Towers Of Hanoi

*a.k.a. Pyramid Piling Puzzle, Rainbow Puzzle, Brahma Puzzle*

Very old design, this puzzle purchased from Bits And Pieces 2007.
(wood stand and seven wood discs, 2.3" by 5.3" base by 3.5" high)

On Post A there are \( n \) rings of different sizes, in the order of the largest ring on the bottom to the smallest one on top. Posts B and C are empty. The object is to move the \( n \) rings from Post A to Post B by successively moving a ring from one post to another post that is empty or has a larger diameter ring on top.

Solution: Since any of the rings 1 through \( n-1 \) can be placed on top of ring \( n \), all \( n \) rings can be moved by invoking the recursive procedure TOWER:

\[
\text{procedure TOWER}(n,x,y,z) \quad \text{if } n>0 \text{ then begin} \\
\quad \text{TOWER}(n-1,x,z,y) \\
\quad \text{write }"\text{Move ring } n \text{ from } x \text{ to } y." \\
\quad \text{TOWER}(n-1,z,y,x) \\
\quad \text{end} \\
\text{end}
\]

\( \text{TOWER}(n,x,y,z) \) makes \( 2^n-1 \) moves; for example, \( \text{TOWER}(3,A,B,C) \) takes 7 steps:

\[
\begin{array}{ccc}
1 & 2 & 2 \\
3 & 3 & 1 \\
A & B & C \\
\end{array} \quad \begin{array}{ccc}
1 & 2 & 1 \\
3 & 3 & 2 \\
A & B & C \\
\end{array} \quad \begin{array}{ccc}
1 & 3 & 2 \\
1 & 3 & 3 \\
A & B & C \\
\end{array} \quad \begin{array}{ccc}
1 & 2 & 1 \\
3 & 1 & 2 \\
A & B & C \\
\end{array} \quad \begin{array}{ccc}
1 & 3 & 2 \\
1 & 3 & 3 \\
A & B & C \\
\end{array}
\]

"Unwinding" the recursion of TOWER, yields the following simple iterative algorithm that moves the discs on post in the clockwise direction:

\[
\text{if } n \text{ is odd then } d := \text{clockwise} \quad \text{else } d := \text{counterclockwise} \\
\text{repeat} \\
\quad \text{Move the smallest ring one post in direction } d. \\
\quad \text{Make the only legal move that does not involve the smallest ring.} \\
\text{until all rings are on the same post}
\]

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The Pyramid Piling Puzzle Version Of Towers Of Hanoi

Pyramid Piling Puzzle, Well-Maid Wood Products, Suffield, CT, unknown age.
(cardboard box 3.7" x 5" x 3/4", wood base and pieces, directions, info. sheet; discs were lost at some point and replaced)

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The Rainbow Puzzle Version Of Towers Of Hanoi

Rainbow Puzzle, unknown age.
(2.5" diameter by 1.6" wood box with three 3/16" diameter 2" long wood pegs, and 8 wood discs, from 11/16" diameter to 1+15/16" diameter; to play, turn box over and put pegs into the holes in the bottom)

Further Reading
Excerpt from J. A. Storer's book.
Claus Page from: http://www.cs.wm.edu/~pkstoc/toh.html
Ajtai Patent, from: www.uspto.gov - patent no. 5,992,851

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