Propositional Logic Equivalences

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Equivalences involving $\wedge$

In the equivalences, A, B, C will denote arbitrary formulas. For short, I will often say 'equivalent' rather than 'logically equivalent'.

1. $A \wedge B$ is logically equivalent to $B \wedge A$ (commutativity of $\wedge$)
2. $A \wedge A$ is logically equivalent to A (idempotence of $\wedge$)
3. $A \wedge \top$ is logically equivalent to A
4. $\bot \wedge A$ and $\neg A \wedge A$ are equivalent to $\bot$
5. $(A \wedge B) \wedge C$ is equivalent to $A \wedge (B \wedge C)$ (associativity of $\wedge$)

Equivalences involving $\vee$

6. $A \vee B$ is equivalent to $B \vee A$
   (commutativity of $\vee$)

7. $A \vee A$ is equivalent to A
   (idempotence of $\vee$)

8. $\top \vee A$ and $\neg A \vee A$ are equivalent to $\top$

9. $A \vee \bot$ is equivalent to A

10. $(A \vee B) \vee C$ is equivalent to $A \vee (B \vee C)$ (associativity of $\vee$)

Equivalences involving $\neg$

11. $\neg\top$ is equivalent to $\bot$

12. $\neg\bot$ is equivalent to $\top$

13. $\neg\neg A$ is equivalent to A

Equivalences involving $\to$

14. $A \to A$ is equivalent to $\top$

15. $\top \to A$ is equivalent to A

16. $A \to \top$ is equivalent to $\top$

17. $\bot \to A$ is equivalent to $\top$

18. $A \to \bot$ is equivalent to $\neg A$

19. $A \to B$ is equivalent to $\neg A \vee B$, and also to $\neg(A \wedge \neg B)$

20. $\neg(A \to B)$ is equivalent to $A \wedge \neg B$.

Equivalences involving $\leftrightarrow$

21. $A \leftrightarrow B$ is equivalent to
    - $(A \rightarrow B) \wedge (B \rightarrow A)$,
    - $(A \wedge B) \vee (\neg A \wedge \neg B)$,
    - $\neg A \leftrightarrow \neg B$.
22. $\neg(A \leftrightarrow B)$ is equivalent to
    - $A \leftrightarrow \neg B$,
    - $\neg A \leftrightarrow B$,
    - $(A \wedge \neg B) \vee (\neg A \wedge B)$.

De Morgan laws

Augustus de Morgan (19th-century logician) did not discover these (they are much older) and indeed he could not even express them in his own notation!

23. $\neg(A \wedge B)$ is equivalent to $\neg A \vee \neg B$
24. $\neg(A \vee B)$ is equivalent to $\neg A \wedge \neg B$

Distributivity of $\wedge, \vee$

25. $A \wedge (B \vee C)$ is equivalent to $(A \wedge B) \vee (A \wedge C)$.
26. $A \vee (B \wedge C)$ is equivalent to $(A \vee B) \wedge (A \vee C)$
27. $A \wedge (A \vee B)$ and $A \vee (A \wedge B)$ are equivalent to A.