The Orb (Orb-It)

This fascinating puzzle is called Orb in the USA and Orb-It in the UK. The puzzle consists of a ball with two small rings of beads near the poles, and two larger rings parallel to and near the equator. The four rings have different colours. The bead rings rotate around the ball. The ball itself is divided vertically into two halves. By rotating one half the rings of beads become either one long spiral, or two loops (looking a bit like a tennis ball). In the solution below the spiral position is never used.

The beads in the polar rings are blue or yellow. The remaining beads are red or green.

It might be interesting to quote from the afterword of the Rubik's Cubic Compendium [p213] written by David Singmaster.

A further type of closely related puzzle is what I call the 'switchable cycle' puzzles. Here one has several disjoint cycles of pieces with some mechanism for splitting the cycles. The best known example was the Orb-It (UK) or Orb (US), which had four parallel tracks on a sphere. One hemisphere could be turned to eight positions with respect to the other, causing the half-tracks to join up in several different ways. This was marketed in both Europe and the United States.

It was invented by Christopher C. Wiggs & Christopher J. Taylor, and they filed for a US patent on 16 February 1982, which was granted in 19 November 1985, US 4,553,754. An earlier British patent, GB 8113543, dates from 1 May 1981.

The number of positions:
There are 8 beads in each of the polar rings and 20 in the equatorial ones, which gives: $56! / (8!^220!^2) = 73,888,773,475,012,113,089,523,051,000$ or $7.4 \cdot 10^{28}$ positions. This considers rotations of the rings as the different positions, and does not include the other loop shapes.
Solution:
Phase 1: Separate the polar/equatorial beads. In this phase the blue/yellow beads are placed in the polar rings, and the red/green beads are therefore displaced to the equatorial rings.

a. Suppose there is a red/green bead in each of the polar rings, and a blue/yellow bead in each of the middle rings. Then rotate the rings to place those four beads at the following positions:
   1. Top ring: at the front, immediately to the left of the seam dividing the two halves of the orb.
   2. Second ring: at the front, immediately to the right of the seam.
   3. Third ring: at the back, next to the seam in the right hand half of the orb.
   4. Bottom ring: at the back, next to the seam in the left hand half of the orb.
b. Rotate the right hand side a quarter turn in either direction, to create the position with two loops.
c. Slide the beads in the right hand loop anti-clockwise so that the four beads cross over the seam.
d. Rotate the right hand side back again.
e. Repeat the above as often as possible.
f. If you now have a position with 4 or more odd-coloured beads but they are not spread over all 4 rings, then rotate any rings with more than one odd bead so that they occur on both halves of the puzzle and any rings with a single odd bead so that it lies in the left hand side. Follow this with a half turn of one side of the puzzle to spread the odd beads over all the rings. Now go back to steps a-d.
g. If you have a position with only 2 odd-coloured beads, then use the method in a-d above to place only one of them correctly. This will disturb three other beads, leaving you with 4 odd-coloured beads which can be solved as above.

Note that the above method doesn't just work with a single bead in each ring. It works just as well with a pair of adjacent beads in each ring, or even three of four.

Phase 2: Separate red and green, and similarly blue and yellow.

a. Find an incorrectly placed bead in each of the equatorial rings.
b. Place the incorrect beads as follows:
   1. First ring: At the front, to the left of the seam.
   2. Second ring: At the front, to the right of the seam.
c. Rotate the right hand side through 180 degrees.
d. Shift the first ring to the right, so that both beads cross the seam.
e. Rotate the right hand side back 180 degrees.
f. Repeat the above till the equatorial rings are correct.
g. Repeat the above for the polar rings.

Note that the above moves don't just work with a single bead in each ring. It works just as well with a pair of adjacent beads in each ring, or even three or more.