# CS112: Resolution for Propositional Logic

James Pustejovsky

September 20, 2004

## CNF and Clausal Form

**Definition 1:** A formula is in *conjunctive normal form* (CNF) iff it is a conjunction of disjunctions of literals.

**Theorem 1:** every formula in the propositional calculus can be transformed into an equivalent formula in CNF.

**Definition 2:** A *clause* is a set of literals which is considered to be an implicit disjunction. A *unit clause* is a clause consisting of exactly one literal. A formula in *clausal form* is a set of clauses which is considered to be an implicit conjunction.

**Example:**

a. $(\neg q \vee p \vee q) \wedge (p \vee \neg p \vee q \vee p \vee \neg p)$
   is equivalent to the clausal form

b. $\{\{\neg q, \neg p, q\}, \{p, \neg p, q\}\}$

## Properties of Clausal Form

**Definition 3:** Let $S$ and $S'$ be sets of clauses. $S \approx S'$ denotes that $S$ is satisfiable if and only if $S'$ is satisfiable.

**Lemma 1:** Suppose that a literal $l$ appears in some clause of $S$, but $l^c$ does not appear in any clause of $S$. Let $S'$ be obtained from $S$ by deleting every clause containing $l$. Then $S \approx S'$.

**Lemma 2:** Let $C = \{l\} \in S$ be a unit clause and let $S'$ be obtained from $S$ by deleting every clause containing $l$ and by deleting $l^c$ from every remaining clause. Then $S \approx S'$.

**Lemma 3:** Suppose that both $l \in C$ and $l^c \in C$ for some $C \in S$, and let $S' = S - \{C\}$. Then $S \approx S'$.

## The Resolution Rule

**Resolution Rule:** Let $C_1$ and $C_2$ be clauses such that $l \in C_1$. $l^c \in C_2$. The clauses $C_1$, $C_2$ are said to be *clashing clauses* and to *clash* on the complementary literals $l$, $l^c$. $C$, the resolvent of $C_1$ and $C_2$, is the clause:

$$Res(C_1, C_2) = (C_1 - \{l\}) \cup (C_2 - \{l^c\}).$$

$C_1$ and $C_2$ are the *parent clauses* of $C$.

**Example:** The pair of clauses $C_1 = \{a, b, \neg c\}$ and $C_2 = \{b, c, \neg e\}$ clash on the pair $c$, $\neg c$, and the resolvent is
$C = (a, b, \neg c - \{\neg c\}) \cup (b, c, \neg e - \{c\}) = \{a, b\} \cup \{b, \neg e\} = \{a, b, \neg e\}$.