

## Ling130 – Lecture Notes for 3/26/10

### ❖ Quick Review

- On Tuesday, we started to use DRT to deal with the anaphora problem
- We wanted a compositional treatment of discourse, but DRT isn't really compositional
  - Ideally, the compositional interpretation and the FOL interpretation should have the same truth values
    - *A man walks in the park. He whistles.*
    - FOL:  $\exists x[man(x) \wedge walk(x) \wedge whistle(x)]$
    - Compositional FOL:  $\exists x[man(x) \wedge walk(x)] \wedge whistle(x)$
    - DRT:  $[x][man(x), walk(x), whistle(x)]$
    - The DRT version works, but is basically no different than the FOL version.
- DPL attempts to deal with the compositional version directly by changing how the logical connectives are interpreted.

### ❖ The Dynamic View on Meaning

- Traditionally, we think of the meaning of a sentence in terms of truth conditions.
- Dynamic treatments of semantics do something different.
  - The meaning of a sentence lies in the way it changes the representation of the information of the interpreter.
  - Think of everything you know at the time you hear an utterance as an information state. When a quantifier or an indefinite description come along, that changes your

information state because you now have a new referent that you might refer to later.

- *Meaning=state transition*
  - An utterance of a sentence brings us from a certain state of information to another one
- From now on, we're going to be dealing with interpretations.
  - In FOL, an interpretation is a model that assigns sets to predicates and so on.
  - In DPL, an interpretation is a set of ordered pairs of assignments, or the set of all its possible input-output pairs (Think of a sentence like a computer program. The input is your current information state. The output is your information state after hearing the sentence.)
    - A pair  $\langle g, h \rangle$  is in the interpretation of a program  $\pi$ , if when  $\pi$  is executed in state  $g$ , a possible resulting state is  $h$

## ❖ **Dynamic Predicate Logic**

- *Vocabulary of DPL*
  - n-place predicates, individual constants, and variables
  - The interpretation function  $F$  does the usual thing (assigns individuals to constants and n-tuples of individuals to n-place predicates)
  - Assignments, denoted by  $g, h$ , etc. are total functions from the set of variables to the domain  $D$
  - $h[x]g$  means that assignment  $h$  differs from  $g$  at most with respect to the value it assigns to  $x$
  - As with program interpretation in DL, the interpretation of a DPL sentence is a set of ordered pairs

- $\langle g, h \rangle$  is in the interpretation of a formula  $\phi$  iff when  $\phi$  is evaluated with respect to  $g$ ,  $h$  is a possible outcome of the evaluation procedure

➤ Problem 1: Cross-sentential Anaphora ( $\exists xPx \wedge Qx$ )

▪ **Dynamic Existential Quantification**

- $\llbracket \exists xPx \rrbracket = \{ \langle g, h \rangle \mid h[x]g \ \& \ h(x) \in F(P) \}$ 
  - ◆ An assignment  $g$  is the interpretation of  $\exists xPx$  iff there is some assignment  $h$  which differs from  $g$  at most with respect to the value it assigns to  $x$ , and which is in the interpretation of  $Px$
  - ◆ *John walks.*  $\rightarrow Wj \rightarrow \exists xWx$ 
    - $\llbracket \exists xWx \rrbracket = \{ \langle g, h \rangle \mid h[x]g \ \& \ h(x) \in F(W) \}$
    - $g: \emptyset, h: \{j\}$

- This doesn't account for the general case of  $\exists x\phi$  yet, but this does:

- ◆  $\llbracket \exists x\phi \rrbracket = \{ \langle g, h \rangle \mid \exists k : k[x]g \ \& \ \langle k, h \rangle \in \llbracket \phi \rrbracket \}$
- ◆ This version allows for  $\phi$  to be anything. Anytime,  $\llbracket \ ]$  is used, this means to continue interpreting what's inside the brackets dynamically.

▪ **Dynamic Conjunction**

- We need to be able to pass variables from the first conjunct to the second one, and these values should also be left available for future conjuncts
  - ◆  $\llbracket \phi \wedge \psi \rrbracket = \{ \langle g, h \rangle \mid \exists k : \langle g, k \rangle \in \llbracket \phi \rrbracket \ \& \ \langle k, h \rangle \in \llbracket \psi \rrbracket \}$
- Note that conjunction (and existential quantification) are internally dynamic because it can pass variable bindings from the left conjunct to the right

- It (they) is also externally dynamic because it can keep passing on bindings to future conjuncts
- $\llbracket \exists x Px \wedge Qx \rrbracket$ 
  - We can almost do this, but we need to know the interpretation of **atomic formulas**:
    - ◆  $\llbracket R t_1, \dots, t_n \rrbracket = \{ \langle g, h \rangle \mid h = g \ \& \ \langle \llbracket t_1 \rrbracket_h, \dots, \llbracket t_n \rrbracket_h \rangle \in F(R) \}$
    - ◆ Notice that the assignments  $g$  and  $h$  are equivalent in this interpretation
      - This characteristic of a test. These formulas have no dynamic effects on their own
      - Tests let assignments that satisfy them through (the second part of the interpretation) and block those that don't
      - We'll see many more examples of tests in the rest of DPL semantics
  - Now, we can work out the interpretation of  $\llbracket \exists x Px \wedge Qx \rrbracket =$ 
    - $\{ \langle g, h \rangle \mid \exists k : \langle g, k \rangle \in \llbracket \exists x Px \rrbracket \ \& \ \langle k, h \rangle \in \llbracket Qx \rrbracket \} =$ 
      - ◆ Begin with the main connective (conjunction)
    - $\{ \langle g, h \rangle \mid \exists k : k[x]g \ \& \ k(x) \in F(P) \ \& \ h = k \ \& \ h(x) \in F(Q) \} =$ 
      - ◆ Give the interpretation for  $\exists x Px$  and use the atomic formula rule on the resulting  $Px$  and  $Qx$
    - $\{ \langle g, h \rangle \mid h[x]g \ \& \ h(x) \in F(P) \ \& \ h(x) \in F(Q) \}$ 
      - ◆ Finally, we get rid of instances of the assignment  $k$  by replacing it with  $h$
- Problem 2: Donkey Anaphora ( $\forall x Px \rightarrow Qx$ )
  - **Dynamic Implication**
    - Implication passes values from the antecedent to the consequent, so it is internally dynamic

- However, implication, in general, doesn't pass values to later sentences, so it is not externally dynamic
  - We will see a counter example to this claim, but for now, we'll just go with it ☺
- Since implication is not externally dynamic, it functions as a test in DPL:

- $\llbracket \phi \rightarrow \psi \rrbracket = \{ \langle g, h \rangle \mid h = g \ \& \ \forall k : \langle h, k \rangle \in \llbracket \phi \rrbracket \Rightarrow \exists j : \langle k, j \rangle \in \llbracket \psi \rrbracket \}$ 
  - ♦ The interpretation of  $\phi \rightarrow \psi$  accepts an assignment  $g$  iff every possible output of  $\phi$  with respect to  $g$  leads to a successful interpretation of  $\psi$ , and it rejects  $g$  otherwise

$$\begin{aligned} \llbracket \exists x Px \rightarrow Qx \rrbracket &= \\ \{ \langle g, h \rangle \mid h = g \ \& \ \forall k : \langle h, k \rangle \in \llbracket \exists x Px \rrbracket \Rightarrow \exists j : \langle k, j \rangle \in \llbracket Qx \rrbracket \} &= \\ \{ \langle g, g \rangle \mid \forall k : \langle g, k \rangle \in \llbracket \exists x Px \rrbracket \Rightarrow \exists j : \langle k, j \rangle \in \llbracket Qx \rrbracket \} &= \\ \{ \langle g, g \rangle \mid \forall k : k[x]g \ \& \ k(x) \in F(P) \Rightarrow k(x) \in F(Q) \} & \end{aligned}$$

- **Dynamic Universal Quantification**

- Externally static, so functions as a test:
  - ♦  $\llbracket \forall x \phi \rrbracket = \{ \langle g, h \rangle \mid h = g \ \& \ \forall k : k[x]h \Rightarrow \exists m : \langle k, m \rangle \in \llbracket \phi \rrbracket \}$
- Big Example (*Every farmer who owns a donkey beats it.*):  
(Really hard, we won't ask you to do anything like this!)

$$\begin{aligned} \llbracket \forall x [ [ Px \wedge \exists y [ Qy \wedge Rxy ] ] \rightarrow Sxy ] \rrbracket &= \\ 1. \{ \langle g, h \rangle \mid h = g \ \& \ \forall k : k[x]h \Rightarrow \exists m : \langle k, m \rangle \in \llbracket [ Px \wedge \exists y [ Qy \wedge Rxy ] ] \rightarrow Sxy \rrbracket \} &= \\ 2. \{ \langle g, g \rangle \mid \forall k : k[x]g \Rightarrow ( \forall j : \langle k, j \rangle \in \llbracket [ Px \wedge \exists y [ Qy \wedge Rxy ] \rrbracket \Rightarrow \exists z : \langle j, z \rangle \in \llbracket Sxy \rrbracket ) \} &= \\ 3. \{ \langle g, g \rangle \mid \forall k : k[x]g \ \& \ k(x) \in F(P) \Rightarrow ( \forall j : j[y]k \ \& \ j(y) \in F(Q) \ \& \ \langle j(x), j(y) \rangle \in F(R) \Rightarrow \langle j(x), j(y) \rangle \in F(S) ) \} &= \\ 4. \{ \langle g, g \rangle \mid \forall h : h[x, y]g \ \& \ h(x) \in F(P) \ \& \ h(y) \in F(Q) \ \& \ \langle h(x), h(y) \rangle \in F(R) \Rightarrow \langle h(x), h(y) \rangle \in F(S) \} & \end{aligned}$$

Line 1: Apply the universal quantification rule

Line 2: Replace occurrences of h with g and apply the implication rule

Line 3: Apply the atomic formula rule along with the conjunction and existential quantification rules

Line 4: Simplify by removing quantifiers where possible

➤ Remaining Connectives

• **Dynamic Negation**

◆  $\llbracket \neg\phi \rrbracket = \{ \langle g, h \rangle \mid h = g \ \& \ \neg\exists k : \langle h, k \rangle \in \llbracket \phi \rrbracket \}$

◆ Big Example #2:

$\llbracket \neg\exists xPx \wedge Qx \rrbracket =$

1.  $\{ \langle g, h \rangle \mid \exists k : \langle g, k \rangle \in \llbracket \neg\exists xPx \rrbracket \ \& \ \langle k, h \rangle \in \llbracket Qx \rrbracket \} =$

2.  $\{ \langle g, h \rangle \mid \exists k : \langle g, k \rangle \in \llbracket \neg\exists xPx \rrbracket \ \& \ h = k \ \& \ h(x) \in F(Q) \} =$

3.  $\{ \langle g, h \rangle \mid \langle g, h \rangle \in \llbracket \neg\exists xPx \rrbracket \ \& \ h(x) \in F(Q) \} =$

4.  $\{ \langle g, h \rangle \mid h = g \ \& \ \neg\exists k : \langle h, k \rangle \in \{ \langle g, h \rangle \mid h[x]g \ \& \ h(x) \in F(P) \} \ \& \ h(x) \in F(Q) \} =$

5.  $\{ \langle g, g \rangle \mid \neg\exists k : k[x]g \ \& \ k(x) \in F(P) \ \& \ g(x) \in F(Q) \}$

Line 1: Apply conjunction rule

Line 2: Apply the atomic formula rule to  $\langle k, h \rangle \in \llbracket Qx \rrbracket$

Line 3: Replace occurrences of k with h

Line 4: Apply the negation rule, leave the second conjunct alone, and, while you're at it, apply the existential and atomic formula rules to that conjunct

Line 5: Replace occurrences of h with g and apply the atomic formula rule

- **Dynamic Disjunction**

- $\llbracket \phi \vee \psi \rrbracket = \{ \langle g, h \rangle \mid h = g \ \& \ \exists k : \langle h, k \rangle \in \llbracket \phi \rrbracket \vee \langle h, k \rangle \in \llbracket \psi \rrbracket \}$
- Disjunction is unique because it is both externally and internally static

- Summary

- Most logical constants in DPL are interpreted as tests (their interpretations include  $h=g$ ). The exceptions are conjunction and existential quantification because they are externally dynamic (they force a dynamic interpretation beyond their own scope).

- ❖ **Concluding Remarks**

- Recall the overall goal: Develop a compositional, non-representation semantics of discourse that enables us to marry the compositional framework of Montague grammar to a dynamic outlook on meaning
  - Empirically, DPL is like Discourse Representation Theory (DRT)
    - The interpretation of a DRT structure is dynamic, but this only comes out in the interpretation of implication
    - So, DRT gets us closer to the dynamic interpretation of anaphora that we want, but isn't compositional
  - Methodologically, DPL is like Montague Grammar because it incorporates compositionality
- So, what's missing?
  - DPL is restricted to an extensional first-order system, but Montague Grammar makes use of intensional higher order logic

- The authors present a solution to this called 'Dynamic Montague Grammar' in another 1990 paper
- DPL has some things in common with Discourse Representation Theory that are controversial
  - There are examples that show that universal quantification, implication, disjunction, and negation are, in some contexts, both internally and externally dynamic
    - ◆ "If a client turns up, you treat him politely. You offer him a cup of coffee and ask him to wait."
    - ◆ "Every player chooses a pawn. He puts it on square one."
  - The authors solution to this problem is to provide a paraphrase of the discourse that gets around the non-dynamic aspects
    - ◆ "If a client turns up, you treat him politely, you offer him a cup of coffee, and ask him to wait."
      - Here the second part of the discourse is folded into the consequent of the conditional to take advantage of the internally dynamic character of implication, or, in general:  $[\phi \rightarrow \psi] \wedge \chi \approx \phi \rightarrow [\psi \wedge \chi]$
    - ◆ "Every player chooses a pawn, and (he) puts it on square one."
      - $\forall x \phi \wedge \psi \approx \forall x [\phi \wedge \psi]$
- The purpose of this solution is to avoid giving dynamic interpretation to logical constants that are not consistently dynamic
- However, it does seem in contrast with the goal of incorporating compositionality!



## ❖ More on DPL and DRT

- Difference 1: How things are interpreted
  - DRT makes both a syntactic and a semantic distinction between conditions and DRSs
    - Conditions are interpreted like FOL sentences (i.e. in terms of their truth values)
    - DRSs are interpreted in terms of their verifying embeddings, which, I believe, is a fancy way of saying what other DRSs are accessible so that they are really interpreted in terms of how they bind anaphora
  - DPL doesn't make this kind of distinction; Everything is interpreted using the dynamic interpretation of the connectives
- Difference 2: What connectives are used
  - DPL uses regular FOL with the exception that unbound variables are ok
  - DRT doesn't have quantifiers or conjunction
    - The discourse referents are how DRT does quantification and the list of conditions is like conjunction
- What the differences mean in the end
  - There are other differences between these approaches, but, in the end, you can show that they're roughly the same
  - The syntax for DRT is somewhat better defined, but the semantics for DPL is better defined
  - The authors claim that DPL is more compositional, but even they can give examples where DPL has to fake the compositionality aspect

- DPL seems to rely on being able to give an (incorrect) FOL representation of the discourse before going through the DPL interpretation
  - *NOTE: DPL gives an interpretation of the discourse while DRT gives a representation of it!*
- DRT doesn't require this and claims to have an algorithmic way of determining what an anaphoric reference refers to
- In conclusion!
  - It seems like DPL is nice for giving an interpretation of a discourse, but it's hard to use.
  - DRT is relatively straightforward and pretty.
  - So, if we all we really care about is giving a representation of a discourse, DRT is the way to go.
  - And, luckily, DRT and DPL can roughly map to each other, so we can still use DPL to get an interpretation!