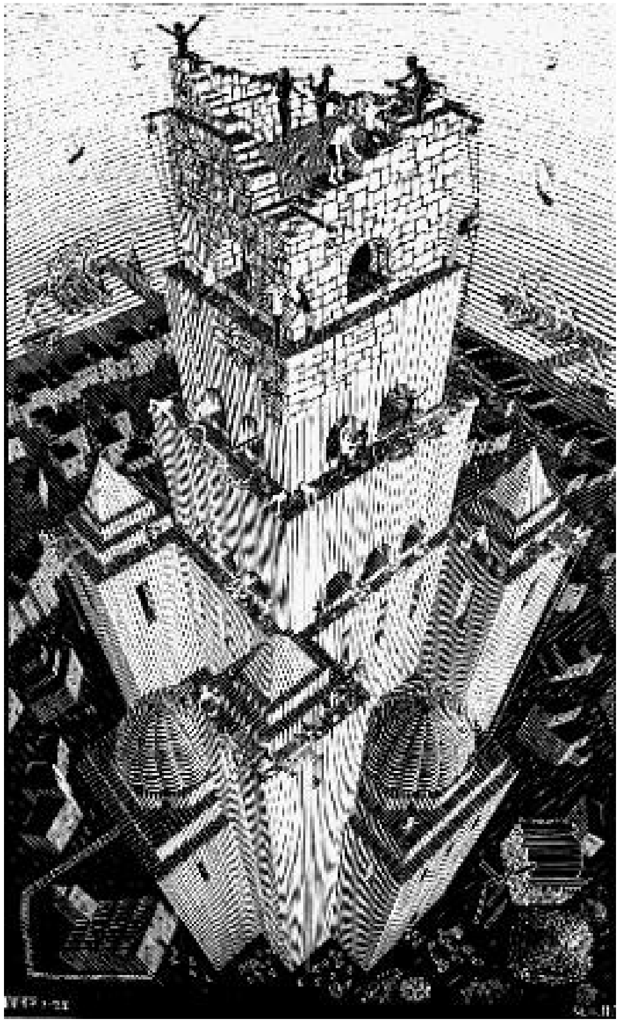
Viewing in 3D

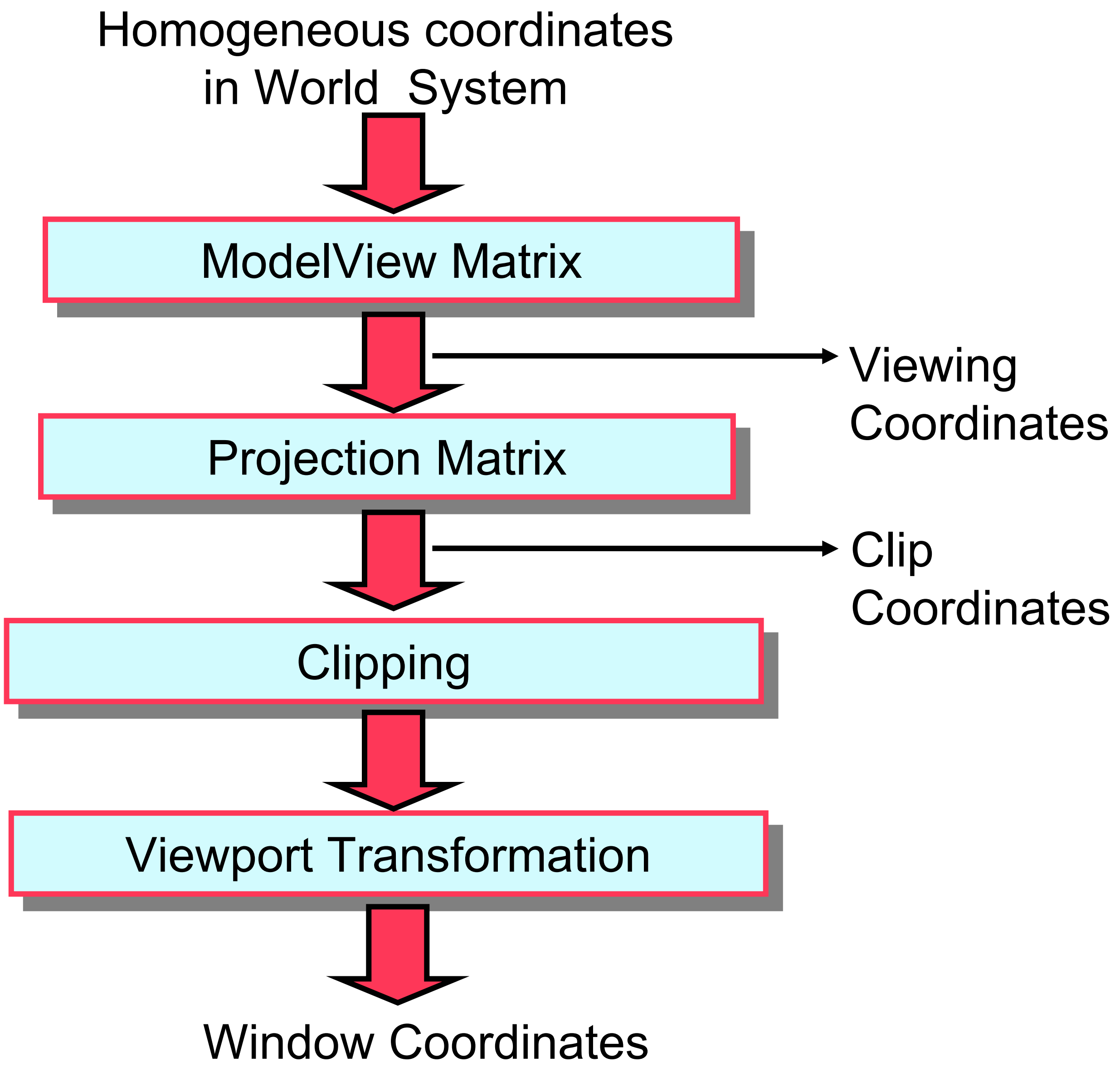
Foley & Van Dam, Chapter 6



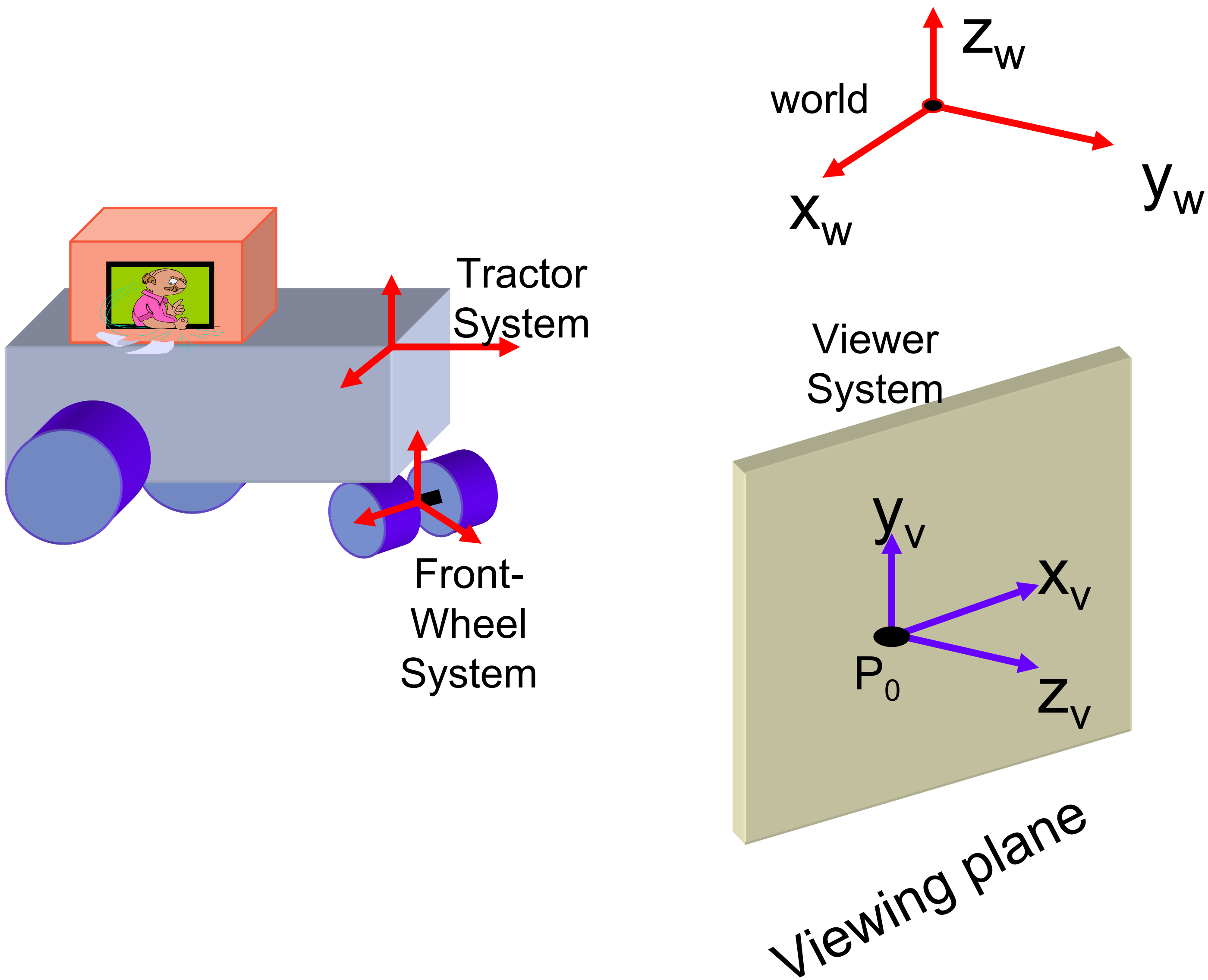
Viewing in 3D

- Transformation Pipeline
- Viewing Plane
- Viewing Coordinate System
- Projections
  - Orthographic
  - Perspective

OpenGL Transformation Pipeline



Viewing Coordinate System



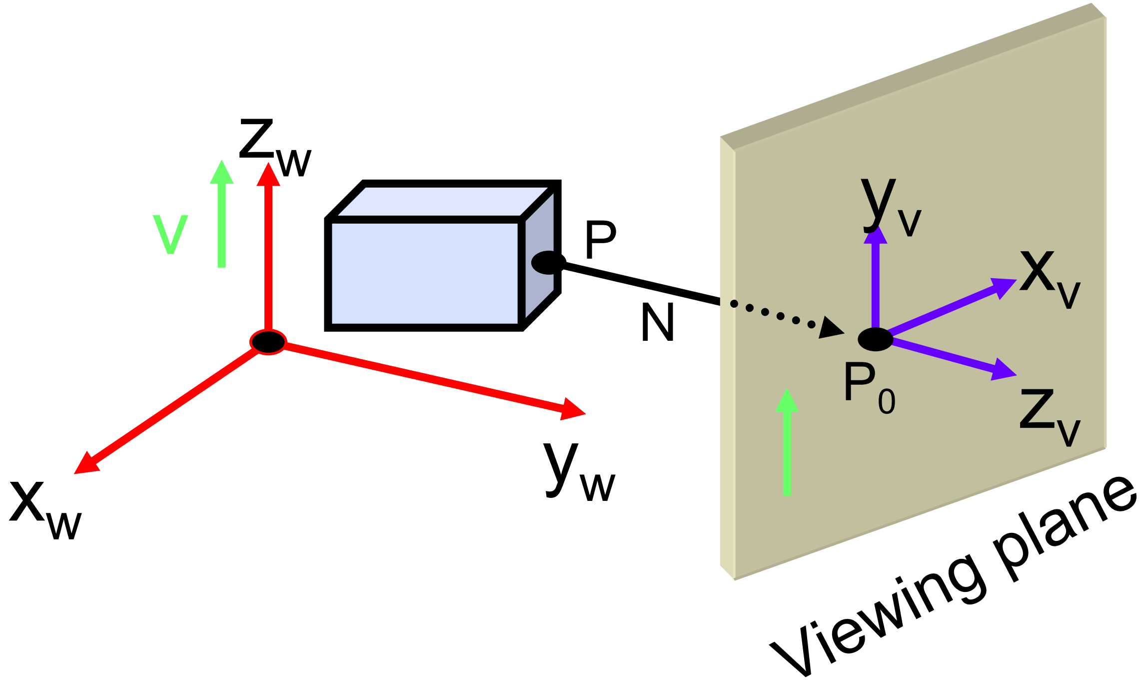
Specifying the Viewing Coordinates

- *Viewing Coordinates system*, $[x_v, y_v, z_v]$, describes 3D objects with respect to a viewer

- A viewing plane (*projection plane*) is set up perpendicular to z_v and aligned with (x_v,y_v)

- In order to specify a viewing plane we have to specify:
  - a *vector* N normal to the plane
  - a *viewing-up vector* V
  - a point on the viewing plane

Specifying the Viewing Coordinates



- $P_0=(x_0,y_0,z_0)$ is the point where a camera is located
- P is a point to look-at
- $N=(P_0-P)/|P_0-P|$ is the view-plane normal vector
- $V=z_w$ is the view up vector, whose projection onto the view-plane is directed up

Viewing Coordinate System

$$z_v = N \quad ; \quad x_v = \frac{V \times N}{|V \times N|} \quad ; \quad y_v = z_v \times x_v$$

- The transformation M, from world-coordinate into viewing-coordinates is:

$$M = \begin{bmatrix} x_v^1 & x_v^2 & x_v^3 & 0 \\ y_v^1 & y_v^2 & y_v^3 & 0 \\ z_v^1 & z_v^2 & z_v^3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -x_0 \\ 0 & 1 & 0 & -y_0 \\ 0 & 0 & 1 & -z_0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = R \cdot T$$

- Defining the camera in OpenGL:

glMatrixMode(GL_MODELVIEW);

glLoadIdentity();

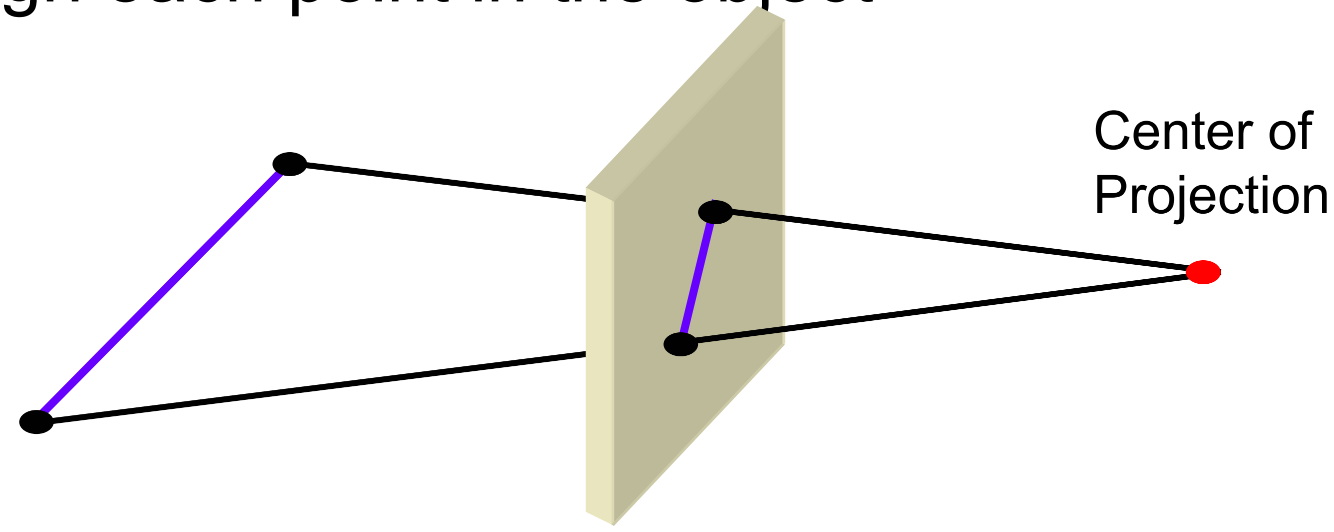
gluLookAt(P_{0x}, P_{0y}, P_{0z}, P_x, P_y, P_z, V_x, V_y, V_z);

Projections

• Viewing 3D objects on a 2D display requires a mapping from 3D to 2D

• A projection is formed by the intersection of certain lines (*projectors*) with the view plane

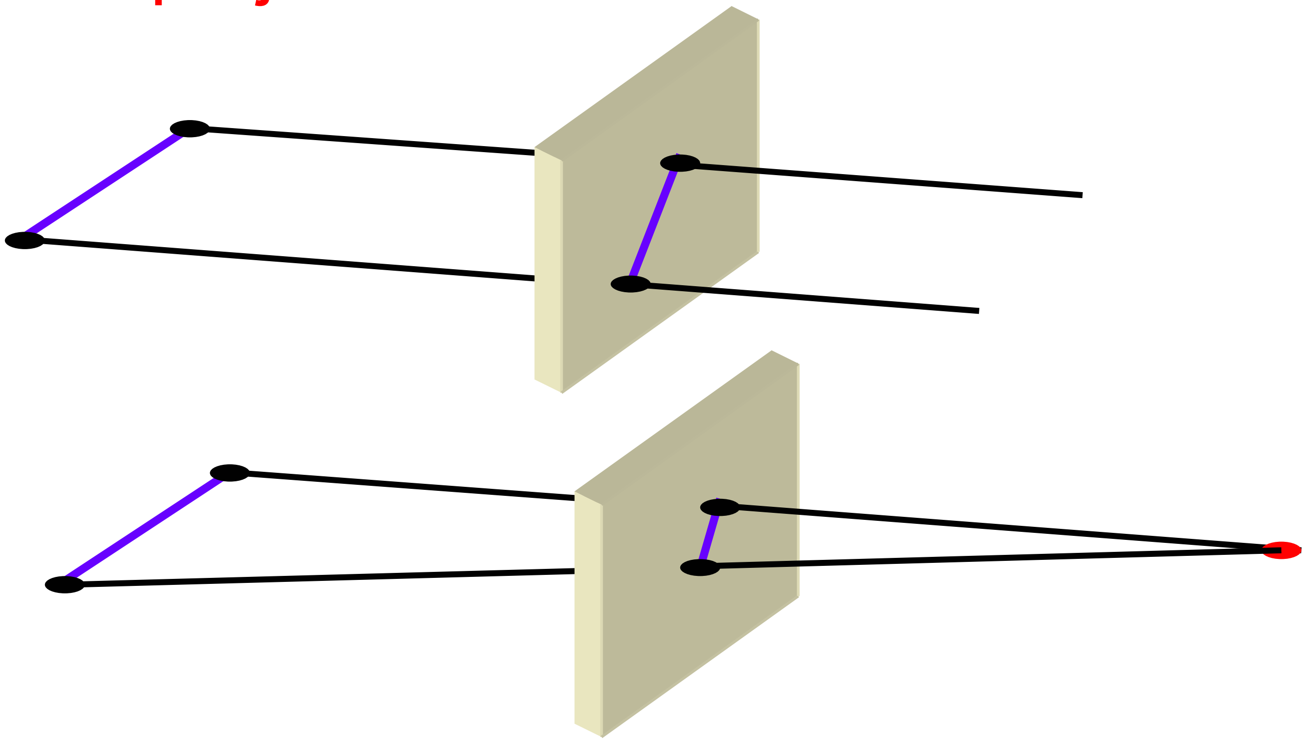
• Projectors are lines from the *center of projection* through each point in the object



Projections

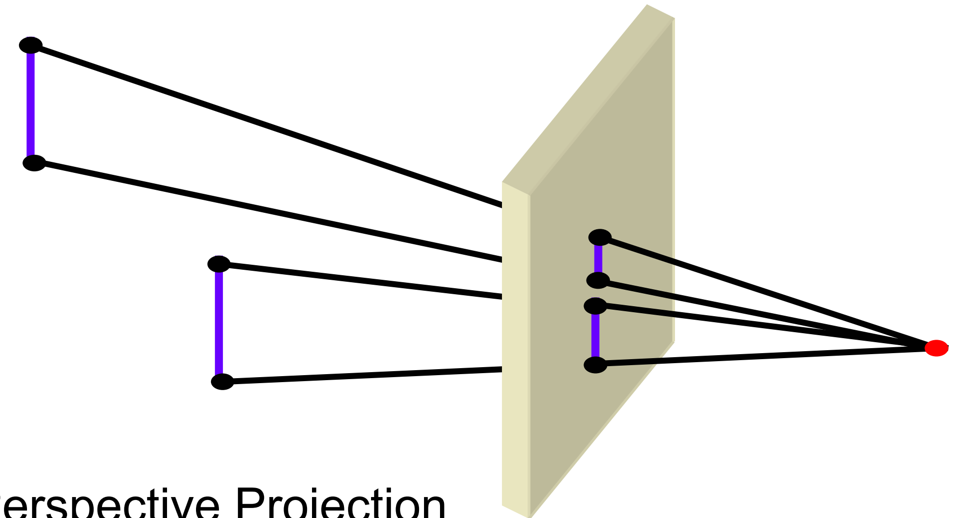
- Center of projection at infinity results with a parallel projection

- A finite center of projection results with a perspective projection



Projections

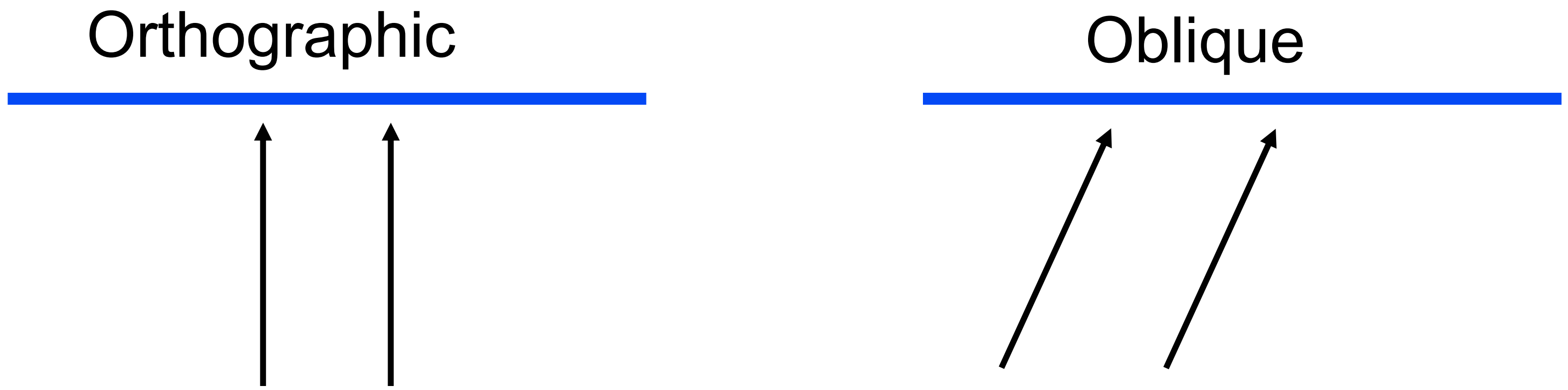
- Parallel projections preserve relative proportions of objects, but do not give realistic appearance (commonly used in engineering drawing)
- Perspective projections produce realistic appearance, but do not preserve relative proportions



Perspective Projection

Parallel Projection

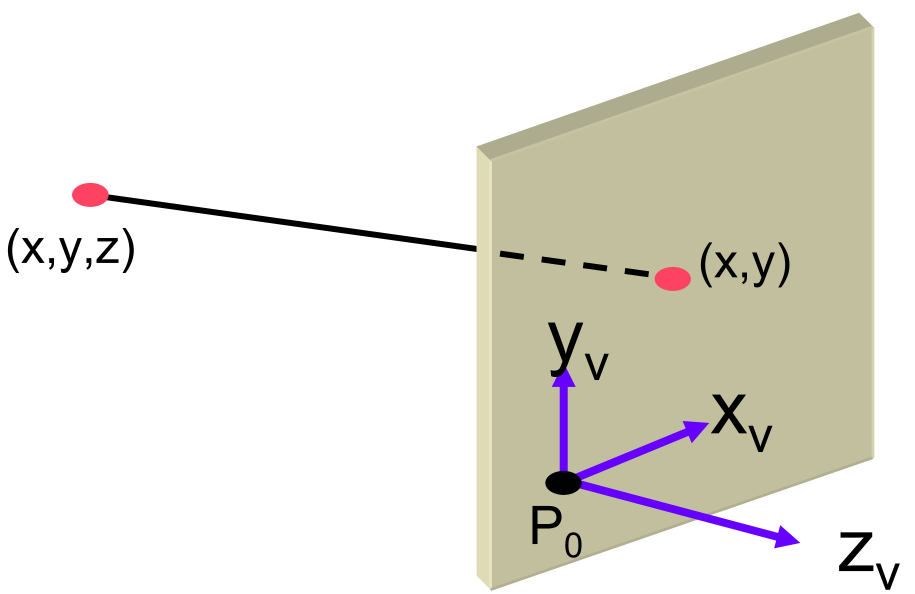
- Projectors are all parallel
- Orthographic: Projectors are perpendicular to the projection plane
- Oblique: Projectors are not necessarily perpendicular to the projection plane



Orthographic Projection

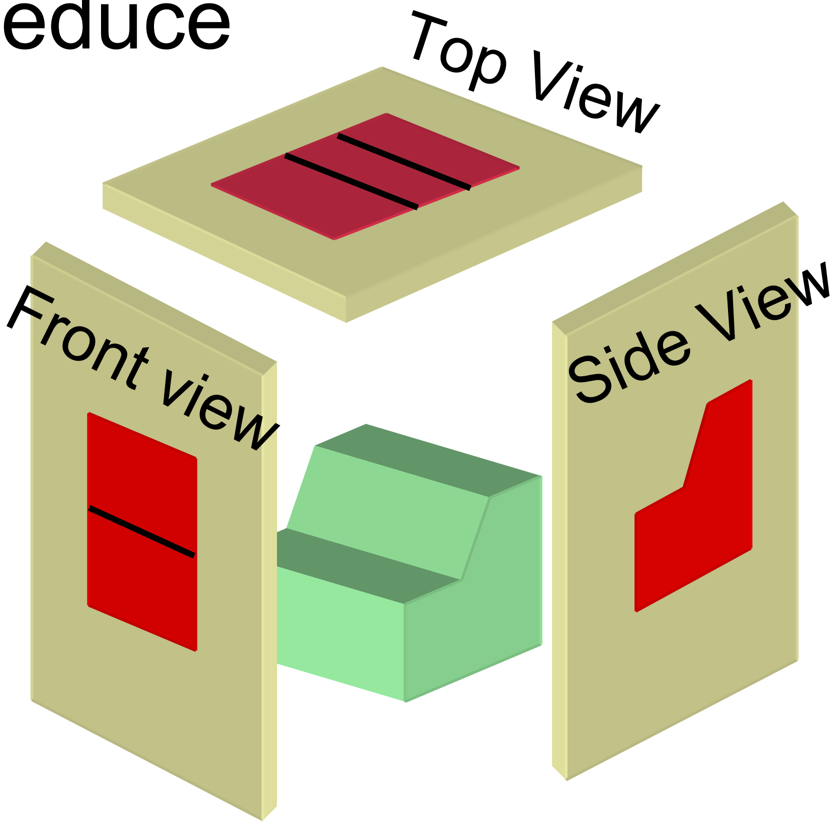
Since the viewing plane is aligned with (x_v, y_v), orthographic projection is performed by:

$$\begin{bmatrix} x_p \\ y_p \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} x_v \\ y_v \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_v \\ y_v \\ z_v \\ 1 \end{bmatrix}$$



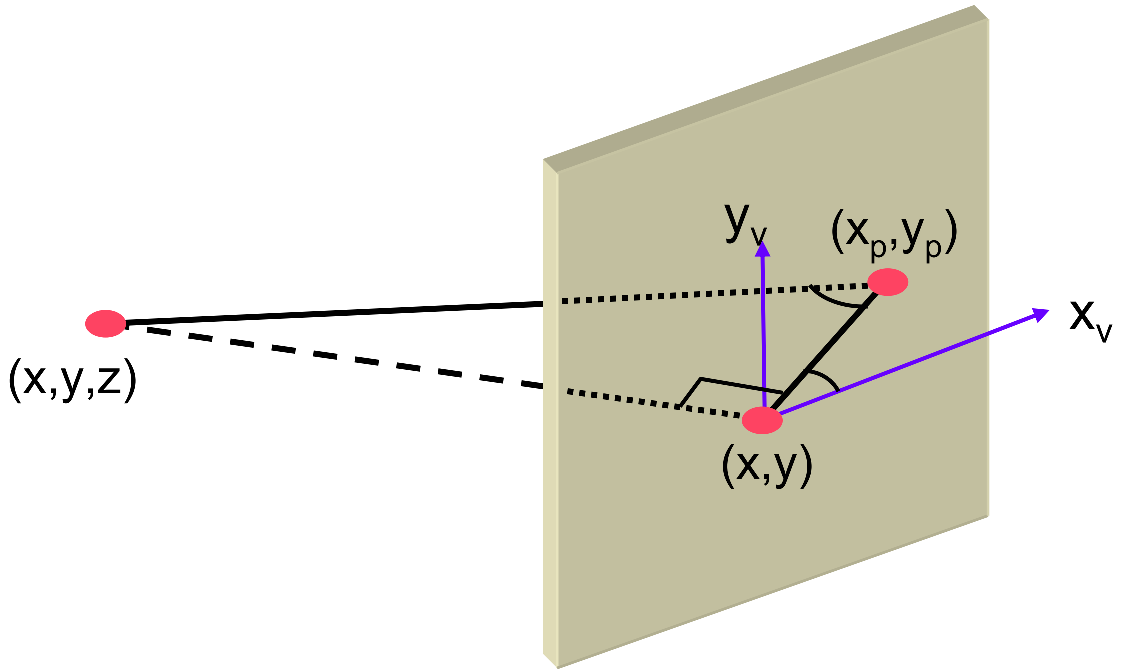
Orthographic Projection

- Lengths and angles of faces parallel to the viewing planes are preserved
- **Problem**: 3D nature of projected objects is difficult to deduce



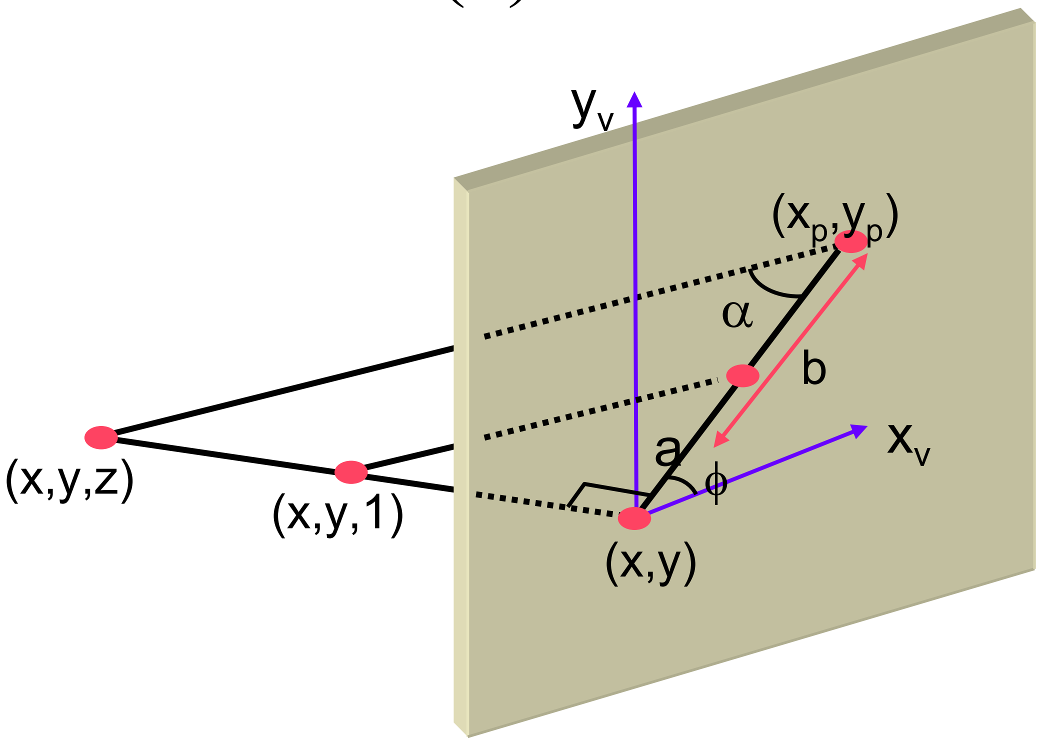
Oblique Projection

- Projectors are ***not*** perpendicular to the viewing plane
- Angles and lengths are preserved for faces parallel to the plane of projection
- Preserves 3D nature of an object



Oblique Projection

- Two types of oblique projections are commonly used:
  - Cavalier: $\alpha = 45^o = \tan^{-1}(1)$
  - Cabinet: $\alpha = \tan^{-1}(2) \approx 63.4^o$



Oblique Projection

$$\begin{bmatrix} x_p \\ y_p \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & a\cos\phi & 0 \\ 0 & 1 & a\sin\phi & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_v \\ y_v \\ z_v \\ 1 \end{bmatrix} = \begin{bmatrix} x_v + z_v a\cos\phi \\ y_v + z_v a\sin\phi \\ 0 \\ 1 \end{bmatrix}$$

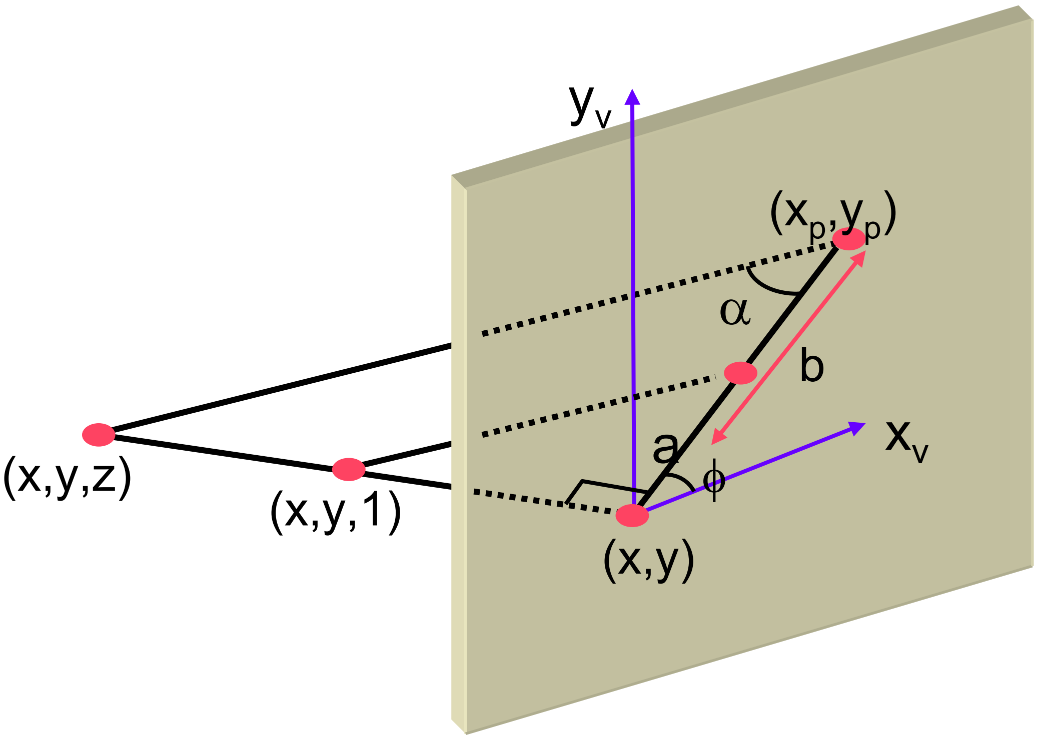
$1/a = \tan(\alpha)$

$z/b = 1/a$

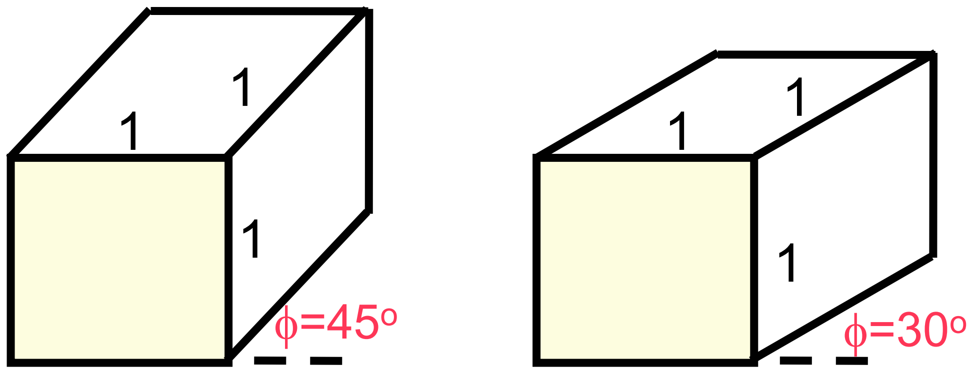
$b = za$

$x_p = z \cdot a \cdot \cos(\phi)$

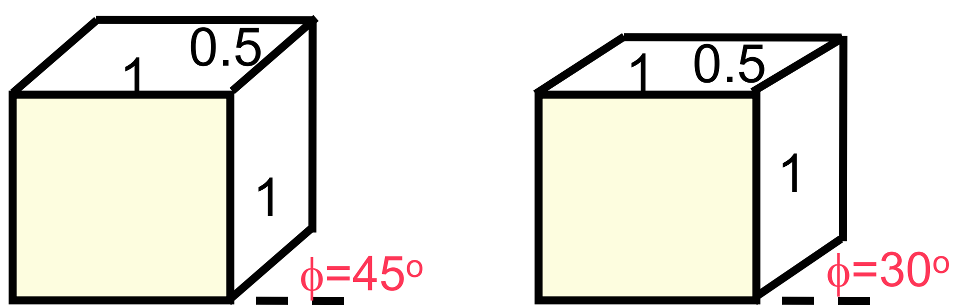
$y_p = z \cdot a \cdot \sin(\phi)$



Oblique Projection



Cavalier Projections of a cube
for two values of angle φ



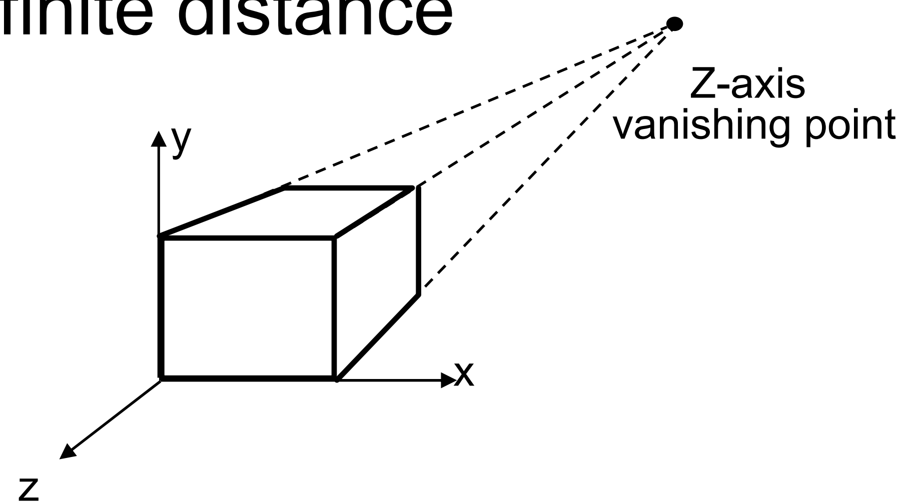
Cabinet Projections of a cube
for two values of angle φ

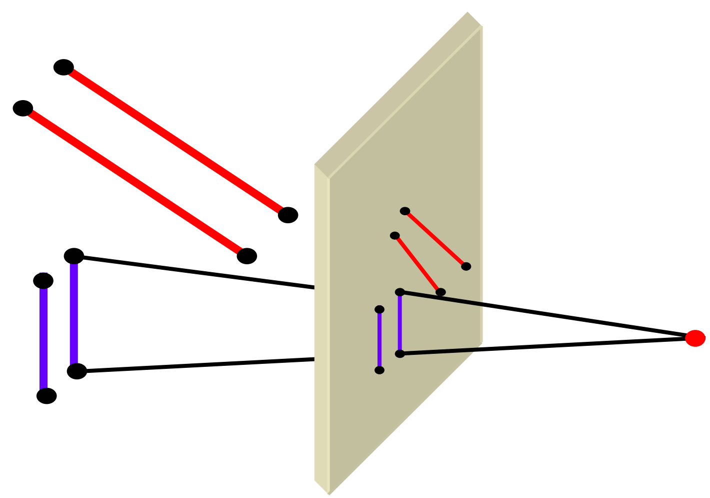
Oblique Projection

- **Cavalier** projection :
  - Preserves lengths of lines perpendicular to the viewing plane
  - 3D nature can be captured but shape seems distorted
  - Can display a combination of front, side, and top views
- **Cabinet** projection:
  - Lines perpendicular to the viewing plane project at 1/2 of their length
  - A more realistic view than the Cavalier projection
  - Can display a combination of front, side, and top views

Perspective Projection

- In a perspective projection, the center of projection is at a finite distance from the viewing plane
- The size of a projected object is inversely proportional to it distance from the viewing plane
- Parallel lines that are not parallel to the viewing plane, converge to a *vanishing point*
- A vanishing point is the projection of a point at infinite distance



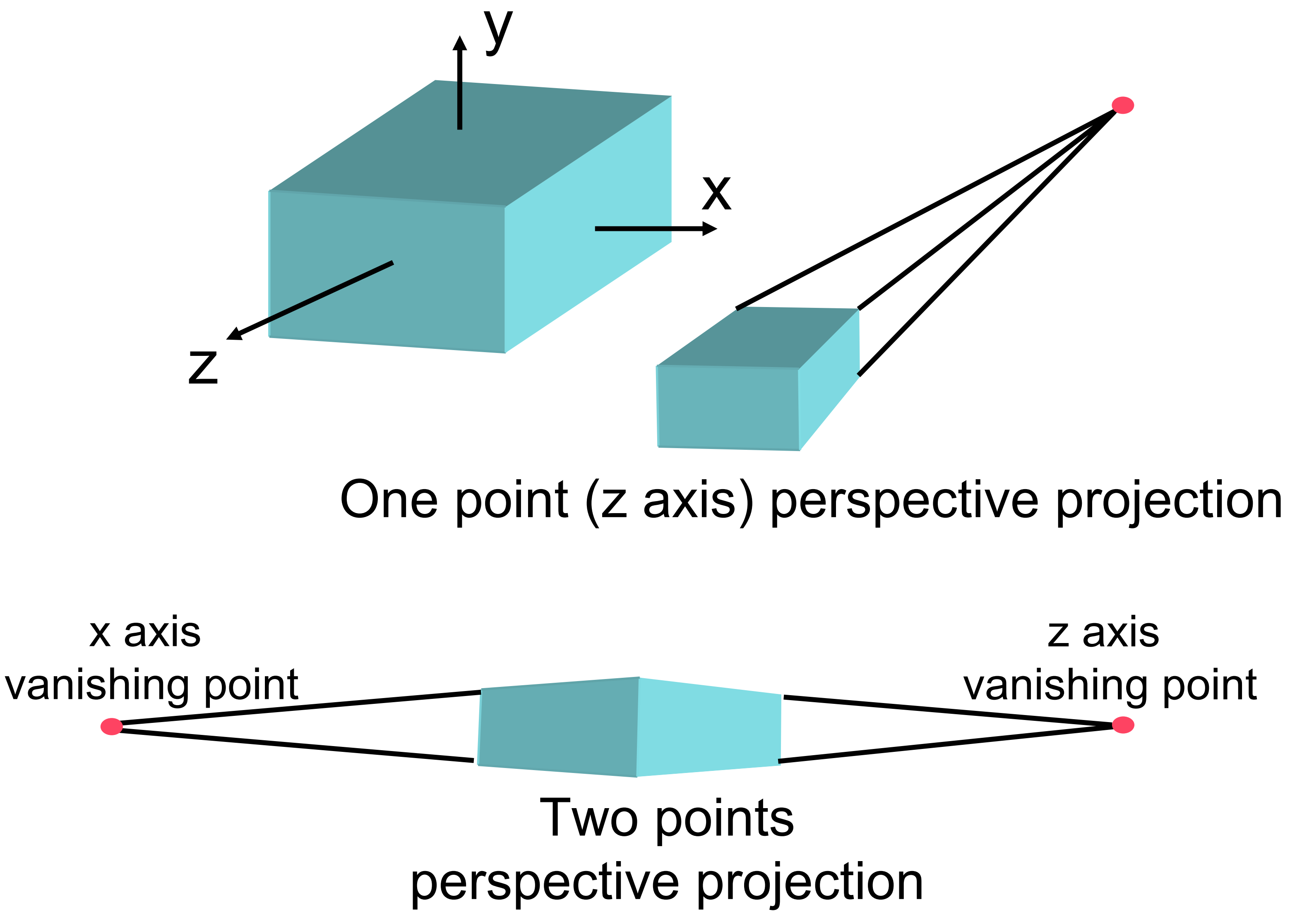


Perspective Projection

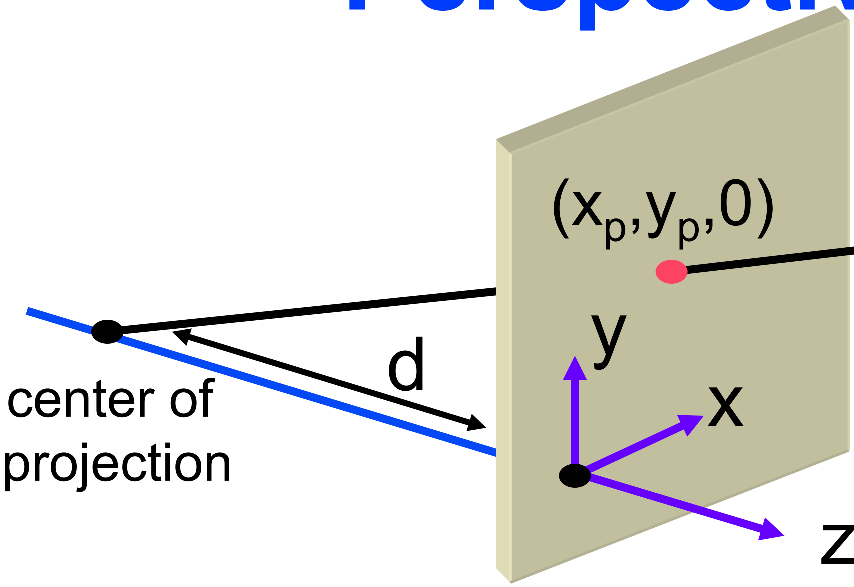
Vanishing Points

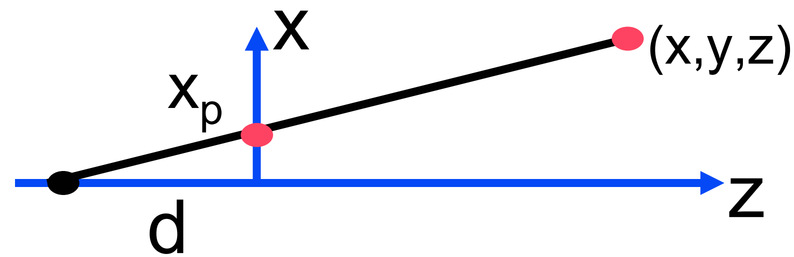
- There are infinitely many general vanishing points
- There can be up to three *principal vanishing points* (axis vanishing points)
- Perspective projections are categorized by the number of principal vanishing points, equal to the number of principal axes intersected by the viewing plane
- Most commonly used: one-point and two-points perspective

Vanishing Points



Perspective Projection





- Using similar triangles it follows:

$$\frac{x_p}{d} = \frac{x}{z+d} \quad ; \quad \frac{y_p}{d} = \frac{y}{z+d}$$

$$x_p = \frac{d \cdot x}{z+d} \quad ; \quad y_p = \frac{d \cdot y}{z+d} \quad ; \quad z_p = 0$$

Perspective Projection

Thus, a perspective projection matrix is defined as:

$$M_{per} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{d} & 1 \end{bmatrix}$$

$$M_{per} P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{d} & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 0 \\ \frac{z + d}{d} \end{bmatrix}$$

$$\Rightarrow \quad x_p = \frac{d \cdot x}{z + d} \quad ; \quad y_p = \frac{d \cdot y}{z + d} \quad ; \quad z_p = 0$$

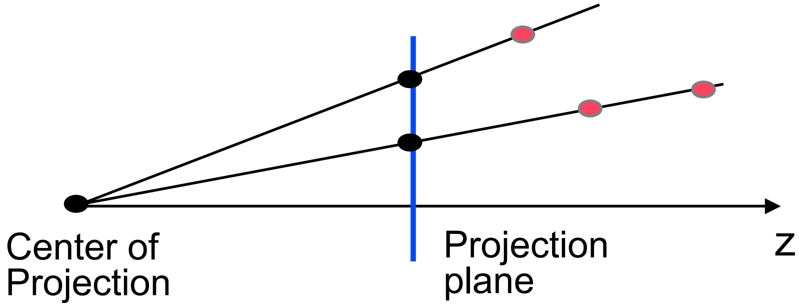
Perspective Projection

- M_{per} is singular ($|M_{per}|=0$), thus M_{per} is a many to one mapping (for example: $M_{per}P=M_{per}2P$)
- Points on the viewing plane ($z=0$) do not change
- The homogeneous coordinates of a point at infinity directed to (U_x,U_y,U_z) are $(U_x,U_y,U_z,0)$. Thus, The vanishing point of parallel lines directed to (U_x,U_y,U_z) is at $[dU_x/U_z, dU_y/U_z]$
- When $d\rightarrow\infty$, $M_{per}\rightarrow M_{ort}$

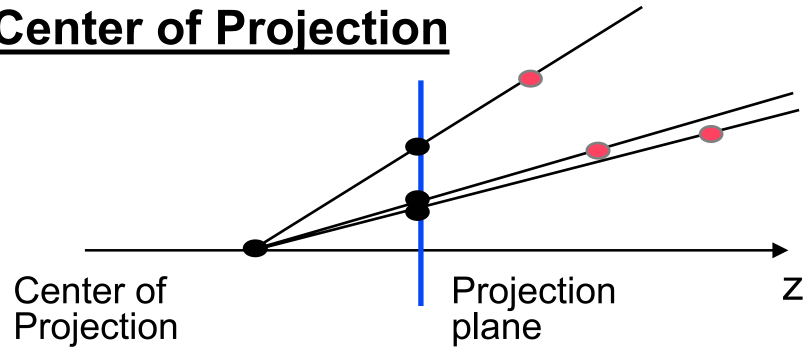
Projections

What is the difference between moving the center of projection and moving the projection plane?

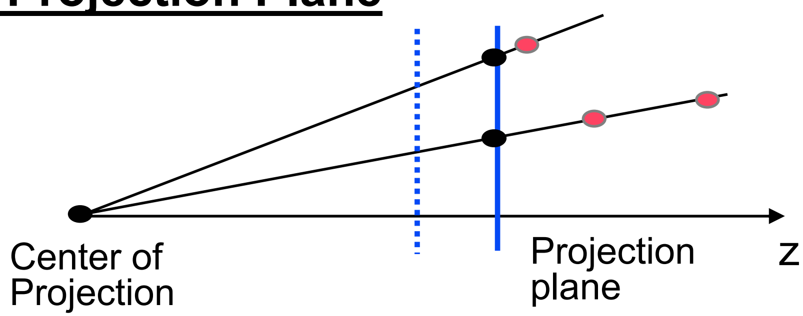
Original



Moving the Center of Projection



Moving the Projection Plane



Projections