewer turns). Assume that you have turned the three apexes containing an orange face so that one face contains three orange triangles, each at 120° to each other. The other three triangles are some other color. You want to place new edge pieces on this surface such that the orange faces of the new edge pieces replace the non-orange colors on this surface and the colors of the faces hanging down match the colors on either side.

This can be done by a simple exchange. Assume an edge piece with a blue color on the orange surface and a green color hanging down between two yellow surfaces. (Check the orientation of the orange/yellow edge piece. It may be found in either of two different ways.) Turn the puzzle so that the top section divides along a diameter and the blue/green edge piece comes down to the lower section of the puzzle. There are two ways you can do this. One way will divide the top along a diameter at the right of the blue/green edge piece. The other will make this division along the left side of the blue/green edge piece. You want the one which will enable the orange/yellow edge piece to replace the blue/green edge piece (by a turn of the bottom section of the puzzle) and then have the orange color come to the top when the original turn is reversed.

When you have replaced the non-orange edge pieces in the orange face with the appropriate orange edge pieces you will be left with three edge pieces out of place. Move them into proper position and orientation as described in the End Game above.

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A COMPARISON OF VARIOUS METHODS

About two decades ago an infernal device appeared on the scene which was destined to rock the international world of puzzles. No doubt more world citizens attempted to conquer Rubik's Cube than any other puzzle known to man. And it is also probable that more people failed in that quest than ever before.

In the period 1980-83 dozens of books were published which attempted to describe solutions in clear and easily understood language. While some were fairly successful with 14 to 16 year olds, most largely failed in the adult world where an amazing number of VCR's blink 12:00 around the clock.

Were you in the group which tried and failed because the descriptions, perhaps, were too obtuse? Or did you actually learn a method, memorizing all of those series of turns of cube faces and then apply it to a scrambled cube? Did you practice diligently and actually get your time down to a minute?

But then you neglected the cube and, over time, forgot some of the series of turns. Perhaps, like the son of a friend of mine, you were embarrassed when your mother called on you to show off before some of her friends and you were unable to do so.

The method described on this web site is for any and all of you. The solution is divided into five parts. It is so simple that, once learned, one cannot forget the moves. You will not have to twist the cube incessantly in order to figure out how the moves are made. However, it is also intended for experienced cubists who have mastered a given method and can still solve a cube in about a minute. That is because, as will be shown, The Ultimate Solution to Rubik's Cube will require only about one-third to two-thirds the number of moves required by most other methods.

Rubik's Cube is enjoying something of a comeback. "Rubik's" is now a trademark of Seven Towns Ltd and the Cube is distributed in the United States by Oddzon Products, Inc.. While the solutions in paperback form are out of print a large number of solutions are readily available on the internet. Clearly these solutions are an effort to improve on others which have gone before. Whatever else may be true, these solutions, and the number of times they are accessed by web users, demonstrate that, after more than two decades, there are thousands of avid fans still interested in Rubik's Cube and other Rubik concoctions.

Nourse's Simple Solution

The majority of solutions on the web appear to trace their lineage back to the solution popularized by James Nourse (The Simple Solution to Rubik's Cube, 1981). Devoid of the shortcuts, either explicit or implied, Nourse built the cube by forming a cross on one face with correctly placed and oriented edge pieces. He then moved corner pieces into correct position and orientation on that same face. He completed the central section of the cube by properly placing and orienting the central edge cubes.

This was accomplished without disturbing the cubes on the first face.

The moves that Nourse used to solve the first face were so simple and straightforward that, once the cubist had practiced them a few times he could carry them out without memorizing a series of moves. However, things became a little more complicated with the placement of edge pieces in the center section.

There were now eight moves in each of two series. While it was possible to follow the progress of the series, it was even easier to memorize each series and make a successful application without understanding the process. If one forgot the series at a later time one had to return to the text to relearn it.

Whatever else may be true, the neophyte, who had been wandering in a maze, could now readily complete two-thirds of the puzzle. To anyone with even a relatively modest interest in the cube this was amazing progress indeed.

But the black box character of the puzzle increased at the next step. Here a 10 move series causes two corner pieces on the bottom face to exchange places. (Nourse, Singmaster, et. al. refer to either a one- quarter rotation or a one-half rotation of a cube face as a "move". Others emphasize "turns". A turn is a one-quarter rotation of a cube face. Hence a one-half rotation of a face is two turns. In this discussion I will use the former convention.) It would be an unusual person indeed who could figure out how the series brought about that result. And he would get no help from Nourse who apparently reasoned that the reader did not need to understand a series, just be able to apply it.

At the next step he does not even tell the reader that a ten move series will cause three of the bottom corner pieces to rotate (in a counterclockwise direction) while remaining in place. Rather, the reader learns how to hold various color patterns on the bottom face and applies the same series over and over again until all bottom corner pieces are properly oriented. The black box nature of Rubik's Cube is firmly established.

It may be inevitable that this type of solution must turn Rubik's Cube into a black box puzzle. Even more of a problem is the fact that the final steps (the placement and orientation of the bottom edge pieces) can require such an inordinately large number of moves. It is possible that these edge pieces could need as many as 63 moves to complete the cube.

Without any of the short cuts this "Simple Solution" requires an average of about 110 moves to complete the cube. Hence, placing and orienting the last four edge pieces might require more moves than the previous 16 pieces combined.

Other Solutions

One would think that there ought to be a shorter way. Many people have looked for it. As a general rule, they accept the Nourse solution for the first two layers and then look for variations in the treatment of the third layer.

Olefsky and Bowen (both on the Internet) and Ostrop (Solving the Cube, 1981) form a cross on the bottom (second) face and then move the edge pieces into proper position (swap). They then move the bottom corners into place and finally rotate the corners. Olefsky makes a change in placing the top corner pieces. He leaves one corner vacant as a "working corner" and places center section edge pieces above this corner using a four move series. When three edge pieces have been placed he puts the fourth corner piece in place and then uses an eight move series to place the fourth center section edge piece.

Using 20 scrambled cubes Olefsky's modification gave a slightly better result than Bowen's, an average of 95 moves to 105 moves. Ostrop's average was a relatively poor 150 moves.

Don Taylor (Mastering Rubik's Cube, 1980) moves corner pieces of the bottom face (final layer) into their correct positions and then moves the edge pieces into position. He then rotates the corner pieces and, finally, flips the edge pieces until all pieces on the bottom layer are correctly placed and oriented. Taylor's method gave an average of 120 moves to solve the scrambled cube.

Minh Thai (The Winning Solution, 1982) places all of the corner pieces before any of the edge pieces. His method appears to be descended from the solution presented by Ideal Toy Company (The Ideal Solution, 1980). In the latter, one places the corner pieces of the first face and then three edge pieces on the first face. This is followed by placing the four corner pieces of the second face and then orienting those pieces. Minh Thai puts in edge pieces only after placing all corner pieces. But he places three edge pieces in the first face and then three edge pieces in the second face in proper position/orientation using the same basic approach. The final two edge pieces (face 1 and face 2) are placed simultaneously. Both approaches complete the cube by orienting the middle edge pieces and then correctly placing them.

Minh Thai's primary contribution was to devise a specific series for every possible situation. Thus he requires 42 series (of up to 14 moves) to solve the cube rather than about 16. But he does it in far fewer moves. Using his method my average for 50 trials was 74 moves with a minimum of 60. The Ideal Solution average was about 130 moves

This approach may be found on the internet on Matt Monroe's cube page. Using his version I solved 20 scrambled cubes with an average of 115 moves.

As a general rule one can reduce the number of moves required by increasing the number of series used in the solution. For example, Nourse's solution requires about a dozen series and gave me an average of about 110 moves to solve the cube. Nourse says that, using his shortcuts, he can get that average down to less than 100 moves. If I use all of Nourse's shortcuts there are about 20 different series and I get an average of 95 moves (in 50 trials) to solve a scrambled cube.

But the ultimate in adding more series to reduce the number of moves in the solution was provided by Jessica Fridrich. Fridrich was the national champion of

Czechoslovakia in 1982 and one of Minh Thai's competitors at the International Rubik's Cube-A-Thon in Budapest.

Fridrich's method is similar to "The Simple Solution" (with shortcuts) for the first two layers. But after that the wheels come off! Jessica will always complete the solution with two series (she calls them algorithms) with an average of 9 and 12 moves respectively. She has determined that there are only 40 possible arrangements (56 counting mirror images) of the corner and edge pieces of the last layer. A specific series may be devised for each which will orient all eight pieces of this layer.

With all pieces oriented we find that some are not in the right position. Again Jessica has devised a specific series for each possible arrangement which will move the pieces into their right positions.

Counting all the possibilities, including the simultaneous placing of corners and edges in the first two layers, one would need to memorize more than 100 cube arrangements and corresponding algorithms in order to use Jessica's method. In addition, to be able to compete with her, you would need to be able to spin cube faces at the rate of about three turns per second. (That let's me out. All I can do is about 1.5 turns per second; but then I am 75 years old.) In addition, I cannot comprehend the monumental memorizing task which she has evidently accomplished.

But I can attest to the fact that it does work. After some preliminary trials to get the hang of it I restored 50 scrambled cubes using Jessica's method. The average number of moves was 58. The minimum was 38.

Ultimately one's personal choice for solving the cube is a matter of taste. If you don't like any of these then check others on the Internet. To see some of what's available, click on [Cubeman's Cube-Related Links](#), [Rubik's Cube Resource List](#) or Matt Monroe's [Rubik's Cube Links](#). (All of the methods from the internet which are referenced here may be reached from one or more of these links pages.)

Many of these are variations on a theme; usually James Nourse's theme. In many instances the contribution of the authors is to include a set of very interesting graphics which may make the solution easier for you to learn. However, if the author is a speed cubist like Lars Petrus (who competed with Thai and Fridrich in Budapest) then the procedure can get very complex in a hurry! Nevertheless you should check out Petrus's solution. It is distinctly different from others you may have seen and the graphics are fantastic!

Petrus has not yet completed his solution on the Web (he has "dozens to hundreds" of examples yet to post) but he claims to average about 60 moves in solving a cube for speed. Hence the 1982 finalists for whom their methods are known average about 58, 60 and 74 moves to solve a scrambled cube. The best that the others can do is about 95 moves. However, these three finalists had to employ 42 to 100+ series and up!

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A COMPARISON OF VARIOUS METHODS

I doubt that anyone will try to outdo Jessica Fridrich by devising a solution with still more algorithms. But there are other approaches. For example, the solution described on this web site places all edge pieces in the correct position/orientation before placing any corner piece. And perhaps the ultimate will not be obtained by increasing the number of algorithms but by reducing them. Would you believe two series?

"The Ultimate Solution to Rubik's Cube" first places all 12 edge pieces using a single series (the [Edge Piece Series](#)). This series is neither new nor unusual. Jake Olefsky uses a similar series in Step 3 of his solution. However, I use a more direct approach. For example: $F R' F' R$ would cause the three edge pieces which are adjacent to the TFR corner to move in a counterclockwise direction about that corner. Its mirror image would cause these same three pieces to move in a clockwise direction. But I deviate even further from Jake and his friends. I use a word description of the series rather than a series of letters.

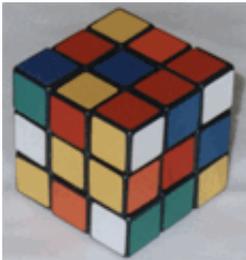
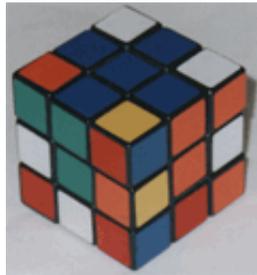
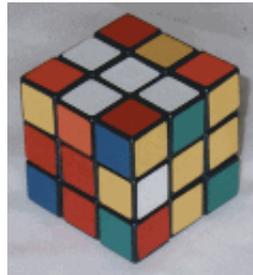


Fig. 1 Start

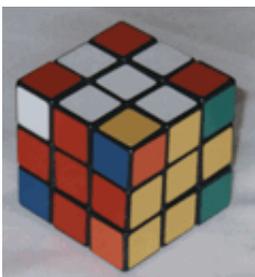


[Fig. 2 Step One](#)



[Fig. 3 Step Two](#)

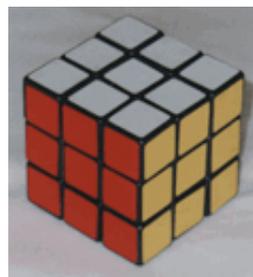
Once all edge pieces are in place the corner pieces are moved into correct position/orientation using a second series (the [Corner Piece Series](#)). This series has eight turns. Again it is not described by a set of letters but by a word description which leads the cubist through the set of turns. It is progressive and symmetrical. One form moves three corner pieces in a clockwise direction about a triangle on the top face of the cube. It's mirror image sends three corner pieces in a counterclockwise direction about a triangle which is a mirror image of the first.



[Fig. 4 Step Three](#)



[Fig. 5 Step Four](#)



[Fig. 6 Step Five](#)

Again it is not new. You can find it in Don Taylor's "Mastering Rubik's Cube" (diagram b, page 24) and Jonathan Bowen's "The Rubik Cube" (part 6 series vi) on the Internet.

Two series. The ultimate solution! Does it work? Of course it does. A consecutive series of 100 scrambled cubes required an average of 65 moves (or 70 turns) to restore the cubes to their original state. The minimum number was 40. The average can be reduced to about 62 moves by using the approach of placing corners and edge pieces simultaneously in the first two layers. But this destroys the simplicity and gains very little. In fact it always takes me longer to solve the cube this way because one must locate two subcubes simultaneously and then ascertain how they will be dealt with as a pair.

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