Sufficient Conditions for Coarse-Graining Evolutionary Dynamics

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Genetic Algorithms and the Building Block Hypothesis

- In 1975 Holland published his seminal work on GAs
  - Description of the Genetic Algorithm
  - A theory of adaptation for GAs
    - which later came to be known as BBH
- GAs useful for adapting solutions to difficult real world problems
- However BBH has drawn considerable skepticism amongst many researchers (e.g. Vose, Wright, Rowe)
- Despite criticism of BBH, no alternate full-fledged theories of adaptation have been proposed

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No Alternate Theories of Adaptation for GAs — Why?

- For genomes of non-trivial length, current theoretical results do not permit the formulation of *principled* theories of adaptation
  - Schema theories only permit a tractable analysis of evolutionary dynamics over a single generation
  - Markov Chain approaches only yield a qualitative description of evolutionary dynamics in the asymptote of time

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- The infinite population assumption is often used to make mathematical models of GA dynamics tractable
- Even with this assumption there are currently no theoretical results which permit a *principled* analysis of any aspect of GA behavior over the "short-term"
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The Promise of Coarse-Graining

- Coarse-graining a very useful technique from theoretical Physics
- If successfully applied to an IPGA it permits a *principled* analysis of certain aspects of the IPGA's dynamics over multiple generations
- Therefore coarse-graining is a promising approach to the formulation of principled theories of evolutionary adaptation

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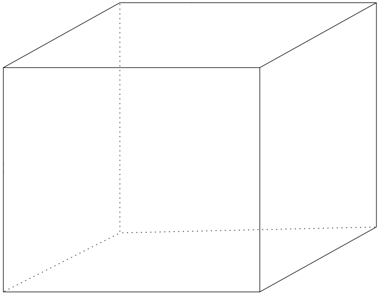
Coarse-Graining by Depiction

$$x_1^{t+1} = \dots$$
$$x_2^{t+1} = \dots$$
$$\vdots$$
$$x_{1,000,000,000,000}^{t+1} = \dots$$

Coarse-Graining by Depiction



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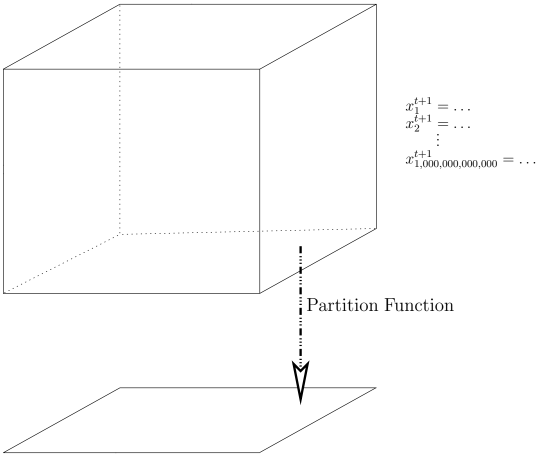

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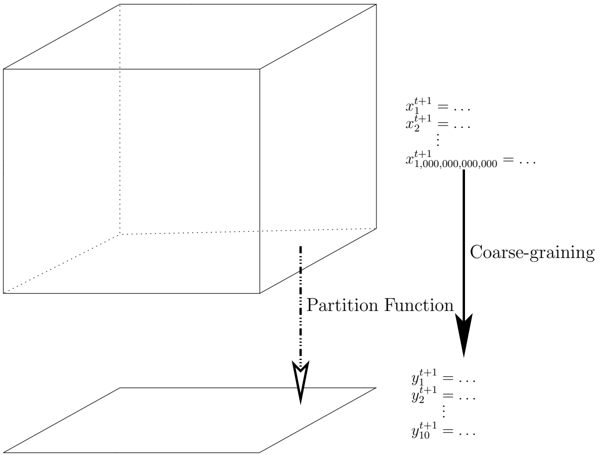

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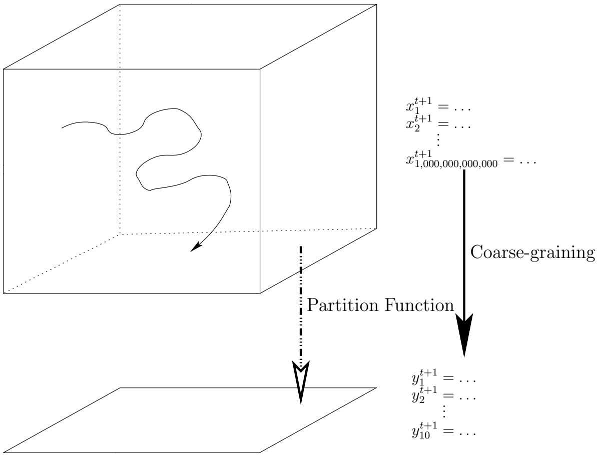

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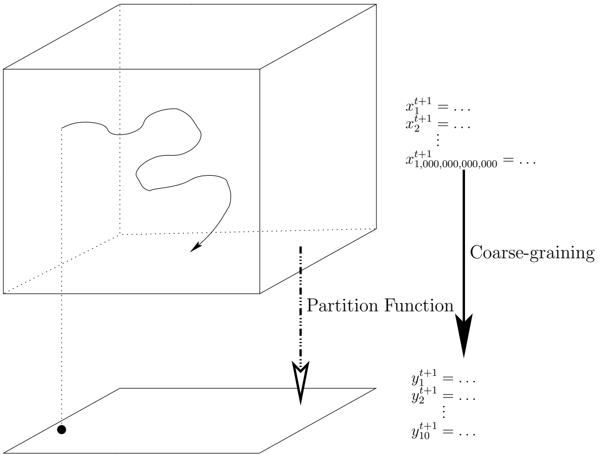

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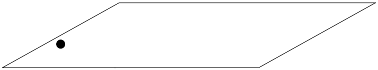

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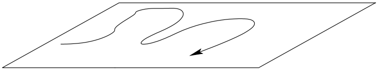

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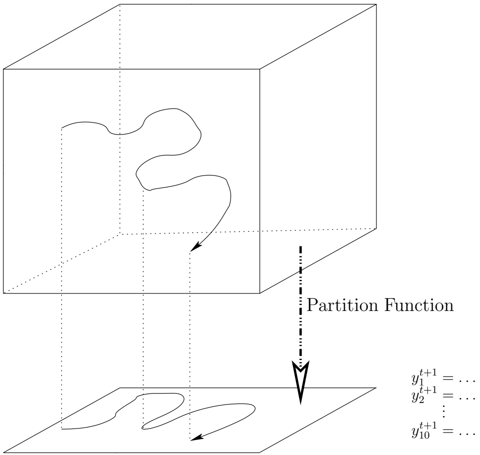

Previous Coarse-Graining Results

- Wright, Vose, and Rowe (Wright et al. 2003) show that any mask based recombination operation of an IPGA can be coarse-grained.
- However they argue that the selecto-recombinative dynamics of an IPGA with an arbitrary initial population cannot be coarse-grained unless the fitness function satisfies a very strong constraint
- I call this constraint *schematic fitness invariance*
  - for some schema partition, and for any schema in the partition, all the genomes in the schema have exactly the same fitness
- Wright et al. argue that schematic fitness invariance is so strong that it renders a coarse-graining of selecto-recombinative dynamics useless

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Comparison between Coarse-Graining Results

- I show that if the class of initial populations is appropriately constrained then it is possible to coarse-grain the selecto-recombinative dynamics of an IPGA for a much weaker constraint on the fitness function
- The constraint on the class of initial populations is not onerous
  - A uniformly distributed population satisfies this constraint
- The constraint on the fitness function is weak enough that it makes the coarse-graining result potentially useful in a theory of adaptation

	Constraint on Initial Population	Constraint on Fitness Function
Wright et al. 2003	None	Severe
Burjorjee 2007	Non-onerous	Weak

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1. Describe an abstract framework for analyzing the dynamics of a selecto-recombinative infinite population EA (IPEA)
2. Describe the theoretical technique used to Coarse-Grain the dynamics of an IPEA
3. Present results that show that these dynamics can be coarse-grained provided that the IPEA satisfies certain abstract conditions
4. Describe how the coarse-graining results can be used to coarse-grain the dynamics of an IPGA with long genomes and a non-trivial fitness functions
5. Present experimental validation of this theory

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Modeling Populations and Operations on Populations

- Modeling scheme based on the one used in (Toussaint 2003)
- Populations modeled as distributions over the genome set
  - Distribution values sum to 1
- Population-level effect of evolutionary operations modeled as the application of *parameterized* mathematical operators to genomic distributions
  - Parameter objects used by the operators (fitness function, transmission function) give individual-level information about the genomes

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Transmission Functions

- Use transmission functions (Altenberg 1994) to represent the variational information at the individual-level
  - Example: $T(g|g_1, \ldots g_n)$ is the probability that parents $g_1, \ldots, g_n$ will yield a child g when recombined

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Modeling the Effect of Variation on Populations

- Given some transmission function T, the effect of variation at the population-level is modeled by the variation operator $\mathcal{V}_T$
- For some population p, if $p' = \mathcal{V}_T(p)$, then, p' is as follows:

$$p'(g) = \sum_{\substack{(g_1,\ldots,g_m) \\ \in \prod_1^m G}} T(g|g_1,\ldots,g_m) \prod_{i=1}^{m} p(g_i)$$

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Modeling the Effect of Selection on Populations

- Given some fitness function $f : G \to \mathbb{R}^+$, the effect of fitness proportional selection is modeled by the selection operator $\mathcal{S}_f$
- For some population p, if $p' = S_f(p)$, then p' is as follows: For any genotype g,

$$p'(g) = \frac{f(g)p(g)}{\mathcal{E}_f(p)}$$

where $\mathcal{E}_f$ is the weighted average fitness of p

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Some Terminology and Notation

- $\beta : G \rightarrow K$ a surjective function
  - Call β a partitioning
  - Call co-domain K the theme set
  - Call the elements of K themes
- $\langle k \rangle_\beta$ denotes the set of all $g \in G$ such that $\beta(g) = k$
- Call $\langle k \rangle_\beta$ the theme class of k under β

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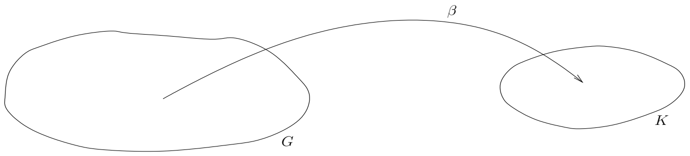
Projection Operator

- Let $\beta : G \to K$ be a partitioning
- A projection operator Ξ_β 'projects' a distribution p_G over G 'through' β to create a distribution $p_K = \Xi_\beta(p_G)$ over the theme set



For any $k \in K$,

$$p_K(k) = \sum_{g \in \langle k \rangle_\beta} p(g)$$

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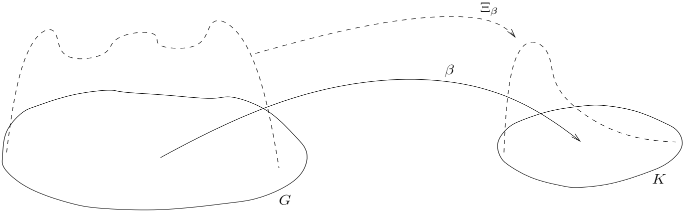

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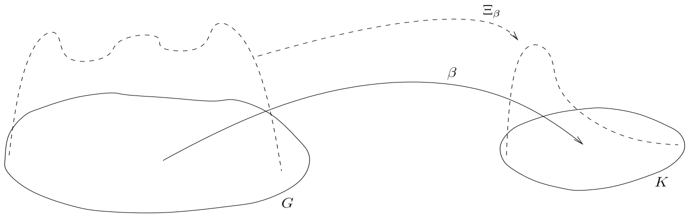

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Semi-Concordance, Concordance, Global Concordance

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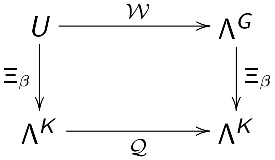

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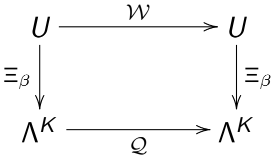

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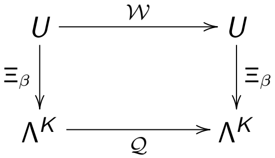

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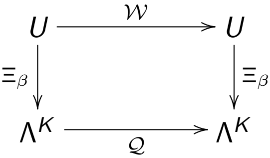

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- Let $\mathcal{G} = \mathcal{V}_T \circ \mathcal{S}_f$
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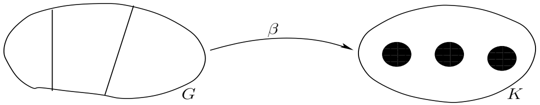
Ambivalence (By Example)

An Ambivalent 2-parent transmission function T



Say that T is *ambivalent under* β

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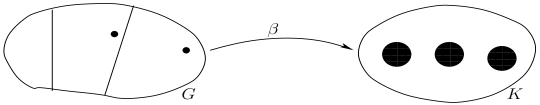

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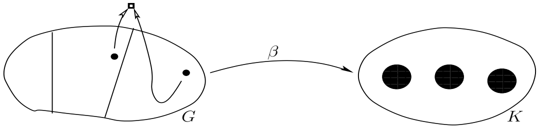

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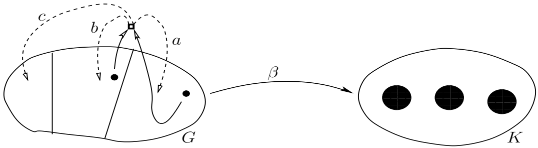

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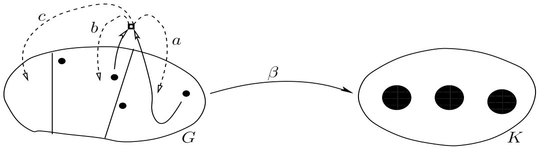

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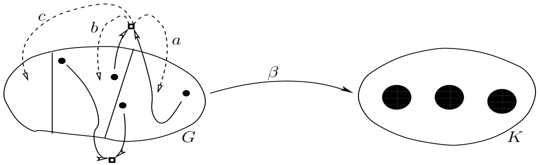

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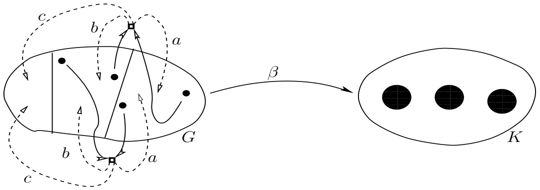

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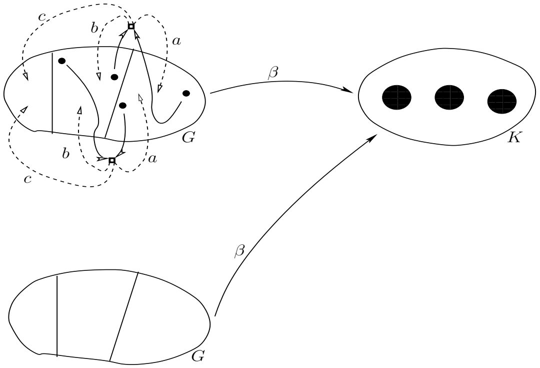

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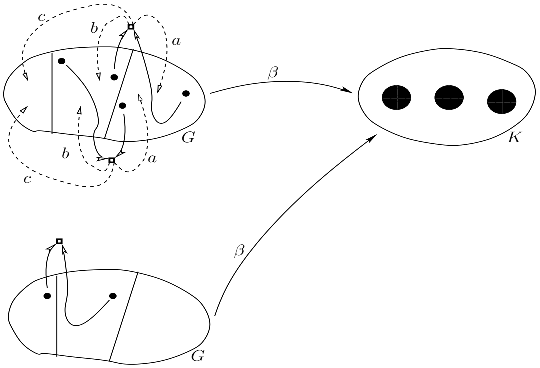

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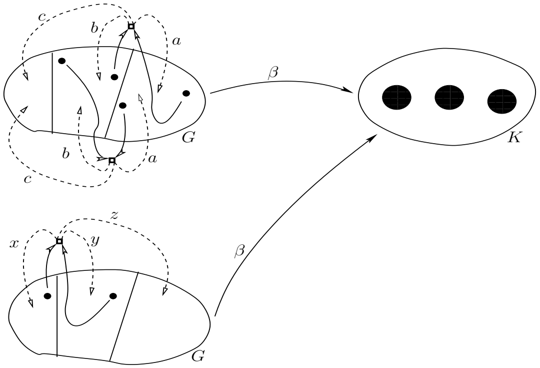

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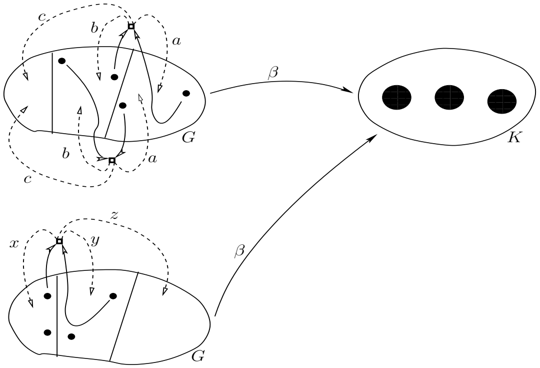

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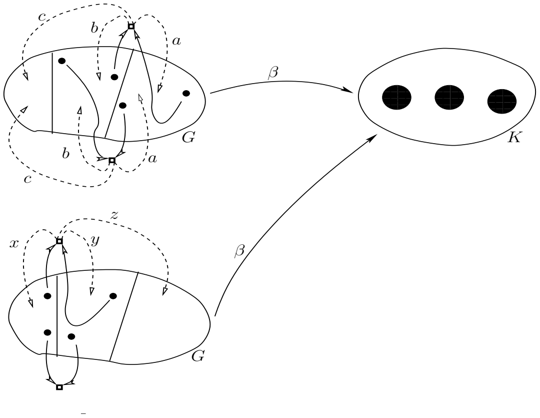

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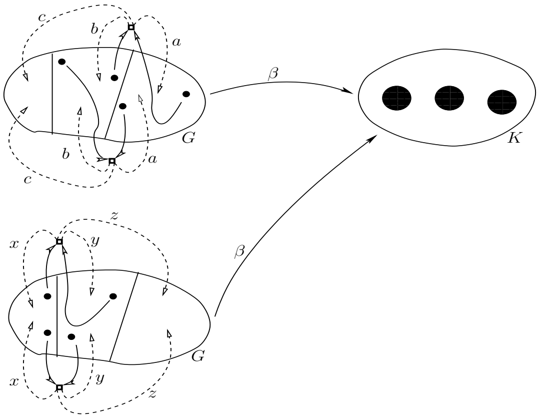

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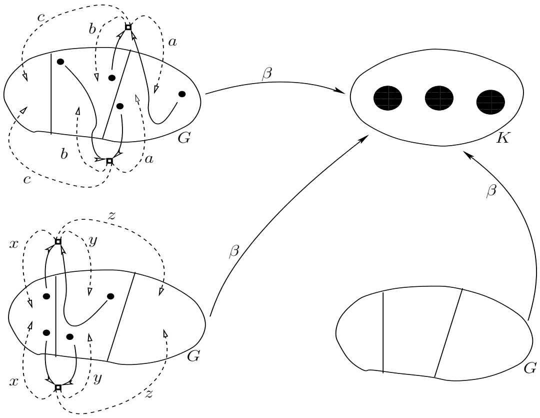

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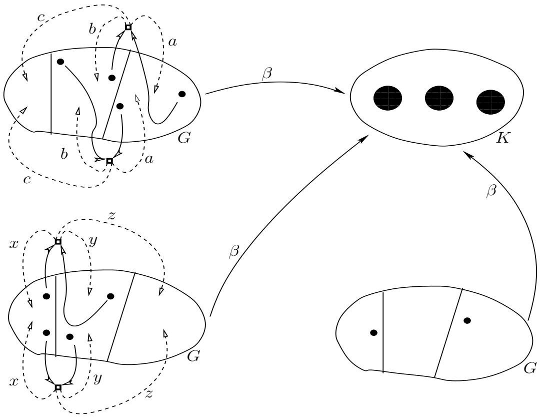

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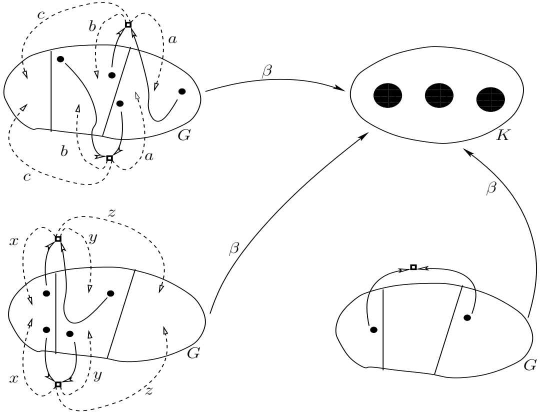

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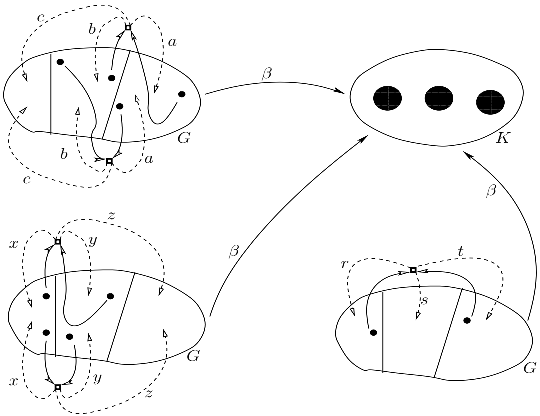

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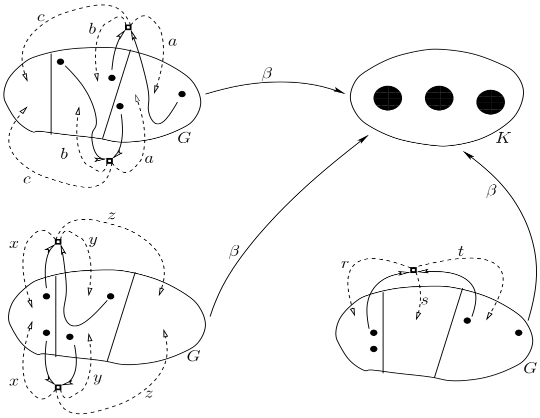

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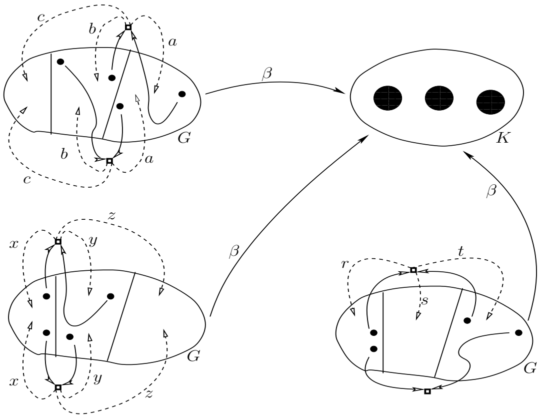

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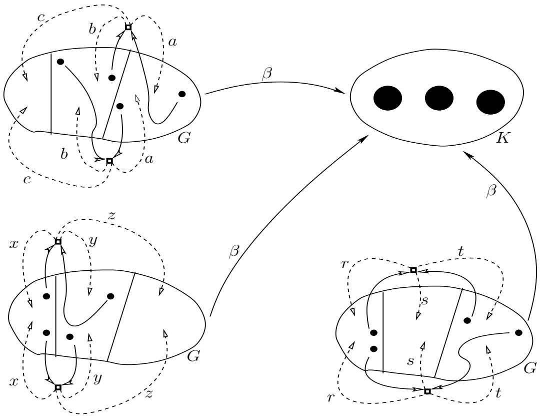

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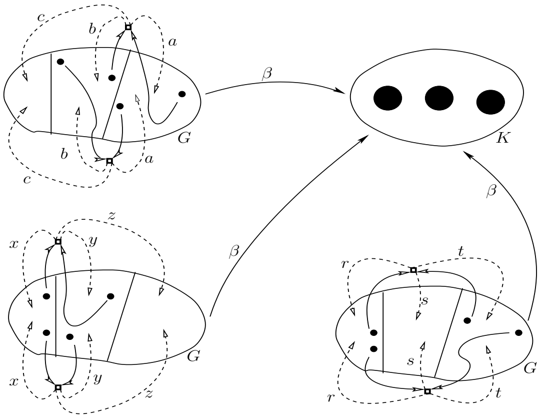

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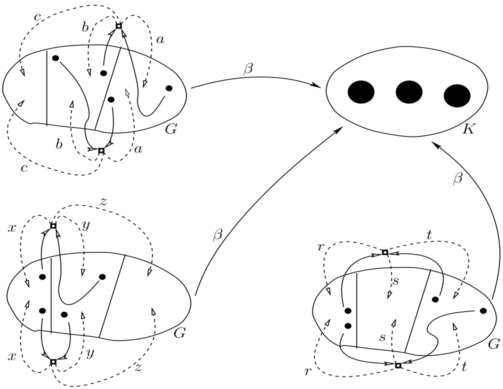

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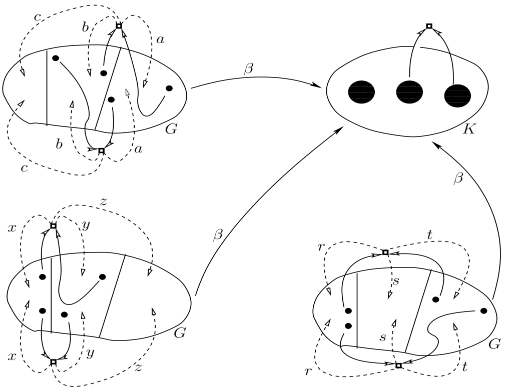

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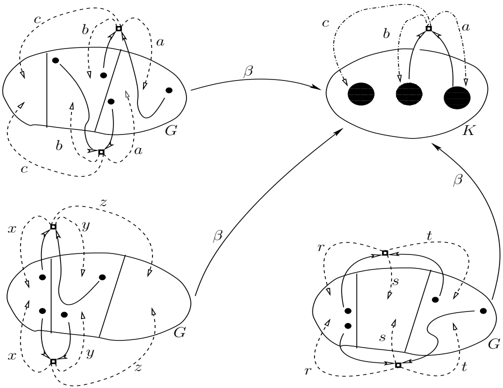

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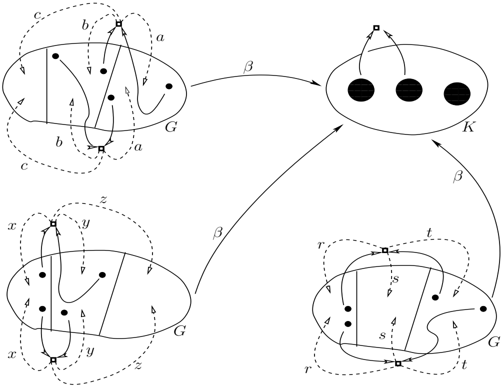

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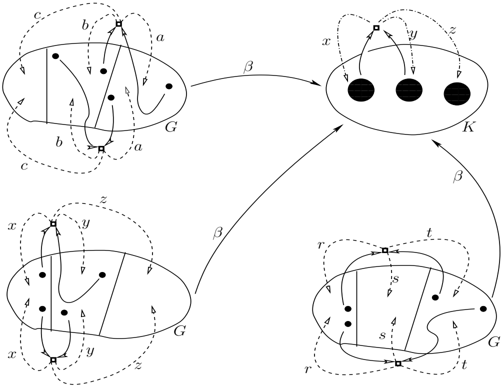

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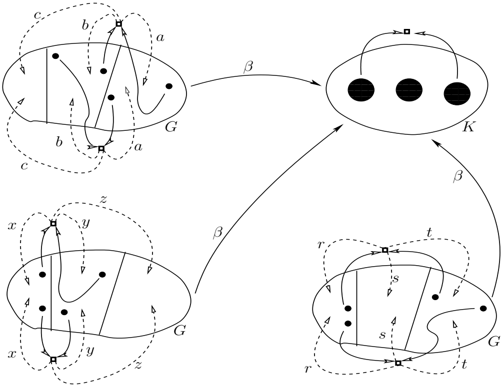

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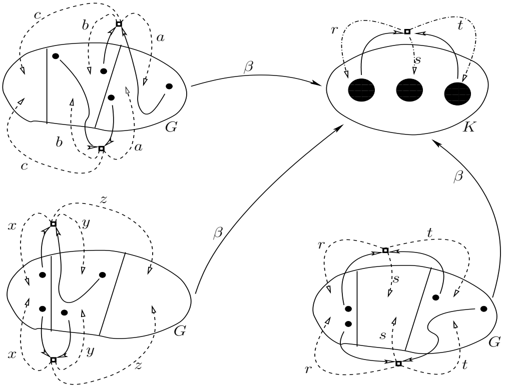

Theorem: Global Concordance of Variation

- G, K countable sets
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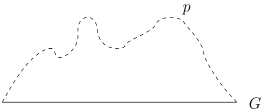
Theme Conditional Operator (By Example)



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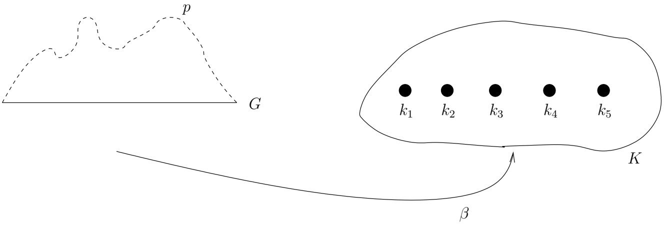

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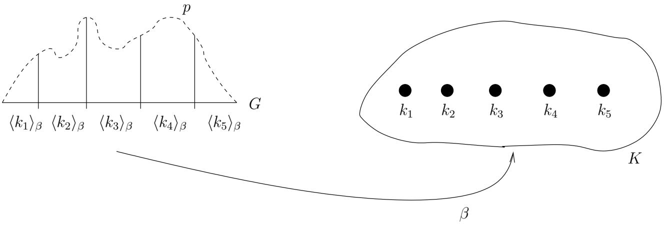

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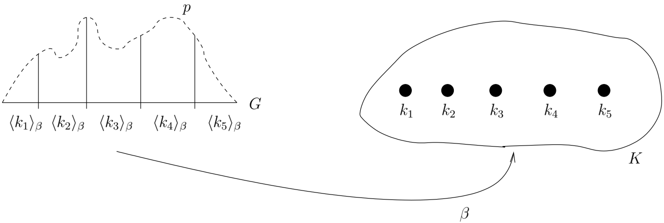

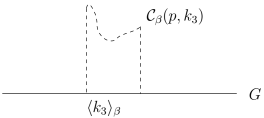

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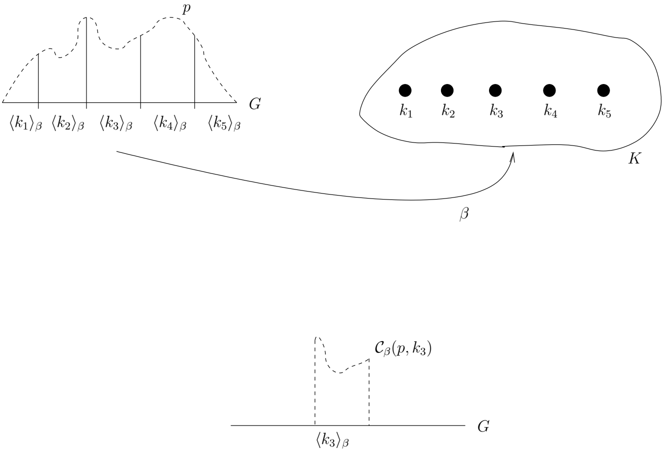

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Bounded Thematic Mean Divergence (By Example)

- G a finite Set
- $\beta : G \longrightarrow K$ a partitioning
- $f^* : K \longrightarrow \mathbb{R}^+$ some function
- $\delta \geq 0$
- $f : G \longrightarrow \mathbb{R}^+$
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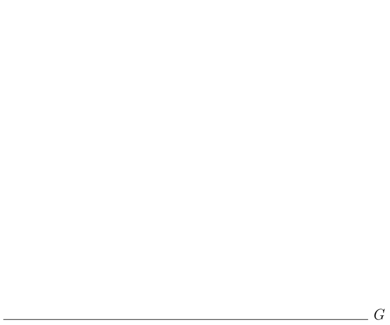

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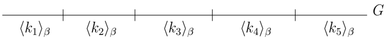

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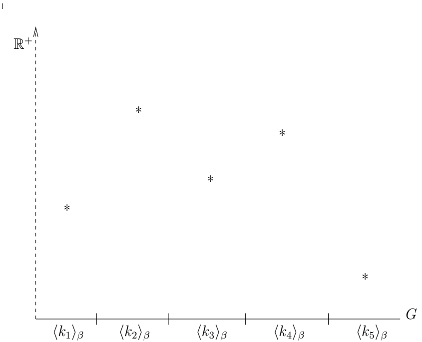

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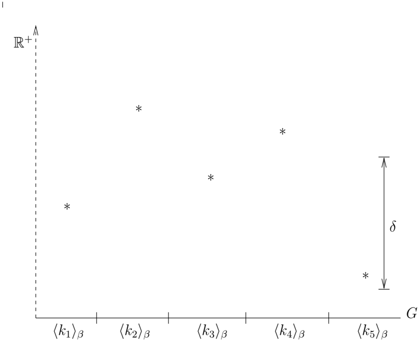

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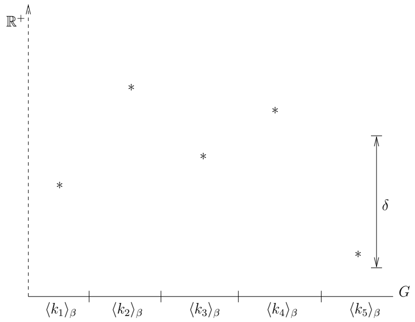

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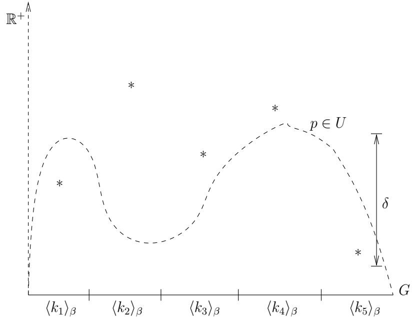

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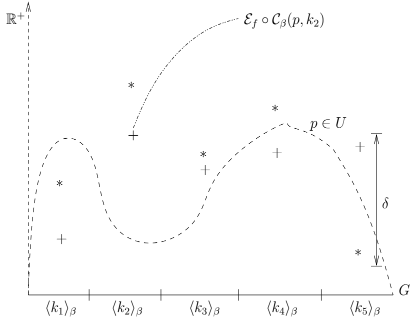

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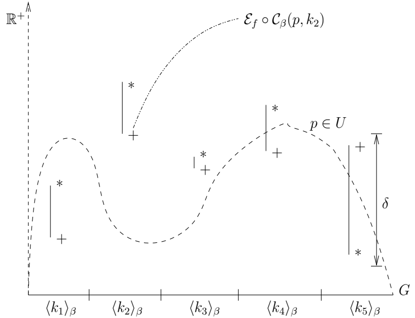

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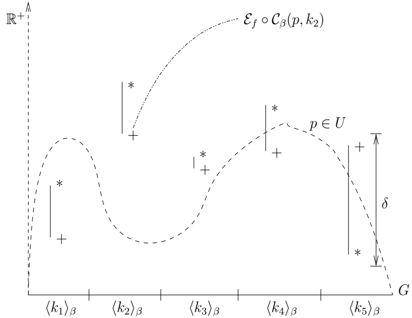

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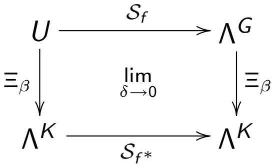

Formalization of an IPEA and its Dynamics

- An Evolution Machine is a 3-tuple (G, T, f) where,
  - G a set
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1. The thematic mean divergence of f w.r.t. f^* on U under β is bounded by δ
2. T is ambivalent under β
3. E is non-departing over U

Let $E^* = (K, T^{\overrightarrow{\beta}}, f^*)$, then for any $t \in \mathbb{Z}^+$,

$$\begin{array}{ccc} U & \xrightarrow{\mathcal{G}_E^t} & U \\ \Big\downarrow \Xi_\beta & \lim_{\delta \to 0} & \Big\downarrow \Xi_\beta \\ \Lambda^K & \xrightarrow[\mathcal{G}_{E^*}^t]{} & \Lambda^K \end{array}$$

Theorem: Limitwise Concordance of Evolution

- $E = (G, T, f)$ an Evolution Machine
- $\beta : G \rightarrow K$ a partitioning of G
- $U \subseteq \Lambda^G$ (such that $\Xi_\beta(U) = \Lambda^K$)
- $f^* : K \rightarrow \mathbb{R}^+$
- $\delta \geq 0$

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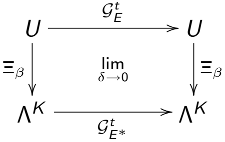

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- Limitwise concordance of evolution theorem is very general
- The result is applicable to any IPEA, provided that 3 abstract conditions are satisfied
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- The fidelity of the coarse-graining depends on the bound δ on the thematic mean divergence
  - Fidelity increases as $\delta \longrightarrow 0$

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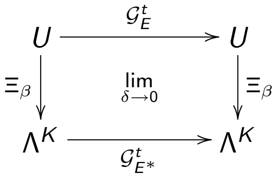

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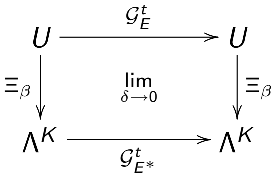

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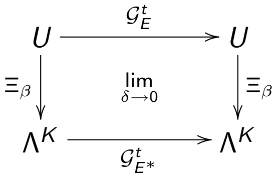

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Implications for Coarse-Graining an IPGA

- Given an IPGA with
  - long genomes
  - uniform cross-over
- And some relatively coarse schema partitioning
- The IPGA will satisfy the 3 abstract conditions if it satisfies the following 2 concrete conditions:
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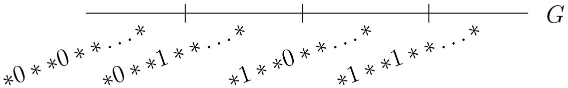

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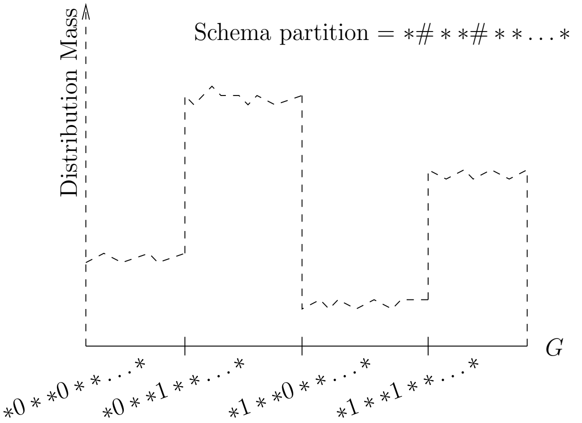

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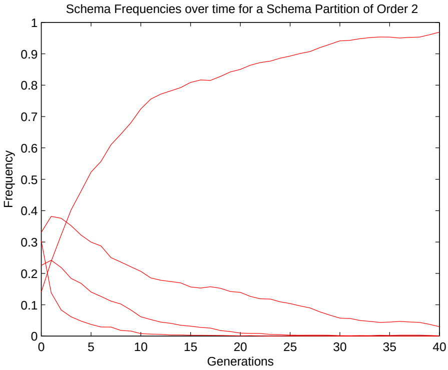

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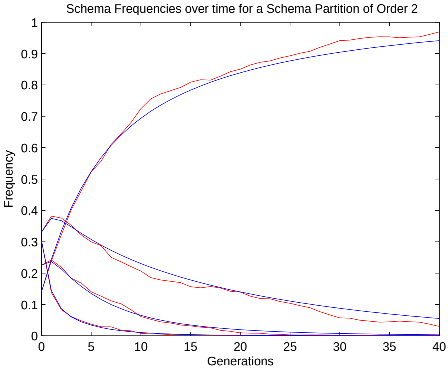

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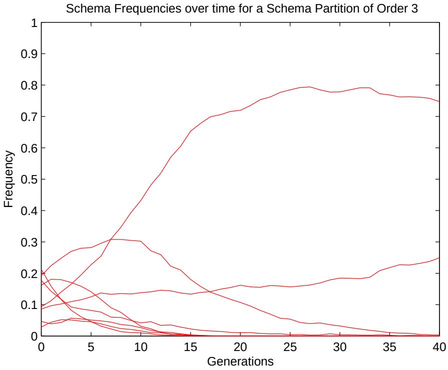

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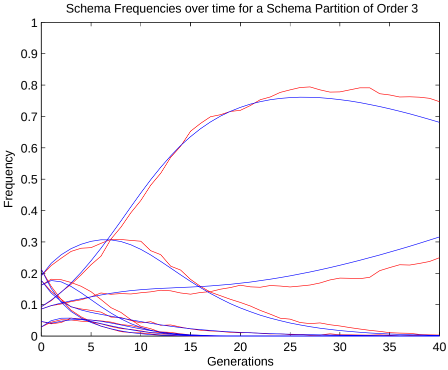

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