Specifying the Viewing Coordinates

- **Viewing Coordinates system**, \([x_v, y_v, z_v]\), describes 3D objects with respect to a viewer.
- A **viewing plane** (*projection plane*) is set up perpendicular to \(z_v\) and aligned with \((x_v, y_v)\).
- In order to specify a viewing plane we have to specify:
  - a vector \(N\) normal to the plane
  - a *viewing-up vector* \(V\)
  - a point on the viewing plane

- \(P_0=(x_0, y_0, z_0)\) is the point where a camera is located
- \(P\) is a point to **look-at**
- \(N=(P_0-P)/|P_0-P|\) is the view-plane normal vector
- \(V=z_w\) is the view up vector, whose projection onto the view-plane is directed up
Viewing Coordinate System

\[ z_v = N \quad ; \quad x_v = V \times N \quad ; \quad y_v = z_v \times x_v \]

- The transformation \( M \), from world-coordinate into viewing-coordinates is:

\[
M = \begin{bmatrix}
    x_1 & x_2 & x_3 & 0 \\
    y_1 & y_2 & y_3 & 0 \\
    z_1 & z_2 & z_3 & 0 \\
    0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
    1 & 0 & 0 & -x_0 \\
    0 & 1 & 0 & -y_0 \\
    0 & 0 & 1 & -z_0 \\
    0 & 0 & 0 & 1
\end{bmatrix} = R \cdot T
\]

- Defining the camera in OpenGL:

```c
glMatrixMode(GL_MODELVIEW);
glLoadIdentity();
gluLookAt(P0x, P0y, P0z, Px, Py, Pz, Vx, Vy, Vz);
```

Projections

- Viewing 3D objects on a 2D display requires a mapping from 3D to 2D

- A projection is formed by the intersection of certain lines (projectors) with the view plane

- Projectors are lines from the center of projection through each point in the object

Projections

- Center of projection at infinity results with a parallel projection

- A finite center of projection results with a perspective projection

Parallel Projection

- Projectors are all parallel

- Orthographic: Projectors are perpendicular to the projection plane

- Oblique: Projectors are not necessarily perpendicular to the projection plane

Orthographic Projection

Since the viewing plane is aligned with \((x_v,y_v)\), orthographic projection is performed by:

\[
\begin{bmatrix}
    x_p \\
    y_p \\
    0 \\
    1
\end{bmatrix}
= \begin{bmatrix}
    x_v \\
    y_v \\
    0 \\
    1
\end{bmatrix}
\begin{bmatrix}
    1 & 0 & 0 & 0 \\
    0 & 1 & 0 & 0 \\
    0 & 0 & 0 & 1 \\
    0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
    x_v \\
    y_v \\
    z_v \\
    1
\end{bmatrix}
\]
**Orthographic Projection**

- Lengths and angles of faces parallel to the viewing planes are preserved
- **Problem**: 3D nature of projected objects is difficult to deduce

**Oblique Projection**

- Projectors are **not** perpendicular to the viewing plane
- Angles and lengths are preserved for faces parallel to the plane of projection
- Preserves 3D nature of an object

**Oblique Projection**

- Two types of oblique projections are commonly used:
  - **Cavalier**: $\alpha=45^\circ = \tan^{-1}(1)$
  - **Cabinet**: $\alpha=\tan^{-1}(2) \approx 63.4^\circ$

**Oblique Projection**

- **Cavalier** projection:
  - Preserves lengths of lines perpendicular to the viewing plane
  - 3D nature can be captured but shape seems distorted
  - Can display a combination of front, side, and top views
- **Cabinet** projection:
  - Lines perpendicular to the viewing plane project at 1/2 of their length
  - A more realistic view than the Cavalier projection
  - Can display a combination of front, side, and top views
**Perspective Projection**

- In a perspective projection, the center of projection is at a finite distance from the viewing plane.
- The size of a projected object is inversely proportional to its distance from the viewing plane.
- Parallel lines that are not parallel to the viewing plane converge to a *vanishing point*.
- A vanishing point is the projection of a point at infinite distance.

**Vanishing Points**

- There are infinitely many general vanishing points.
- There can be up to three *principal vanishing points* (axis vanishing points).
- Perspective projections are categorized by the number of principal vanishing points, equal to the number of principal axes intersected by the viewing plane.
- Most commonly used: one-point and two-points perspective.

**Perspective Projection**

- Using similar triangles it follows:
  \[
  \frac{x_p}{d} = \frac{x}{z + d} ; \quad \frac{y_p}{d} = \frac{y}{z + d} \\
  x_p = \frac{d \cdot x}{z + d} ; \quad y_p = \frac{d \cdot y}{z + d} ; \quad z_p = 0
  \]

Thus, a perspective projection matrix is defined as:

\[
M_{\text{per}} = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & \frac{1}{d} & 1
\end{bmatrix}
\]

\[
M_{\text{per}} \cdot P = \frac{x}{z + d} \quad \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & \frac{1}{d} & 1
\end{bmatrix} = \begin{bmatrix}
x \\
y \\
z \\
z + d
\end{bmatrix}
\]

\[
\Rightarrow \quad x_p = \frac{d \cdot x}{z + d} ; \quad y_p = \frac{d \cdot y}{z + d} ; \quad z_p = 0
\]
**Perspective Projection**

- $M_{\text{per}}$ is singular ($|M_{\text{per}}|=0$), thus $M_{\text{per}}$ is a many to one mapping (for example: $M_{\text{per}}P = M_{\text{per}}2P$)
- Points on the viewing plane ($z=0$) do not change
- The homogeneous coordinates of a point at infinity directed to $(U_x, U_y, U_z)$ are $(U_x, U_y, U_z, 0)$. Thus, The vanishing point of parallel lines directed to $(U_x, U_y, U_z)$ is at $[dU_x/U_z, dU_y/U_z]$
- When $d \to \infty$, $M_{\text{per}} \to M_{\text{ort}}$

**Projections**

What is the difference between moving the center of projection and moving the projection plane?

**Projections**

Planar geometric projections

- Parallel
- Perspective
- Oblique
- Orthographic
- One point
- Top
- Front
- Two point
- Side
- Other
- Three point
- Cavalier
- Cabinet

**Moving the Center of Projection**

**Moving the Projection Plane**