Ling 130 Notes: Semantics of First-order Logic

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A model M for a language L of predicate logic consists of a domain D (called A in the book) and an interpretation function, I (called F_M in the book). When we view the interpretation function as relative to an assignment, g, of specific values to variables, then we speak of an interpretation function, $[\![\ ]\!]_{\mathbf{M},g}$, in a model M under an assignment g.
This allows us to generalize first order objects as *terms*, defined below:
Definition 1:

(i) $[\![t]\!]_{\mathbf{M},g} = I(t)$ if t is a constant in L;

(ii) $[\![t]\!]_{\mathbf{M},g} = g(t)$ if t is a variable.

We can now give the full definition of expressions in the predicate calculus, within a model, **M**, and assignment function, g:

Definition 2:

(i) If $A(a_1, \ldots, a_n)$ is an atomic sentence in L, then
$V_{\mathbf{M},g}(A(t_1, \ldots, t_n) = 1$ iff $\langle [\![t_1]\!]_{\mathbf{M},g}, \ldots [\![t_n]\!]_{\mathbf{M},g} \rangle \in I(A)$.

(ii) $[\![\neg\phi]\!]_{\mathbf{M},g} = 1$ iff $V_{\mathbf{M},g}(\phi) = 0$

(iii) $[\![\phi \wedge \psi]\!]_{\mathbf{M},g} = 1$ iff $V_{\mathbf{M},g}(\phi) = 1$ and $V_{\mathbf{M},g}(\psi) = 1$

(iv) $[\![\phi \vee \psi]\!]_{\mathbf{M},g} = 1$ iff $V_{\mathbf{M},g}(\phi) = 1$ or $V_{\mathbf{M},g}(\psi) = 1$

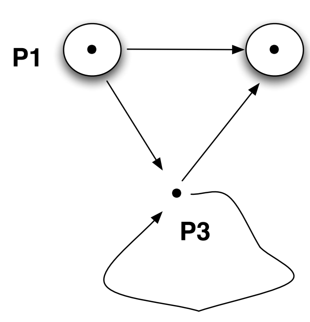
(v) $[\![\phi \rightarrow \psi]\!]_{\mathbf{M},g} = 1$ iff $V_{\mathbf{M},g}(\phi) = 0$ or $V_{\mathbf{M},g}(\psi) = 1$

(vi) $[\![\phi \leftrightarrow \psi]\!]_{\mathbf{M},g} = 1$ iff $V_{\mathbf{M},g}(\phi) = V_{\mathbf{M},g}(\psi)$.

(vii) $[\![\forall v\phi]\!]_{\mathbf{M},g} = 1$ iff for all $d \in \mathcal{D}$: $V_{\mathbf{M},g[v/d]}(\phi) = 1$

(viii) $[\![\exists v\phi]\!]_{\mathbf{M},g} = 1$ iff there is at least one $d \in \mathcal{D}$: $V_{\mathbf{M},g[v/d]}(\phi) = 1$

Assume a model, where constants and predicates have the following interpretation:



Model M_1:

$I(a) = P_1, I(b) = P_2, I(c) = P_3, I(A) = \{P_1, P_2\}$
$I(R) = \{\langle P_1, P_2\rangle, \langle P_1, P_3\rangle, \langle P_3, P_2\rangle, \langle P_3, P_3\rangle\}$

Now let us use this definition to determine the truth value of the following sentences:

1. $\forall x \exists y \exists z[(R(x, y) \wedge R(z, x)) \rightarrow \neg A(x)]$
2. $\exists x \exists y \exists x[R(x, y) \wedge A(y) \wedge R(x, z) \wedge \neg A(z)]$

What are the steps in calculating the interpretation of this expression for the above model M_1?