The Gear Cube Extreme

The Gear Cube Extreme, or Anisotropic Gear Cube, was invented by Oskar van Deventer, based on an idea by Bram Cohen. It is like a 3×3×3 Rubik's Cube where eight edge pieces are cogs that turn when the outer layers are moved. If you give any side face a half turn, the middle layer moves a quarter turn due to the edge cog wheels. The four edges in the horizontal middle slice are normal, so the top and bottom faces can be turned normally.

The puzzle is manufactured and sold by Uwe Meffert. It is a very hard puzzle. On the standard version the edge pieces only have stickers on the rotating cog, but it is possible to put stickers on the non-rotating base part of the edge pieces to make it even harder.

This puzzle should not be confused with the much easier normal Gear Cube, in which all 12 edges are cogs.

The number of positions:

Instead of the centres or corners, lets use a middle layer edge piece as a fixed reference point. The eight corners cannot be twisted, but can be rearranged in 8! ways.
The eight cog edges can also be arranged in all 8! ways and individually twisted in 3 ways each.
The face centre pieces of course move as a unit as they are fixed on the axes of the mechanism, but all 24 orientations of the axes are possible.
The locations of the three moving edge in the middle layer are fully determined by the permutation of the face centres.
This makes the total number of positions equal to 8!·8!·3^{8·24} = 255,989,602,713,600.

If the bases of the eight cog edges also have stickers on them, then their orientation becomes visible. Any even number of them can be flipped. This multiplies the number of positions by 2^7 for a total of 8!·8!·3^{8·24·2^7} = 32,766,669,147,340,800 positions.
Links to other useful pages:

Uwe Meffert's pages. He produces and sells the Gear Cube as well as many other puzzles.

Notation:

The puzzle will always be held such that the layer without cogs is horizontal. Let F denote a clockwise half turn of the front face, keeping the rear face stationary. Similarly, let R denote a clockwise half turn of the Right face. A clockwise quarter turn of the upper face is denoted by U, and a quarter turn of the lower face by D. Anti-clockwise turns are denoted by an apostrophe, e.g. R’ denotes an anti-clockwise half turn of the Right face.

Solution:

Phase 1: Solve the corners.
This is equivalent to solving the corners on a Rubik's Domino.

- a. UBL - Whatever corner lies at the UBL location, consider it to be correct and use it as a reference point for the rest of the corners.
- b. UBR - If the corner that belongs at the UBR location (relative to the solved UBL corner) lies in the U layer, then do F to put it in D layer. Rotate D until it is at DFR and then do R.
- c. UFL - If in U layer but not correct then do F to put it in D layer. Rotate D until it is at DFR and then do F.
- d. UFR - If not correct then rotate D until it is at DFR and do R D’ R D R.
- e. Turn the D layer until at most two corner pieces are incorrect.
- f. Use one of the following to swap two incorrect corners in the D layer:
  1. To swap adjacent corners DBL and DBR, do RF D’ FDR D’ FDR D’.
  2. To swap diagonally opposite corners DFL and DBR, do F DD RDF DD RDR DD F D’.

Phase 2: Position the cog edges.
Their orientation is ignored for now.

- a. Solve the DB edge as follows:
  1. Find the edge that belongs at the DB location.
  2. If it is in the bottom layer, do F F R R to bring it to the top layer.
  3. Turn the U layer to put the edge piece at the UF location.
  4. Do R R to solve the edge piece.
  5. Optionally, turn the U layer back to how it was.
- b. Solve the DF edge as follows:
  1. Find the edge that belongs at the DF location.
  2. If it is in the bottom layer, do F F to bring it to the top layer.
  3. Turn the U layer to put the edge piece at the UB location.
  4. Do R U R U R U R’ U R U’ to solve the edge piece. This useful move sequence will be used again later. It swaps two edges UB and DF.
  5. Turn the U layer back to how it was.
- c. Turn over the puzzle and hold it so that the two solved edges are at the UL and UR locations.
- d. Solve the remaining edges using the following procedure:
  1. If the edge at the UB location belongs in the bottom layer, go to step 5.
  2. If the edge at the UF location belongs in the bottom layer, do U U and go to step 5.
  3. Turn the D layer so that any incorrect edge lies at the DF location.
  4. Do R U R U R U R’ U R U’ to swap the edges at the DF and UB locations.
  5. Look at the colours of the edge at UB, and turn D to bring the two corners with those colours to the front.
  6. Do R U R U R U R U U’ to swap the edges at the DF and UB locations.
  7. Repeat steps 1-6 until all the cog edges match the colours of the adjacent corners.
  8. Turn the U and/or D layers so their colours match each other.
Phase 3: Solve the middle layer edges.

a. Shift the middle layer until the correct edge lies at the BL location.
b. If the BL edge is flipped, then
   1. Shift the middle layer to the left so that the BL edge moves to the BR location
   2. do an R move.
   3. Shift the middle layer a half turn so that the BL edge is now correct
   4. Do an R' move.
c. If the edge that belongs at FL lies at FR, then do UD' F U'D R'. If instead the edge belonging at FL lies at BR then do R' UD' F U'D.
d. If the FR and BR edges need to be swapped, then do R U U R U R U U.

Phase 4: Orient the cog edges.

a. If three or four of the edges in the R middle slice are twisted (the UF, UB, DF, DB edges), then do R R R R once or twice until at most two of them remain twisted.
b. If three or four of the edges in the F middle slice are twisted (the UL, UR, DL, DR edges), then do F F F F once or twice until at most two of them remain twisted.
c. Hold the puzzle with any twisted edge at the UL location and do the sequence
   
   (U' R'R'R') x4 (R' U) x4 (R') x4
   You may have to repeat it if the edge is not yet correctly oriented.
d. Repeat step c for each of the twisted edge cogs.

Another useful move sequence for the above phase, one which twists both UF and UB, is (RRRR U R'R' U') x2.

Phase 5: Flip the edges

This phase is only necessary on the version of the puzzle with the extra stickers on the base parts of the edges.

a. If six or eight of the edges are flipped, then do R F R F R F R to flip all of them, leaving at most two flipped.
b. If four edges are flipped then split them in two pairs, each of which you can solve with the steps below.
c. To flip one edge in the top layer and one in the bottom layer,
   1. Hold the puzzle so that the DF edge is one that needs to be flipped.
   2. Turn the U layer so that a flipped edge lies at the UF location.
   3. Do the sequence RRR U RRR U R'R'R' U R U U R' R U' R U R U R' R'R'R' U
   4. Turn the U layer back to how it was.
d. To flip two adjacent edges in the same layer,
   1. Hold the puzzle with the two edges to be flipped at the UF and UL locations
   2. Do F U.
   3. Do the sequence in step c3 above
   4. Do U' F'.
e. To flip two non-adjacent edges in the same layer,
   1. Hold the puzzle with the two edges to be flipped at the UF and UB locations
   2. Do R'
   3. Do the sequence in step c3 above
   4. Do R.

Phase 6: Solve the centres.

a. If all the side centres are incorrect, do RR UD' F'F' U'D.
b. If only the Front/Back centres are correct, do R UD' F'F' U'D R.