Do: See the solutions file for these.
• 2.23
• 3.7a
• 4.5c
• 5.4a,c Write the frequencies simply as integers; e.g. .04 → 4

7.1 Discussed:
Denote squares of the input tape by $I[j]$ and those of the output tape by $O[j]$.

1) Give pseudocode for a fixed-length program that copies its input tape to its output tape and halts.

   \[ \text{j} \leftarrow 1 \]
   \[ \text{while} \ I[j] \neq \text{BLANK} \]
   \[ \text{O}[j] \leftarrow I[j] \]
   \[ j \leftarrow j+1 \]
   HALT

2) Can this program be the prefix of any other program?
   No, because it has halted.

3) Give an upper bound on the length of minimal programs that print the string $x_k^1$ and halt. $\ell(x_k^1) + c = k + c$, $c$ independent of $x$.

4) Can the program in the proof of C&T Theorem 7.2.2 be the prefix of a longer program? Yes; further instructions could follow.

5) Given a set $S$, prove that there is at least one member whose program length is at least $\log |S|$.

   The number of binary strings of length less than $\log |S|$ is

   \[ \sum_{i=0}^{\log |S|-1} 2^i = 2^{\log |S|} - 1 = |S| - 1 \]

   so at least one element must have length $\log |S|$.

6) Given a string $z$ of length $n$, how many distinct pairs $(x, y)$ are there such that $z = xy$?

   There are $(n + 1)$ ways to parse $z = xy$ into $x$ and $y$.

7) Conclude that $\exists x, y : K(x, y) \geq K(x) + K(y) + \log n$.

   There are $2^n$ strings $z$ and $n+1$ ways to parse each. By Part 5, a program of length at least $\log(n + 1)2^n \geq \log n + n$ is needed; by Part 3 the costs of $x$ and $y$ don’t exceed $n + c$. 
Chapter 8:

(1) If you earlier reported experiencing some difficulty in Chapter 8, resolve that difficulty now. Otherwise, discuss briefly why channel capacity is the same with and without feedback on the erasure channel. If an erasure channel has error rate $\alpha$, then $n\alpha$ of every $n$ bits sent provoke a repeat transmission. Of these, $n\alpha^2$ must be retransmitted, of these $n\alpha^3$, and so on. The total number of bits sent is $n(1+\alpha+\alpha^2+\cdots) = n/(1-\alpha)$. So the rate of “good” bits received per bits sent is $n(1-\alpha) = C$.

(2) Create a simple noisy channel, showing two typical transmitted sequences, two typical received sequences, a jointly typical pair of sequences, and a pair of typical sequences which are not jointly typical. Assume a BSC with error rate 1/8, and use four-bit words. Then if 0000 and 1111 are two transmitted code words, 1000 and 1110 are two typical received words. The pair (0000, 1000) is jointly typical; the pair (0000, 1110) is not.

(3) Do Problem 8.11 but with these error-transition probabilities:

\[
p(y|x) = \begin{cases} 
1/2 & \text{if } y = x \\
1/2 & \text{if } y = x + 2 \mod 5 \\
0 & \text{otherwise.}
\end{cases}
\]

The capacity is the same as it was before, namely $\lg 5 - \lg 2 \doteq 1.236\ldots$. A code of one-symbol words can achieve rate 1 but no more. There are codes of two-symbol words, the same ones as for the original problem, that achieve rate $(\lg 5)/2$. 