COSCI 21a, Assignment W1

Directions: To receive full credit:

• Place your name at the top of each page.
• Start each problem on a new page.

1. Prove each by applying directly the definitions given in class of $O$, $\Omega$, and $\Theta$:
   A. $83n$ is $\Theta(n)$
   B. $n^2$ is $\Omega(22n)$
   C. $n^3/1000$ is $\Omega(1000n)$
   D. $2n^4 - 3n^2 + 32n\sqrt{n} - 5n + 60$ is $\Theta(n^4)$
   E. $2n^2 \sqrt{n}$ is not $\Omega(n^3)$

2. Using the definitions given in class of $O$ and $\Omega$, prove that:
   A. $f(n)$ is $O(g(n))$ and $g(n)$ is $O(h(n))$ implies $f(n)$ is $O(h(n))$
   B. $f(n)$ is $\Omega(g(n))$ and $g(n)$ is $\Omega(h(n))$ does not imply $f(n)$ is $\Omega(h(n))$

3. Let $n > 1$ be an integer:
   A. Prove that $\lceil \log_2(n) \rceil + 1$ is the number of bits required to represent $n$ in binary.
   B. Describe in English and give pseudo-code to compute $\lceil \log_2(\lceil \log_2(n) \rceil) \rceil$.

4. Binary search worked by dividing the problem in half. Given and array $A[1] ... A[n]$ of distinct integers that is sorted (i.e., $A[i] < A[i+1]$) and an integer $x$, describe in English and give pseudo-code for ternary search that works as a generalization of binary search algorithm presented in class to find the position of $x$ in $A$ (or determines that $x$ is not in $A$) by dividing the problem into three parts (that is, it works by making at most two comparisons and then narrows the range to size $n/3$).