Example: Non-Recursive Towers of Hanoi Algorithm

Idea: If we unwind the recursion, it is not hard to see that the recursive Towers of Hanoi algorithm alternates between moving the smallest ring and one of the other rings, and that the smallest rings moves in a regular clockwise or counterclockwise fashion.

Lemma 1: In any minimal length solution to the Towers of Hanoi puzzle, the first and every other move is with the smallest ring.

Proof: Two consecutive moves with the smallest ring could be combined into a single move. Two consecutive moves of a larger ring must be from a peg back to itself (since the third peg must have the smallest peg on top), and hence could be eliminated.
Lemma 2: In any minimal length solution to the Towers of Hanoi puzzle, for odd $n$, the small ring always moves in a clockwise direction ($A$ to $B$ to $C$ to $A$ ...) and for even $n$ in a counterclockwise direction ($A$ to $C$ to $B$ to $A$ ...).

Proof: The move in the TOWER procedure goes from Peg $x$ to Peg $y$. For $n=1$, $x,y=A,B$ and the move is clockwise. If we inductively assume the lemma true for $n–1$ rings, then it follows for $n$ by observing that both recursive calls reorder the pegs so that the relationship between the source and destination pegs (clockwise or counterclockwise) reverses.
Algorithm to move rings clockwise one post:

if $n$ is odd then $d := \text{clockwise}$ else $d := \text{counterclockwise}$
repeat
Move the smallest ring one post in direction $d$.
Make the only legal move that does not involve the smallest ring.
until all rings are on the same post

Time: $\Theta(2^n)$ — The same sequence of moves as the recursive version.

Space: $O(n)$ — Unlike the recursive version, the configuration of the rings must be explicitly represented; it suffices to use 3 stacks, each holding a maximum of $n$ rings.

Observation: Both the mechanical translation of the recursive program to a non-recursive one and the "hand coded" version employ the stack data structure and use $O(n)$ space, although the constants in practice for both time and space are likely to be smaller for the hand-coded version.