COSCI 21a, Assignment W8

Directions: To receive full credit:

• Place your name at the top of each page.
• Start each problem on a new page.

1. Describe in English and give pseudo-code to find a cycle in an undirected connected graph that passes through every edge exactly once in each direction; a representation of the cycle should be output (e.g., a list of the edges visited, or a list of the vertices visited, when the cycle is traversed).

   Hint: Consider employing a depth-first search.

2. A graph $G$ is bipartite if its vertices can be partitioned into two sets so that every edge has one of its end points in each set.

   A. Prove that if an undirected connected graph $G$ is bipartite, it has no odd length cycles.
   B. Prove that if an undirected connected graph $G$ has no odd-length cycles, it is bipartite.

   Hint: Perform a breadth-first search to partition the vertices into even and odd depth.

3. Consider the following four types of graphs:
   • directed graphs
   • undirected graphs
   • bi-connected graphs
   • trees (where edges are directed from a vertex to its children)

   Assuming that a depth-first search may start at any vertex, for each of these types of graphs, which of the following can occur in a depth-first search forest; justify your answers:
   • back edges
   • forward edges
   • cross edges
   • more than one child of the vertex used to start the depth-first search
   • more than one tree in the forest

4. Given an $n$ by $n$ matrix $M$ of integers, describe an $O(n^2)$ algorithm for determining whether there exists a permutation $P$ such that permuting the rows of $M$ and the columns of $M$ by $P$ results in an upper triangular matrix (i.e., the entry for the $i^{th}$ row and $j^{th}$ column is 0 if $i>j$).

   Note: For an example of applying a permutation to both the rows and columns of a matrix, consider $n=3$ and the permutation $P$ that exchanges the first and third items and leaves the second item where it was. Permuting columns followed by rows, or rows followed by columns, gives the same result:

   \[
   \begin{pmatrix}
   1 & 2 & 3 \\
   4 & 5 & 6 \\
   7 & 8 & 9 \\
   \end{pmatrix} \rightarrow \begin{pmatrix}
   3 & 2 & 1 \\
   6 & 5 & 4 \\
   9 & 8 & 7 \\
   \end{pmatrix} \rightarrow \begin{pmatrix}
   9 & 8 & 7 \\
   6 & 5 & 4 \\
   3 & 2 & 1 \\
   \end{pmatrix}
   \]

   or

   \[
   \begin{pmatrix}
   1 & 2 & 3 \\
   4 & 5 & 6 \\
   7 & 8 & 9 \\
   \end{pmatrix} \rightarrow \begin{pmatrix}
   7 & 8 & 9 \\
   1 & 2 & 3 \\
   3 & 2 & 1 \\
   \end{pmatrix} \rightarrow \begin{pmatrix}
   9 & 8 & 7 \\
   6 & 5 & 4 \\
   3 & 2 & 1 \\
   \end{pmatrix}
   \]

   Hint: Consider employing a topological sort.