Example: "Towers of Hanoi" Puzzle

**Problem:** You are given three posts labeled $A$, $B$, and $C$.

On Post $A$ there are $n$ rings of different sizes, in the order of the largest ring on the bottom to the smallest one on top.

Posts $B$ and $C$ are empty.

The object is to move the $n$ rings from Post $A$ to Post $B$ by successively moving a ring from one post to another post that is empty or has a larger diameter ring on top.
Inductive proof that the puzzle has a solution:

(basis) One ring can always be moved.

(inductive step) Assume that we can move \(n-1\) rings from any given post to any other post. Since any of the rings 1 through \(n-1\) can be placed on top of ring \(n\), all \(n\) rings can be moved as follows:

1. Move \(n-1\) rings from Post A to Post C.
2. Move ring \(n\) from Post A to Post B.
3. Move \(n-1\) rings from Post C to Post B.

Example: The solution for 3 disks takes 7 steps:

```
1 3 3
2 1 2
3 2 1
B C A
C A B
B C A
C A B
```
Recursive Algorithm for Towers of Hanoi

The proof by induction is constructive because it tells you how to solve a problem with $n$ rings in terms of solutions to problems with $n–1$ rings; it corresponds directly to a recursive program.

$\text{TO}W\underline{E}R(n,x,y,z)$ moves $n$ rings from Post $x$ to Post $y$ (using Post $z$ as a "scratch" post):

procedure $\text{TO}W\underline{E}R(n,x,y,z)$
if $n>0$ then begin
\begin{itemize}
  \item TOWER($n–1,x,z,y$)
  \item write "Move ring $n$ from $x$ to $y$."
  \item TOWER($n–1,z,y,x$)
\end{itemize}
end
end