LING 130:
Guide to Quantifier Embedding

James Pustejovsky

March 15, 2010

In this note, we examine how a quantifier embedded within another quantified expression can be interpreted. We learn that substitutions can take different scopes over the expressions
This technique also works for embedded quantiers and coordinate NP constructions, which we will get to shortly. Now consider a QNP embedded within another QNP, such as that shown below.

(1) John bought a picture of every student.

Assume that there are two readings: (2a) with wide-scope on *every student*; and (2b) with wide-scope on *a picture*.

(2) a. $\forall x[student(x) \rightarrow \exists y[picture(y) \wedge of(y,x) \wedge buy(j,y)]]$
b. $\exists y[picture(y) \wedge \forall x[student(x) \rightarrow of(y,x)] \wedge buy(j,y)]]$

Most speakers think that the first interpretation above (2a) is much more natural, and that the "single picture" reading is sort of hard get, without additional work or context, as shown below:

(3) a. John bought a picture of every student together.
b. John bought a picture of all the students.

In any case, let's go ahead and do the substitutions for the two quantifiers. First, the one that is embedded.

(4) a. $\langle$"every student", $(e \rightarrow t) \rightarrow t$, $\lambda P \forall x[student(x) \rightarrow P(x)]\rangle$
b. QS: $\langle$ "every student", e, $C_1\rangle$, $[C_1/\lambda P \forall x[student(x) \rightarrow P(x)]]_{\sigma_1}$

Now let us derive the interpretation for the complex QNP containing C_1 under substitution σ_1.

(5) a. $\langle$"picture", $e \rightarrow (e \rightarrow t)$, $\lambda x \lambda y[picture(y) \wedge of(y,x)]\rangle$
b. $\langle$"picture of C_1", $e \rightarrow t$, $\lambda y[picture(y) \wedge of(y,C_1)]\{\sigma_1\}\rangle$
c. $\langle$"a picture of C_1", $(e \rightarrow t) \rightarrow t$, $\lambda P \exists x[picture(x) \wedge of(x,C_1) \wedge P(x)]\{\sigma_1\}\rangle$

Notice where the substitution σ_1 ended up in (5c). Up till now, we haven't really had to worry about the scope of the substitution itself: that is, how big of an expression does a substitution σ attach to, anyway? According to our rule of Quantifier Substitution, repeated in (6),

(6) Quantifier Substitution:
For every expression, γ, in a sentence, we associate a body, α, and the set of quantifier substitutions, Σ, where $\Sigma = \{[C_1/Q_1]_{\sigma_1}, [C_2/Q_2]_{\sigma_2}, \ldots, [C_n/Q_n]_{\sigma_n}\}$
$\gamma = \alpha\{\Sigma\}$

σ attaches to the entire expression, but it seems as though it can take a narrow attachment (or scope) as well; namely, attaching to the literal that contains the constant, C_i; that is, σ_i can attach to either the expression within which the substitution was made, α, or to a smaller literal within this expression; namely (7).

(7) $\langle$"picture of C_1", $e \rightarrow t$, $\lambda y[picture(y) \wedge of(y, C_1)\{\sigma_1\}]\rangle$

This in turn would give a "narrower scope" to the substitution when combined with the quantifier. So, rather than (5c), we derive (8).

(8) $\langle$"a picture of C_1", $(e \rightarrow t) \rightarrow t$, $\lambda P \exists x[picture(x) \wedge of(x, C_1)\{\sigma_1\} \wedge P(x)]\rangle$

Let's derive the wide-scope reading for *every student* first.

(9) STEP-BY-STEP:
a. John bought a picture of every student.
b. buy: $\lambda x \lambda y[buy(y, x)]$
c. picture: $\lambda x \lambda y[picture(y) \wedge of(y, x]$
d. every student: $\lambda P \forall x[student(x) \rightarrow P(x)]$
e. Quantifier Substitution (QS): $C_1 : e, [C_1/\lambda P \forall x[student(x) \rightarrow P(x)]]_{\sigma_1}$
f. Function Application: $\lambda x \lambda y[picture(y) \wedge of(y, x) : e \rightarrow (e \rightarrow t), C_1 : e \Longrightarrow$
$\lambda y[picture(y) \wedge of(y, C_1)]\{\sigma_1\} : e \rightarrow t$
g. a picture: $\lambda P \exists y[picture(y) \wedge of(y, C_1) \wedge P(x)]\{\sigma_1\}$
h. Quantifier Substitution (QS): $C_2 : e, [C_2/\lambda P \exists y[picture(y) \wedge of(y, C_1) \wedge P(x)]\{\sigma_1\}]_{\sigma_2}$
i. Function Application: $\lambda x \lambda y[buy(y, x)] : e \rightarrow (e \rightarrow t), C_2 : e \Longrightarrow$
$\lambda y[buy(y, C_2)] : e \rightarrow t$
j. Function Application: $\lambda y[buy(y, C_2)] : e \rightarrow t, j : e \Longrightarrow$
$buy(j, C_2)\{\sigma_2\}$
k. Substitution Application: $\lambda P \exists y[picture(y) \wedge of(y, C_1) \wedge P(x)]\{\sigma_1\}(\lambda z[buy(j, z)]) \Longrightarrow$
l. $\exists y[picture(y) \wedge of(y, C_1) \wedge buy(j, y)]\{\sigma_1\}$
m. Substitution Application: $\lambda P \forall x[student(x) \rightarrow P(x)](\lambda w \exists y[picture(y) \wedge of(y, w) \wedge buy(j, y)])$
n. Function Application: $\forall x[student(x) \rightarrow \exists y[picture(y) \wedge of(y, x) \wedge buy(j, y)]]$
o. ☺

Now let us derive the narrow-scope reading for *every student*.

(10) STEP-BY-STEP:

a. John bought a picture of every student.
b. buy: $\lambda x \lambda y[buy(y, x)]$
c. picture: $\lambda x \lambda y[picture(y) \wedge of(y, x]$
d. every student: $\lambda P \forall x[student(x) \rightarrow P(x)]$
e. Quantifier Substitution (QS): $C_1 : e, [C_1/\lambda P \forall x[student(x) \rightarrow P(x)]]_{\sigma_1}$
f. Function Application: $\lambda x \lambda y[picture(y) \wedge of(y, x) : e \rightarrow (e \rightarrow t), C_1 : e \Longrightarrow$
$\lambda y[picture(y) \wedge of(y, C_1)\{\sigma_1\}] : e \rightarrow t$
g. a picture: $\lambda P \exists y[picture(y) \wedge of(y, C_1)\{\sigma_1\} \wedge P(x)]$
h. Quantifier Substitution (QS): $C_2 : e, [C_2/\lambda P \exists y[picture(y) \wedge of(y, C_1)\{\sigma_1\} \wedge P(x)]]_{\sigma_2}$
i. Function Application: $\lambda x \lambda y[buy(y, x)] : e \rightarrow (e \rightarrow t), C_2 : e \Longrightarrow$
$\lambda y[buy(y, C_2)] : e \rightarrow t$
j. Function Application: $\lambda y[buy(y, C_2)] : e \rightarrow t, j : e \Longrightarrow$
$buy(j, C_2)\{\sigma_2\}$
k. Substitution Application: $\lambda P \exists y[picture(y) \wedge of(y, C_1)\{\sigma_1\} \wedge P(x)](\lambda z[buy(j, z)]) \Longrightarrow$
l. $\exists y[picture(y) \wedge of(y, C_1)\{\sigma_1\} \wedge buy(j, y)]$
m. Substitution Application:
$\exists y[picture(y) \wedge \lambda P \forall x[student(x) \rightarrow P(x)](\lambda z[of(z, x)]) \wedge buy(j, y)])$
n. Function Application:
$\exists y[picture(y) \wedge \forall x[student(x) \rightarrow of(y, x)] \wedge buy(j, y)]$
o. ☺

To sum up, we can see that a quantified expression within another quantified expression has two options for recording the substitution:

(11) For every expression, γ, containing a body, α, and a quantifier substitution, $[C_i/Q_i]_{\sigma_i}$, γ can be encoded as either:

a. $\alpha\{\sigma_i\}$
b. $[\ldots \alpha_j\{\sigma_i\} \ldots]_\alpha$, where α_j contains C_i.