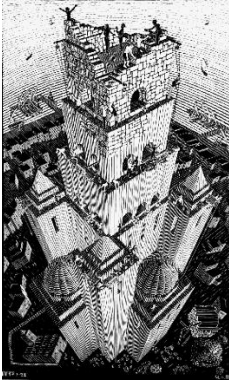


## Viewing in 3D

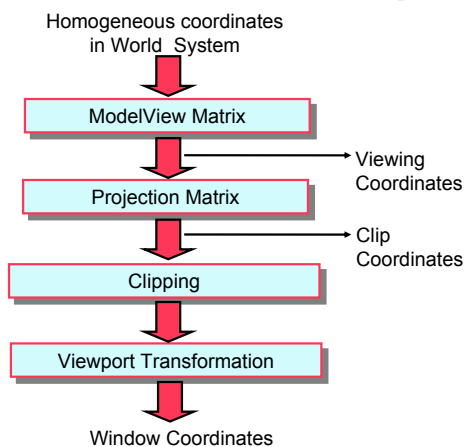
Foley & Van Dam, Chapter 6



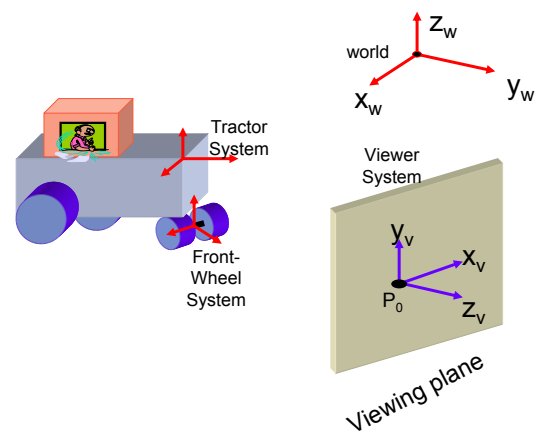
## Viewing in 3D

- Transformation Pipeline
- Viewing Plane
- Viewing Coordinate System
- Projections
  - Orthographic
  - Perspective

## OpenGL Transformation Pipeline



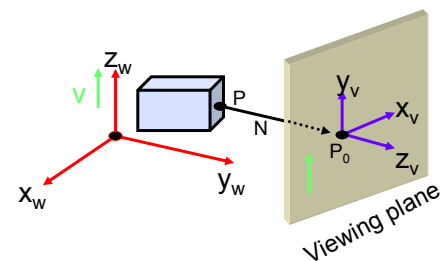
## Viewing Coordinate System



## Specifying the Viewing Coordinates

- *Viewing Coordinates system*,  $[x_v, y_v, z_v]$ , describes 3D objects with respect to a viewer
- A **viewing plane** (*projection plane*) is set up perpendicular to  $z_v$  and aligned with  $(x_v, y_v)$
- In order to specify a viewing plane we have to specify:
  - a **vector N** normal to the plane
  - a **viewing-up vector V**
  - a point on the viewing plane

## Specifying the Viewing Coordinates



- $P_0=(x_0, y_0, z_0)$  is the point where a camera is located
- P is a point to **look-at**
- $N=(P_0-P)/|P_0-P|$  is the view-plane normal vector
- $V=z_w$  is the view up vector, whose projection onto the view-plane is directed up

## Viewing Coordinate System

$$z_v = N \quad ; \quad x_v = \frac{V \times N}{|V \times N|} \quad ; \quad y_v = z_v \times x_v$$

- The transformation M, from world-coordinate into viewing-coordinates is:

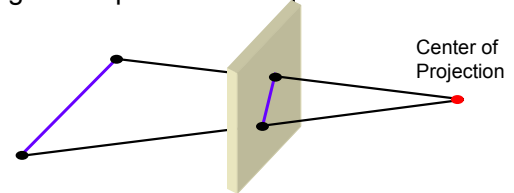
$$M = \begin{bmatrix} x_v^1 & x_v^2 & x_v^3 & 0 \\ y_v^1 & y_v^2 & y_v^3 & 0 \\ z_v^1 & z_v^2 & z_v^3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -x_0 \\ 0 & 1 & 0 & -y_0 \\ 0 & 0 & 1 & -z_0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = R \cdot T$$

- Defining the camera in OpenGL:

```
glMatrixMode(GL_MODELVIEW);
glLoadIdentity();
gluLookAt(P0x, P0y, P0z, Px', Py', Pz', Vx', Vy', Vz');
```

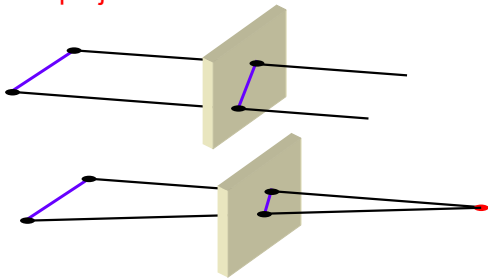
## Projections

- Viewing 3D objects on a 2D display requires a mapping from 3D to 2D
- A projection is formed by the intersection of certain lines (*projectors*) with the view plane
- Projectors are lines from the *center of projection* through each point in the object



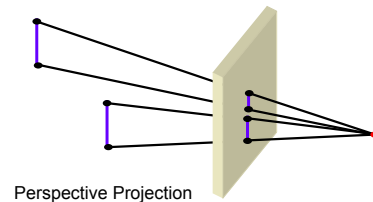
## Projections

- Center of projection at infinity results with a **parallel projection**
- A finite center of projection results with a **perspective projection**



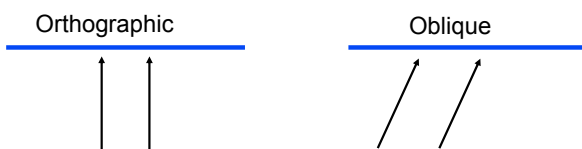
## Projections

- Parallel projections** preserve relative proportions of objects, but do not give realistic appearance (commonly used in engineering drawing)
- Perspective projections** produce realistic appearance, but do not preserve relative proportions



## Parallel Projection

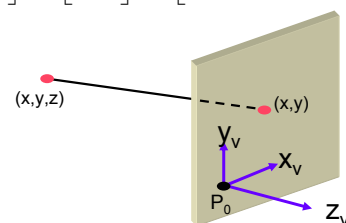
- Projectors are all parallel
- Orthographic**: Projectors are perpendicular to the projection plane
- Oblique**: Projectors are not necessarily perpendicular to the projection plane



## Orthographic Projection

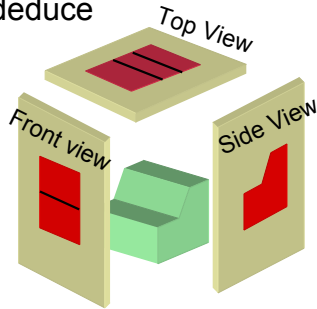
Since the viewing plane is aligned with  $(x_v, y_v)$ , orthographic projection is performed by:

$$\begin{bmatrix} x_p \\ y_p \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} x_v \\ y_v \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_v \\ y_v \\ z_v \\ 1 \end{bmatrix}$$



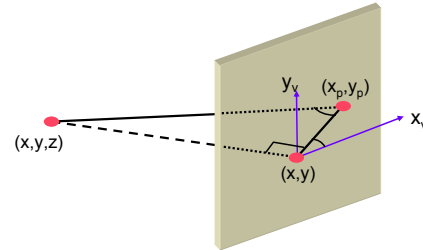
## Orthographic Projection

- Lengths and angles of faces parallel to the viewing planes are preserved
- **Problem:** 3D nature of projected objects is difficult to deduce



## Oblique Projection

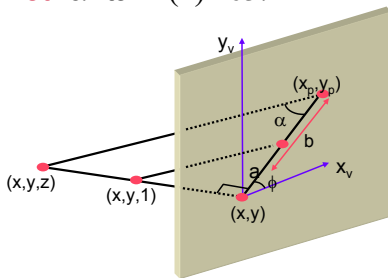
- Projectors are **not** perpendicular to the viewing plane
- Angles and lengths are preserved for faces parallel to the plane of projection
- Preserves 3D nature of an object



## Oblique Projection

• Two types of oblique projections are commonly used:

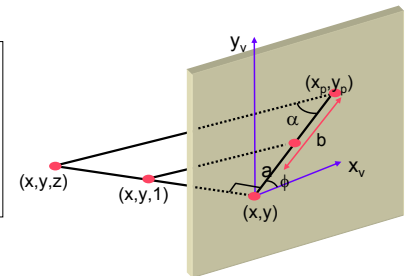
- **Cavalier:**  $\alpha = 45^\circ = \tan^{-1}(1)$
- **Cabinet:**  $\alpha = \tan^{-1}(2) \approx 63.4^\circ$



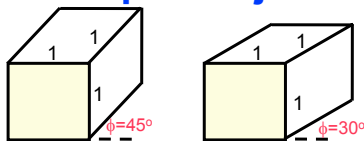
## Oblique Projection

$$\begin{bmatrix} x_p \\ y_p \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & a \cos \phi & 0 \\ 0 & 1 & a \sin \phi & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_v \\ y_v \\ z_v \\ 1 \end{bmatrix} = \begin{bmatrix} x_v + z_v a \cos \phi \\ y_v + z_v a \sin \phi \\ 0 \\ 1 \end{bmatrix}$$

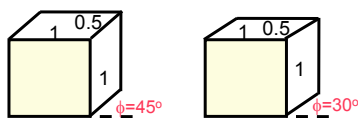
$$\begin{aligned} 1/a &= \tan(\alpha) \\ z/b &= 1/a \\ b &= za \\ x_p &= z \cdot a \cdot \cos(\phi) \\ y_p &= z \cdot a \cdot \sin(\phi) \end{aligned}$$



## Oblique Projection



Cavalier Projections of a cube for two values of angle  $\phi$



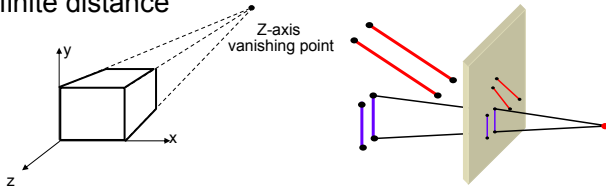
Cabinet Projections of a cube for two values of angle  $\phi$

## Oblique Projection

- **Cavalier** projection :
  - Preserves lengths of lines perpendicular to the viewing plane
  - 3D nature can be captured but shape seems distorted
  - Can display a combination of front, side, and top views
- **Cabinet** projection:
  - Lines perpendicular to the viewing plane project at 1/2 of their length
  - A more realistic view than the Cavalier projection
  - Can display a combination of front, side, and top views

## Perspective Projection

- In a perspective projection, the center of projection is at a finite distance from the viewing plane
- The size of a projected object is inversely proportional to its distance from the viewing plane
- Parallel lines that are not parallel to the viewing plane, converge to a *vanishing point*
- A vanishing point is the projection of a point at infinite distance



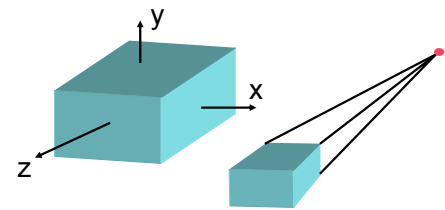
## Perspective Projection



## Vanishing Points

- There are infinitely many general vanishing points
- There can be up to three *principal vanishing points* (axis vanishing points)
- Perspective projections are categorized by the number of principal vanishing points, equal to the number of principal axes intersected by the viewing plane
- Most commonly used: one-point and two-points perspective

## Vanishing Points

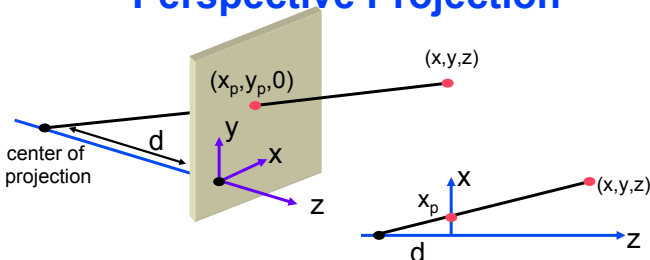


One point (z axis) perspective projection



Two points perspective projection

## Perspective Projection



- Using similar triangles it follows:

$$\frac{x_p}{d} = \frac{x}{z+d} \quad ; \quad \frac{y_p}{d} = \frac{y}{z+d}$$

$$x_p = \frac{d \cdot x}{z+d} \quad ; \quad y_p = \frac{d \cdot y}{z+d} \quad ; \quad z_p = 0$$

## Perspective Projection

Thus, a perspective projection matrix is defined as:

$$M_{per} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{d} & 1 \end{bmatrix}$$

$$M_{per} P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{d} & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 0 \\ \frac{z+d}{d} \end{bmatrix}$$

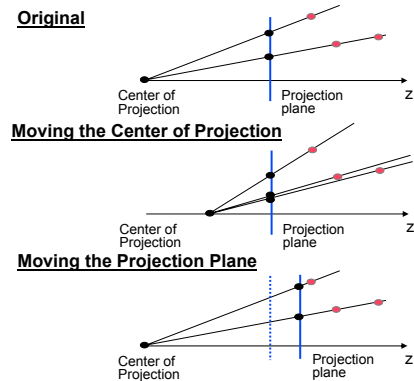
$$\Rightarrow x_p = \frac{d \cdot x}{z+d} \quad ; \quad y_p = \frac{d \cdot y}{z+d} \quad ; \quad z_p = 0$$

## Perspective Projection

- $M_{per}$  is singular ( $|M_{per}|=0$ ), thus  $M_{per}$  is a many to one mapping (for example:  $M_{per}P=M_{per}2P$ )
- Points on the viewing plane ( $z=0$ ) do not change
- The homogeneous coordinates of a point at infinity directed to  $(U_x, U_y, U_z)$  are  $(U_x, U_y, U_z, 0)$ . Thus, The vanishing point of parallel lines directed to  $(U_x, U_y, U_z)$  is at  $[dU_x/U_z, dU_y/U_z]$
- When  $d \rightarrow \infty$ ,  $M_{per} \rightarrow M_{ort}$

## Projections

What is the difference between moving the center of projection and moving the projection plane?



## Projections

### Planar geometric projections

