Pentominoes

Complementing several of the other sections of my website (Puzzles and Games, Puzzles Parties, and even some of my LEGO pages), this subpart will focus on my strong interest in a particular geometric topic: pentominoes.

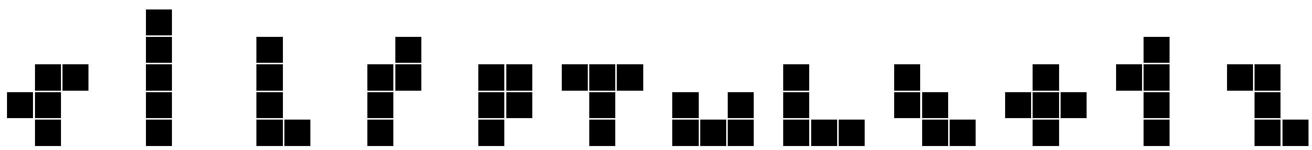
News and Updates

Looking for information about the **Pentomino Daily Calendar**?

Article: *Longest Narrow Path*
Article: *Neighborly Pentominoes*
Article: *Pentominic Surfaces*

(more articles below)

A pentomino is a geometric shape formed by adjoining five squares with one another edge to edge. There are twelve unique ways to do this (not counting rotations and reflections). Those twelve pieces are pictured here and make up the "classic" set of pentominoes. Traditionally each piece is named using the letter of the alphabet which most closely resembles its shape:



F I L N P T U V W X Y Z

The concept of pentominoes (and their more general cousins: polyominoes -- made from any number of squares joined) have been a part of recreational mathematics for over a hundred years, but it was the 1965 book *Polyominoes* by American Professor Solomon W. Golomb which formally defined the shapes and made a rigorous, documented study of them.

I received my first set of pentominoes when I was about 9 years old (it was sold by the company Gabriel under the named *Hexed* -- pictured, right). The lid of the box proclaimed that there were 2339 ways to fit the pieces in the box (a 6x10 rectangle). Boy, was I determined to find all of them. Even at that young age, I was pretty focused. I played with those pentominoes often, and each time I found a solution, I would record it on a sheet of graph paper using colored pens (after having checked against the previous solutions).

This, of course, was all before the "computer age". At some point I found other things to do with my time, but I had, by then, discovered 59 of the answers (I still have the graph paper with my answers, over 20 years later). Since that time I have drifted back to the 12 wonderful geometric shapes, and they have always caught my eye in toy and games stores.

These days I have a number of different sets. Several basic plastic sets, a set of wooden, "solid" ones (meaning they have a thickness dimension as if made from cubes, not flat squares -- so they may be assembled into 3D figures), many sets that I have made from LEGO bricks, and a set of solid steel pieces.

I expect that I will augment my collection in the future... I'd certainly like sets made of brass, crystal, and even a large stone-carved set to stand in my backyard.

With the prominence of computers and the internet these days, information about these objects is readily available. A search on Google turns up quite a bit.

My most recent pentomino project has been the creation of the Pentomino Daily Calendar (year 2006).

I will also include links to various articles about pentominoes that I write. While most of the fun with pentominoes revolves around filling or building certain configurations, there are many other fascinating questions that can be asked about the pieces. As I get time, I will write webpages about some of the questions I have thought about, and provide answers when I know them. Here is a current list of articles:

1. *Enclosing Extra Pieces*
2. *Landlocked Pentominoes*
3. *Numerous Solutions*
4. *Transmuting Pentominoes*
5. *Pentominoes As Letters*
6. *Pentominic Surfaces*
7. *Neighborly Pentominoes*
8. *Longest Narrow Path*

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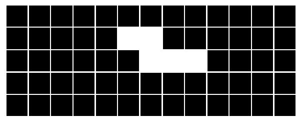
Enclosing Extra Pieces

[Note: I am purposely *not* including the solutions to the diagrams on this page. The reader is encouraged to discover them on his or her own.]

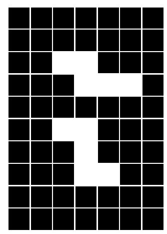
I am pleased and honored to mention that the "Enclosed Triplets" problem has won the first ever "Gamepuzzles Annual Pentomino Excellence Award", bestowed by Kadon Enterprises, Inc.

While compiling and inventing designs for the Pentomino Daily Calendar I included twelve configurations which are often thought of as "missing piece" designs.

Since each of the 12 pentominoes is made from 5 squares, any design based upon all of them will be composed of 60 squares. One can concoct designs based on a 5x13 rectangle (65 squares) which has a "hole" in its interior. This hole can take the shape of any of the 12 pentominoes. For example, here is a solvable design, a "missing N" configuration:



I furthered this idea of holes shaped like pieces by using a 7x10 rectangle and including *two* holes in the interior, each hole the shape of a pentomino (possibly the same shape). For example, a "missing N and Z" form:



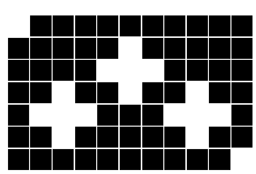
It turns out that you can create a solvable configuration consisting of two holes within the interior of a 7x10 box for any piece-shaped holes. When I say "within the interior" I mean it properly: each hole is at least one square away from the edge of the enclosing box, and the two holes do not touch each other, not even at corners.

Including using two holes of the same shape (say two "X" holes) there are 78 different 2-hole combinations.

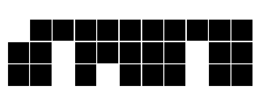
I included 66 such designs in my Calendar puzzles and then focused on bounding multiple holes of the same shape. Would it be possible, say, to create a rectangle which enclosed *three* holes each shaped like the "W" piece? What about the other shapes?

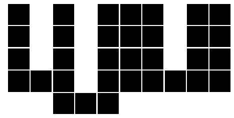
One needs a rectangle sized 5x15 (75 squares minus the 15 determining the holes leaves the required 60). No other true rectangle is reasonable (since 75 has no other factors besides: 3*5*5, and obviously a 3x25 rectangle would not work). Unfortunately this rectangle won't do the trick. The I-pentomino is the troublesome piece. There is no way to include three long I-holes in the interior of a 7x15 box so that the configuration is solvable with the 12 pentominoes.

I then decided to modify the bounds a bit. I used a 7x11 box with two opposite corners removed. This, too, failed for the I-holes. So, while I could manage to form "missing triplets" for 11 of the pentominoes in a 7x11 box with opposite corners removed, the I-pentomino once again caused trouble (pictured below is the example of the X-configuration -- for which there is 1 solution).



I did, however, find a bounding design that *did* allow three I-pentominoes in its interior. It is not quite as aesthetically pleasing as a rectangular box, but it is solvable (there is only a single solution):

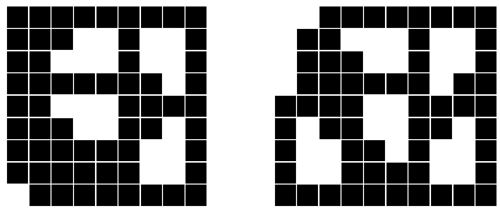




Problem was, I could not find a way to enclose three X's in that bounding shape.

So now, the question is posed: **Can one find a single bounding shape which can enclose any triplet of pentominoes?** I don't know. The I- and X-pentominoes seem to be the troublesome two. If a bounding configuration affords one, it seems to fail on the other. If anyone discovers such a shape, I'd love to see it.

Finally, I also managed to discover figures which enclosed a specific quartet and even quintet. Both cases involve the friendliest of pentominoes: the "P" shaped one. For each design there is only one solution:



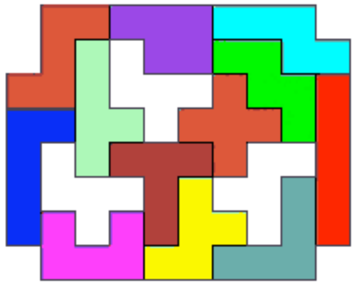
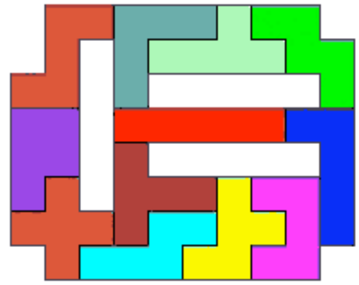
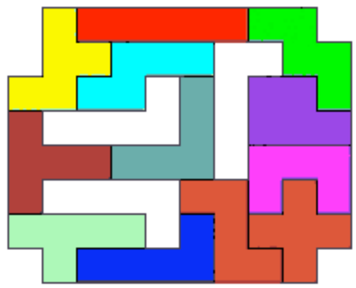
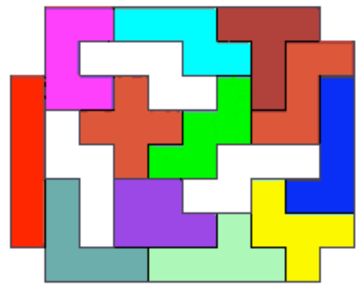
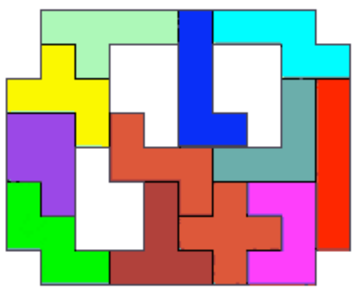
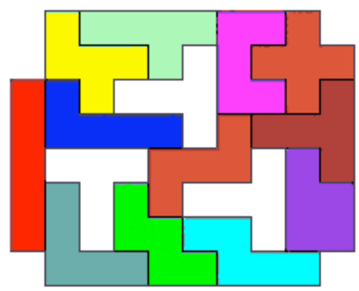
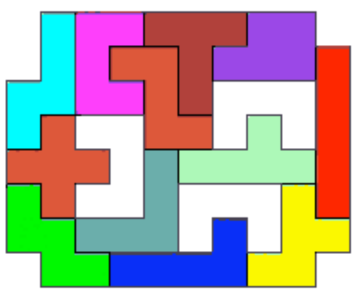
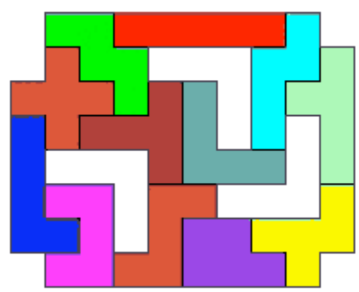
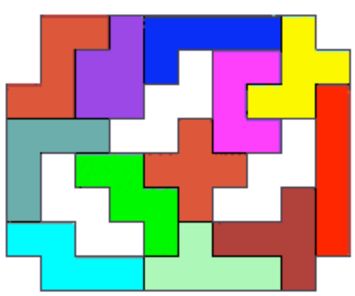
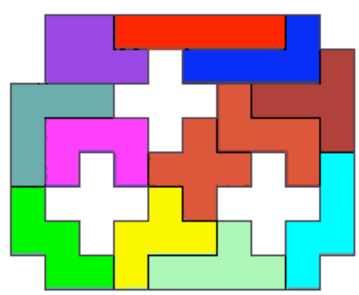
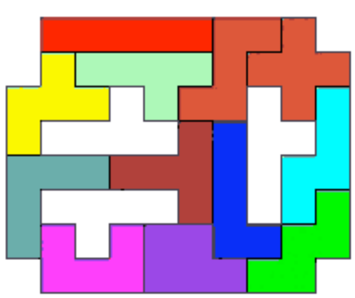
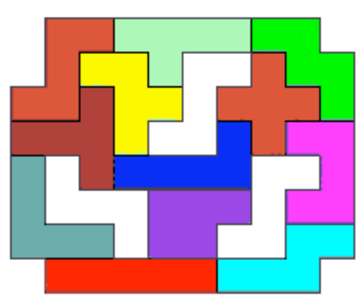
I do not know whether quartets or quintets of other shaped pieces may be enclosed. And I conjecture that no sextet of pieces can be enclosed.

- Eric Harshbarger, 2 Jan 2005

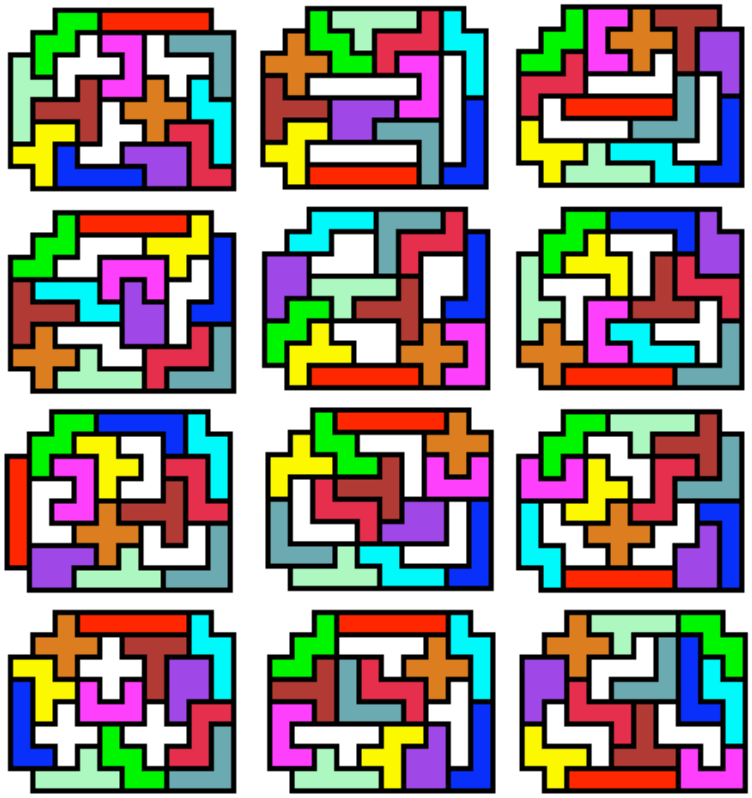
Update (14 Jan 2005): Michael Keller and Aad van de Wetering both attacked my triplet question above and within a day of each other sent me emails. The former's included the following kind words:

> It's too bad there isn't an annual award for best new pentomino problem. Eric Harshbarger would be my nominee for 2005, even though the year has just begun...
>
> About an hour ago, with the help of Aad van de Wetering's polyomino solver (to quickly test various positions), I found a shape which would work for both I and X. It took 45 minutes or so to find solutions for the other ten pentominoes. For eight of the shapes, the first solvable position I found had a unique solution. LNPU had multiple solutions: 18 for P. With a bit more work, I found hole positions for all 12 which had unique solutions. I attach only one of the 12 solutions, for I-I-I.
>
> If Eric wants to post the solution on his page, he can surely do so. (I can provide all 12 solution diagrams). I would like to include a link from my polyform pages...
>
> Michael Keller
> http://home.earthlink.net/~fomalhaut/polyenum.html

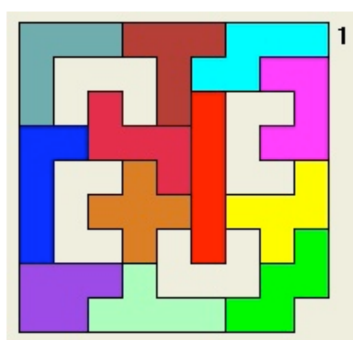
Michael Keller's solution

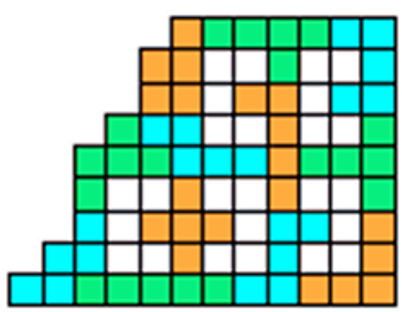
Aad van de Wetering's solution



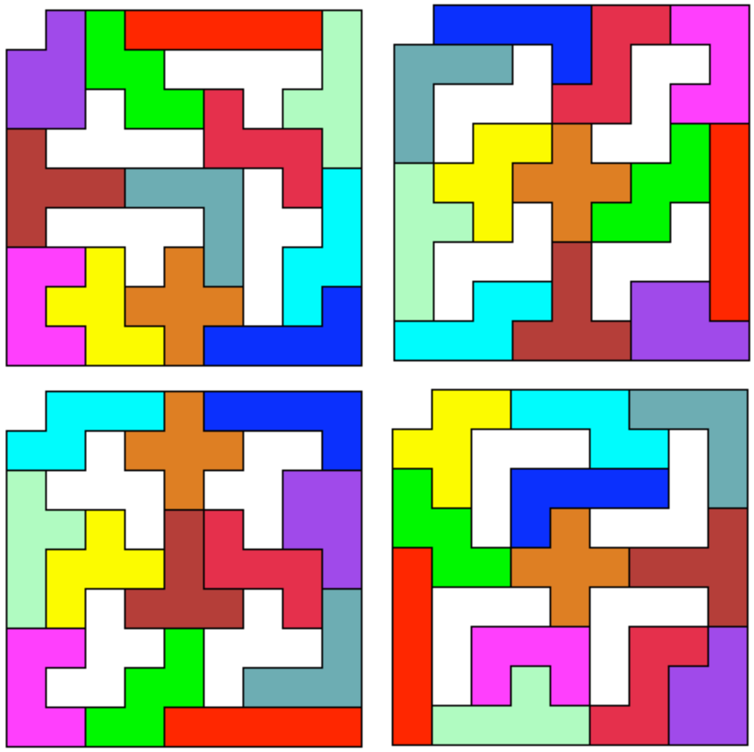
Update (18 Jan 2005): Michael Keller has been busy. He has now provided me with a "missing quartet" configuration for a shape other than the P-pentomino. Below is one involving the U-.

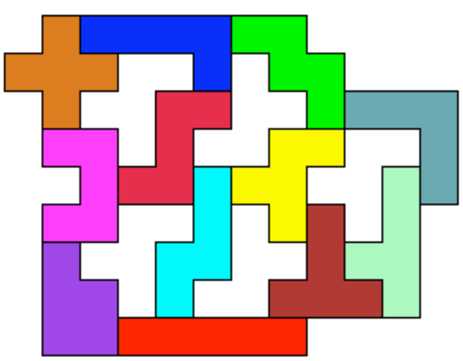
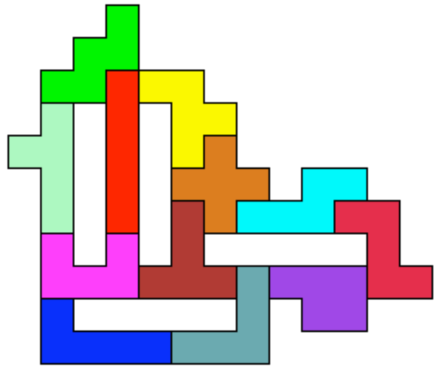
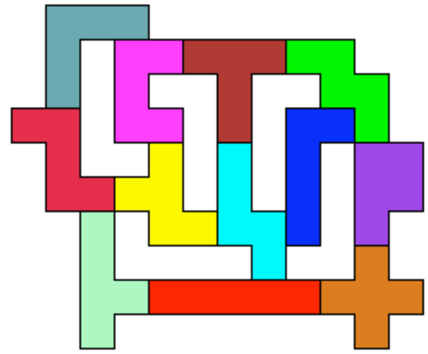
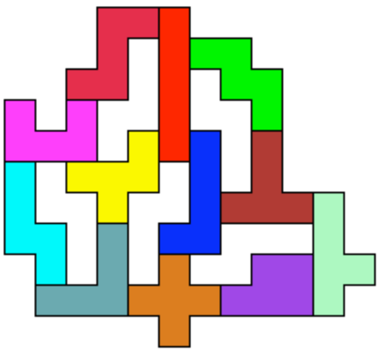
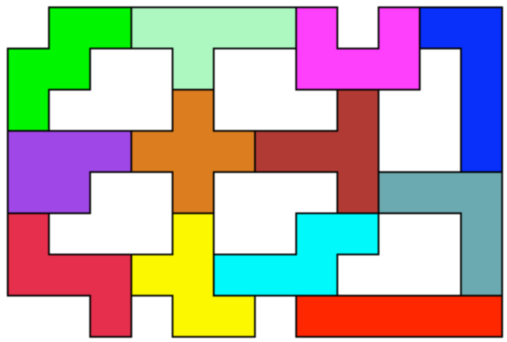
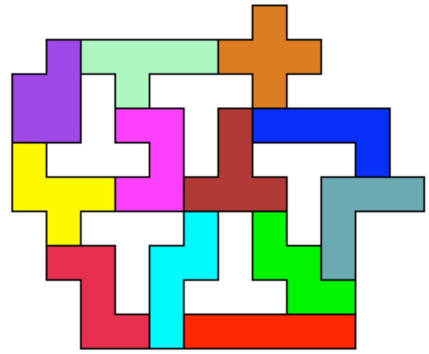
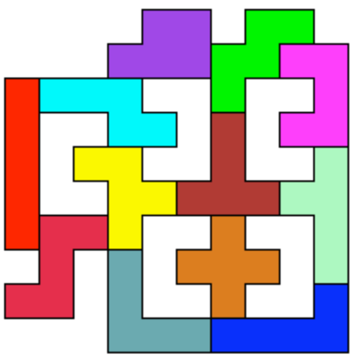
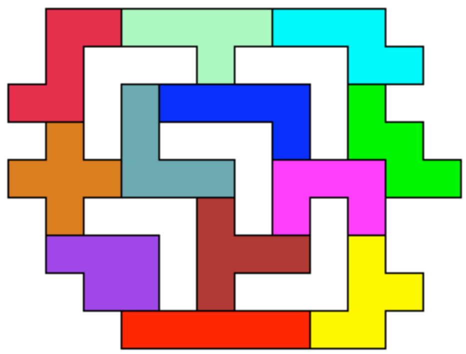
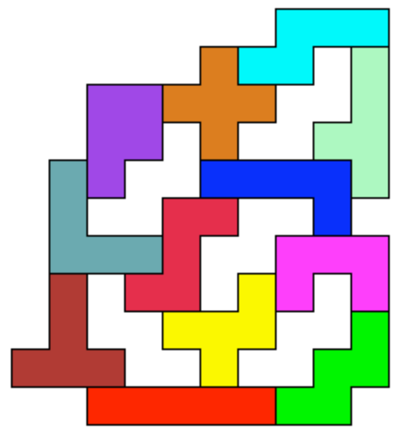
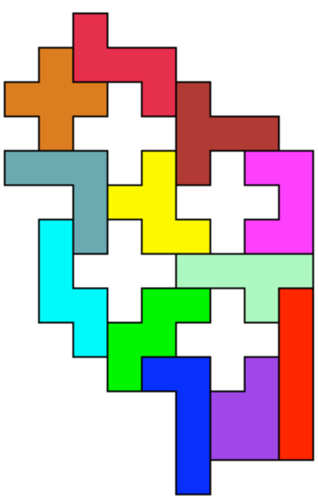
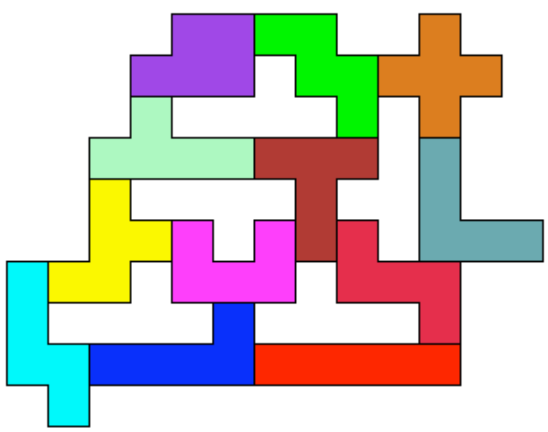
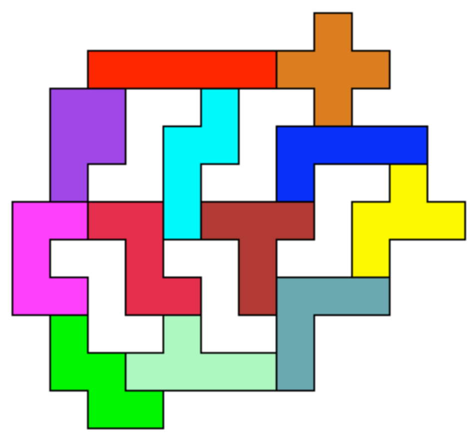


Update (29 Jan 2005): More submissions. The first is from Kate Jones of Kadon Enterprises. She illustrates an enclosure of 5 U-pentominoes:



Next, Aad van de Wetering returns with many solutions to 4, 5, and even a 6 enclosure of various pieces:



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Landlocked Pentominoes

One of the most common puzzles for the 12 pentomino pieces is to try to arrange them into a 6x10 rectangle. There are 2339 ways to do this. Here is one example:

One will note that in this solution three of the pieces, the T-, W-, and Z- pentominoes, do not touch the edge of the the 6x10 box; they are properly in the interior of the figure. I refer to these pieces as being "landlocked".

I began thinking about this landlocked property, and wondered how the other 2338 solutions compared to this one. Were there any solutions with 4 landlocked pieces? Even 5? and how about at the other extreme: were there any solutions which had only 1 landlocked piece? Or even any with no landlocked pieces whatsoever?

I posed this question to a fellow whom I knew to be a pentomino enthusiast. A short time later this person, Tom Saxton of Idle Loop software answered my question. After some computer analysis on his part, he provided the following information: there are no 6x10 solutions with 5 or more landlocked pieces. Below that number the breakdown is as follows:

LL	# of SOLUTIONS
4	207
3	1111
2	864
1	155

And, to my surprise, there were some solutions that have NO landlocked pieces, but only a couple. Here is the image Tom sent to me:

As you can see, in each solution all of the pieces extend out to the edge of the rectangle.

I guess that settles that.

Thanks, Tom.

- Eric Harshbarger, 3 January 2005

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Numerous Solutions

Below is just one way to arrange the twelve pentominoes into a 6x10 rectangular shape. There are, in fact, 2339 different ways to do this.

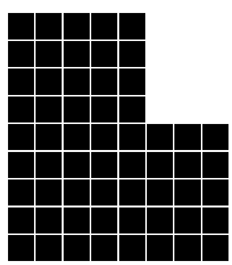
Computers have made solving pentomino configurations nearly a trivial matter. Although it is still very enjoyable to solve such puzzles when purposely avoiding the help of machines, the internet can provide access to various pentomino solvers, for those so inclined.

One might fear, then, that there are few questions left unanswered concerning pentominoes.

Hardly. One simply needs to think of new (or tougher) questions.

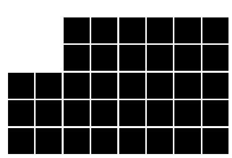
Here's one. The above rectangle has 2339 different solutions. **Is there some configuration that affords more answers?**.

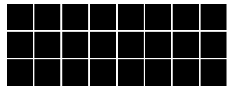
The answer is "yes", but here is an example of a question that would be almost pointless to ask unless access to computers was available. To try to find thousands of solutions by hand of a particular configuration would be ridiculously difficult. But with computer analysis examples can be found. The next figure has 3763 solutions:



Now, actually finding this particular configuration was not a trivial task. Determining how many solutions was easy enough, but how did I decide to test that shape in the first place? Well, I really just stumbled upon it while creating the Pentomino Daily Calendar (it is a figure entitled *Letter "L"*, used for 25 July).

Another configuration produced even more solutions:



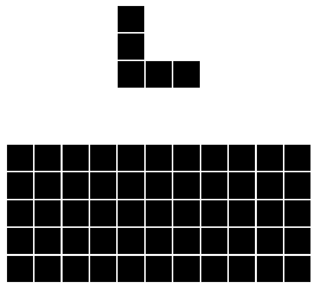


This shape (used on 15 March of the Calendar) has an amazing 5027 solutions.

Is there something with more?

Specifically, the following question can be posed: **What configuration of 60 squares affords the most pentomino solutions?**

The potential candidates don't even have to use fully connected squares. For example:



has 932 solutions (called, simply enough, *5x11Box, Plus One (V)*). Of course, it is no where near the 5,000+ number from above.

Can someone provide a configuration with more solutions? Surely so. I suppose a computer program could be written to generate and test every potential candidate, but I'll leave that to someone else.

Believe it or not, I do have other things to do with my time.

If anyone does get higher numbers, please let me know, though; I'll be happy to post the results here.

- Eric Harshbarger, 5 January 2005

P.S.: Another interesting question related to this topic: **can we find a sequence of configurations such that the number of solutions for the first is 1, the second is 2, the third 3, and so forth? How high can we proceed sequentially?**

Good thing we have those computer solvers handy...

Update (14 Jan 2005): Aad van de Wetering of Holland sent me this list of figures, each with over 10,000 solutions (he did not guarantee this was a complete list):

```
7x9 with vertical domino at top left, monomino bottom right - 16720
8x8 with L-tetromino at a corner - 15512
7x9 with vertical domino at top left, monomino top right - 15482
8x8 with dominos on top two corners, one horizontal - 14791
7x9 with horizontal domino at top left, monomino top right - 14657
8x8 with L-tromino at a corner, adjacent corner - 14541
```

```
7x9 with horizontal domino at top left, monomino bottom right - 14191
7x9 with horizontal domino at top left, monomino bottom left - 13822
7x9 with L tromino at top left - 12945
7x9 with vertical domino at top left, monomino bottom left - 11381
7x9 with vertical tromino at top left - 10130
7x9 with horizontal tromino at top left - 10063
```

Thanks!

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Transmuting Pentominoes

Even the 12 pentominoes themselves can be quite fascinating, ignoring how they might be assembled into a shape. The fact that there are 12 different pentomino shapes no doubt has helped them gain popularity among puzzlers. A dozen pieces is enough to earn a complexity that tantalizes someone, but not so great that the number of combinations becomes daunting.

Each being composed of 5 squares thus affords a total of 60 cells when building shapes. 60 is quite a friendly number in the sense that it has many divisors and forms many possible rectangles: 6x10, 5x12, 4x15, and 3x20 -- all of which can, in fact, be formed with the pieces.

I've also been interested in how the pieces, their shapes, relate to one another. I'm sure there's a game or a puzzle waiting to happen here, but I haven't yet figured it out. But it's still worth noting how easily one pentomino may be "transmuted" into the others.

I'll define "transmuting a pentomino" as taking said shape, removing a single square from it, and then replacing that square so as to form another pentomino (if placed in the original position, the original pentomino is reformed, of course).

I-pentomino

For example, the long, straight I-pentomino can be transmuted by taking one of its squares off its end and reattaching the square elsewhere. There are three (non-trivially) distinct possibilities:

1. Place the square back on one of the ends (either the original place or the other end of the currently 4-long line of squares). This obviously reforms the I-pentomino.
2. Place the fifth square adjacent to one of the end squares, but on the "side" of the 4-long line of squares. This will form the L-pentomino in some orientation.
3. Place the fifth square along the edge but closer to the center of the line of 4 squares remaining. This will form a Y-pentomino in some orientation.

L-pentomino

Y-pentomino

These examples are illustrated to the right (the red square is the replaced piece -- the symmetric cases should be clear).

Note that the L- and Y-pentominoes are the only ones that can be transmuted from the I-pentomino in one step (other than reverting back to the I- itself). I will say that those two pieces are "1 degree (of transmutation)" away from the I-pentomino (the I-piece is 0 degrees away from itself).

This terminology allows us to then show how closely all of the pieces are "related" to one another in terms of degrees of transmutation. Here is a table:

Degrees of Transmutation Between Pentominoes

	F	I	L	N	P	T	U	V	W	X	Y	Z	sum
F	0	2	1	1	1	1	1	1	1	1	1	1	12
I	2	0	1	2	2	2	2	2	3	2	1	2	21
L	1	1	0	1	1	1	1	1	2	2	1	1	13
N	1	2	1	0	1	1	1	1	1	2	1	1	13
P	1	2	1	1	0	1	1	1	1	1	1	1	12
T	1	2	1	1	1	0	2	1	2	1	1	1	14
U	1	2	1	1	1	2	0	1	2	2	1	1	14
V	1	2	1	1	1	1	1	0	1	2	1	1	13
W	1	3	2	1	1	2	2	1	0	2	2	1	18
X	1	2	2	2	1	1	2	2	2	0	1	2	18
Y	1	1	1	1	1	1	1	1	2	1	0	1	12
Z	1	2	1	1	1	1	1	1	1	2	1	0	13

The sum column to the right gives an indication of the overall "friendliness" of a piece to its peers. The F-, P-, and Y- shapes are most easily transmuted to all of the other shapes, while the I- is certainly the most difficult. The I- and W- pentominoes are the most distantly related: the only combination requiring 3 steps of transmutation.

Again, I'm not sure where all of this information leads, but it does keep my mind occupied at times.

- Eric Harshbarger, 12 January 2005

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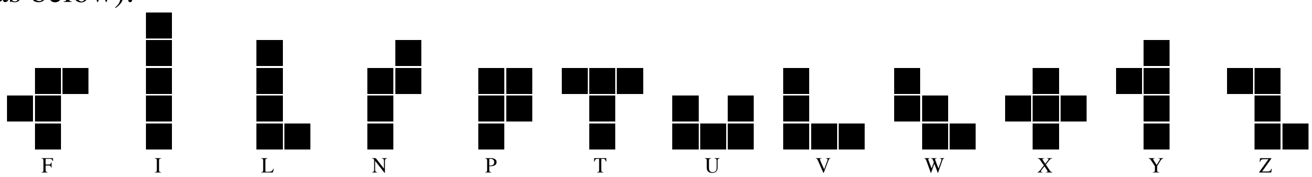
Pentominoes As Letters

This page is more about wordplay rather than geometry. Both topics are of interest to me, so this short article should not be a great surprise.

The fact that the twelve pentominoes are assigned 12 letters of the alphabet for easy distinction (as below):



leads to a very obvious question: **what is the longest word that can be formed using only "Pentomino Letters"?**

"Word" is a rather subjective, um, word... but let's not get too technical. I'll settle for an entry in *Webster's Third New International Dictionary*.

These types of puzzles can be facilitated greatly by using searchable Dictionaries-on-CDs and such, but I always like to at least start considering them using only my inherent vocabulary (and simply checking them against the dictionary).

Having only two vowels (and the "I" and "U", at that) is not very helpful, but some words come pretty easily: ZIPPY, FUZZY, and such.

Turns out FUZZY's adverbial form works as well: FUZZILY.

FLUFFILY is even better ("in a fluffy manner" reads the dictionary... um, okay).

That's eight letters long... now it starts getting tougher...

Allowing hyphens in the words, WILLY-NILLY is a nice find. And LILLYPILLY is good with no hyphen.

But what about PITIFUL?

Only seven.

And PITIFULLY? Yep, that's good.

Nine letters long.

And finally, it turns out, UNPITIFULLY is a valid word as well (a synonym of PITILESS).

So, that's as far as I've gotten without writing a program to scan word lists. I'll update the page after I do that. In the meantime, if anyone thinks of any more pentomino words 11 letters or longer, please let me know.

- Eric Harshbarger, 12 January 2005

P.S.: all of the examples above repeated at least one of their letters. **What's the longest pentomino word you can think of that does not use a letter more than once?** Obviously 12 letters long is the theoretical maximum -- and that is also clearly impossible. The best I can see is the 6-letter-long: UPLIFT.

Update: friend, Tracy Cobbs, discovered UNFITLY... a no-repeat 7 letter word.

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Pentominic Surfaces

[Note: this webpage uses Java applets to illustrate the three dimensional nature of the figures discussed. If you don't see colorful objects that are mouse-draggable, then your web browser is not java enabled, and you are missing out... If you do have trouble viewing the applets, I would recommend you upgrade your web browser to the latest Firefox offered at Mozilla.org]

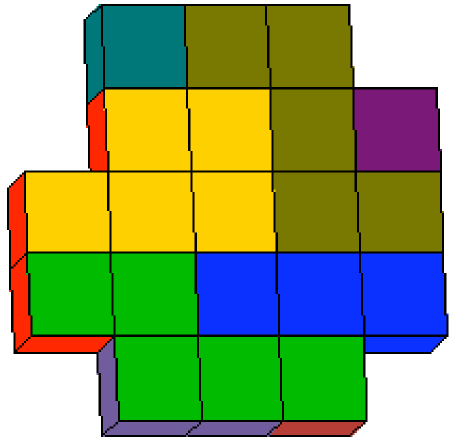
Often, when people discuss pentominoes they are working with flat, two dimensional versions of the shapes, and the figures that are filled in or assembled are likewise two dimensional.

One variation to this is the notion of "solid pentominoes". If one assumes that the 12 pieces have a thickness equal to the unit cube's side length, then the pieces take on a third dimension (the I-pentomino is then considered to be constructed of 5 1x1 cubes stacked together in a line). One still retains the 12 shapes, they are now just blocks instead of thin "paper" (note that when extended to three dimensions, it is possible to create new shapes by adjoining five cubes in any direction -- these are usually referred to as 'penta-cubes' and are obviously a superset of the solid pentominoes). The 12 solid pentominoes also lend themselves to interesting assemblies -- now in three dimnesions. For example a 3x4x5 Box (60 unit cubes) can be constructed.

This article, however, is about a variation of pentominoes that I have not ever seen discussed. I will assume that the 12 pieces are flat (not solid), but I will also assume that they are made of a foldable material such as paper, and that the pieces may be creased along any edge where two of the unit squares meet.

This folding allows us to approach three dimensional figures, but instead of filling them with cubes, we are simply trying to cover the surface of the figures with folded versions of the pentominoes.

For example, below is a three dimensional shape that has a surface area of 60 unit squares. It is covered by the 12 pentominoes (you should be able to rotate the figure by click-dragging your mouse pointer over it).

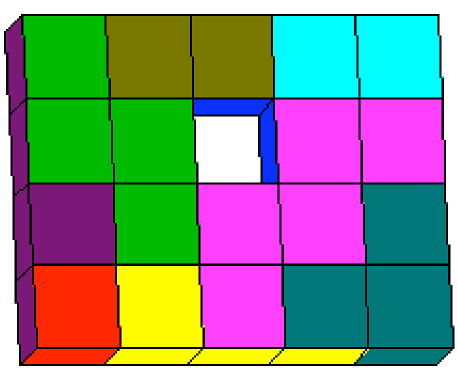


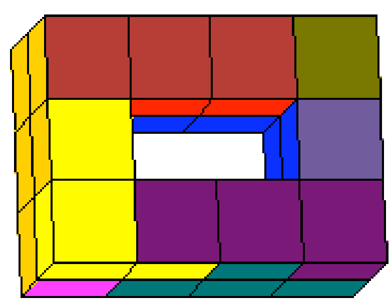
It might be a bit tough to recognize some of the pentominoes in their folded state, but I do use a consistent color scheme:

F I L N P T U V W X Y Z

The above shape is fairly simple. It would have been nice to have covered the surface area of a proper rectangular box, but no such box with a surface area of 60 unit squares exists.

Topologically speaking the next two figures are different than the first: they have one hole each. Both are shown to be solvable.



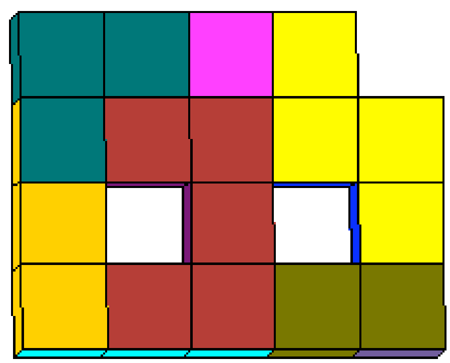


Having never seen a discussion of this variation of pentominoes, I have just started fiddling about with them. These are the only examples I have solved. I would welcome any contributions from others.

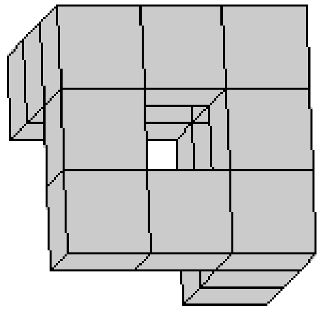
Obviously, finding potentially solvable figures, and then actually solving them, is a non-trivial matter. I used a "flat" pentomino solving computer program after mentally "unfolding" the target figure's surface. After the program found a solution, I refolded the answer into the 3D shape. I wrote the 3D viewing Java applet in a few hours.

The writing of a computer solver that would accommodate various surfaces (rather than just flat figures) is on my pentomino program "wishlist" (more on that in a later article).

Finally, here is a covered object with two holes in it.



Can anyone find one with three holes? For that matter, how many figures of surface area 60 with three proper holes even exist? Below is one possibility, but I have yet to figure out if it is coverable with pentominoes.



- Eric Harshbarger, 18 January 2005

Update (19 Jan 2005): Kate Jones of Kadon Enterprises, Inc. mentioned to me that in 1982 she was given a cube exactly covered by the 12 pentominoes. Michael Waitsman (1946-1995) gave it to her. At first I was questioning how this could be done since the pentominoes cover 60 squares, and a cube, with 6 faces (assuming of 3x3 unit squares each) would only require 54 squares total.

Quickly, though, I realized it might be possible if the edge lengths of the cube were not 3 units, but sqrt(10) units (the hypotenuse of a right triangle with legs 1 and 3 units). Sure enough, that's the trick. Such a configuration is mentioned in Golomb's *Polyominoes*; I had forgotten about it.

I soon made my own version out of folded paper. It is presented below (click for larger picture):

While this covering does not conform exactly to the guidelines I established above (this example does not fold along edges where unit squares meet), it is still an amazing configuration.

Update (20 Jan 2005): I have found a solution to the "3 ring" figure above.

Update (24 Jan 2005): Related to the first update above (concerning the covering of a cube with the 12 pentominoes): Today at a local coffeeshop I doodled around for about an hour and found a configuration of the 12 pieces which fold up into *two* separate cubes, each with side length of sqrt(5). Here is the example:

I have created a PDF file available for download (the color scheme is a bit different). Nethier the single cube nor two smaller cubes discussed in these updates are "nice"; they have corners at which a pentomino folds around and touches itself. There are known to be examples of a single cube covering where this "niceness" is obtained, and there might be such examples for the two smaller cubes, but since I have never seen the two cube problem discussed before, I'm not sure (and I have not taken the time to find one myself, yet).

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Neighborly Pentominoes

The 6x10 Rectangle

Let us revisit a classic pentomino configuration: the 6x10 rectangle. Below is one solution.

An interesting way to codify the arrangement is to consider each piece and count how many other pieces it touches ("count its neighbors"):

```
F - 3
I - 4
L - 3
N - 4
P - 3
T - 7
U - 3
V - 2
W - 5
X - 4
Y - 6
Z - 4
```

Now, let's sort that arrangement numerically (greater numbers to lesser) rather than alphabetically. We can construct a 12 digit number like so: 765444433332 (yes, we are losing some information here, namely which letters went with which numbers, but I'm just interested in the numerical part).

Many questions come immediately to mind; the most obvious of which is: how does that 12 digit number compare to likewise generated numbers from other 6x10 rectangle solutions?

Here's another one:

It generates the following 12 digit number: 544444443222.

So we know with certainty that the generated numbers are not all going to be the same when considering the 6x10 rentangles. But I seriously doubt each is unique.

So what are all of the associated 12 digit numbers? This is all just another way to classify those 2339 solutions to the 6x10.

What is the largest such number generated?

How about the smallest?

If anyone out there wants to do such an analysis, please let me know the results... I'll happily post them here (with credit to you).

Instead of listing the digits out to form a 12 digit number, what if we summed them up?

The first example adds up to 48, the second: 42 (we know the sums always have to be even since neighbors always come in pairs: if a pentomino touches another, then that "touching" will be counted a second time when the second pentomino is looked at).

How do the sums of all 2339 solutions compare? It seems possible that the highest 12 digit number very well may *not* lend itself to the highest sum... but is this the case?

Other Configurations

But why restrict ourselves to just the 6x10 retangle? Let's throw these questions wide open. Let's consider all fully connected configurations (all of the pentominoes must properly touch one another).

1. What is the greatest 12 digit number that can be generated?
2. What is the least 12 digit number that can be generated?
3. Greatest sum?
4. Least sum?

Interestingly, the first of those questions is going to necessitate an unusual concession: we can't make due with base-10 digits. Why not? Well, some of the pentominoes can, in fact, be made to have more than 9 neighbors.

Observe:

Having 10 or 11 neighbors throws a wrench in that base-10 generator. It can easily be remedied, of course, if one is willing to accept a base-12 notation where "A" stands for "10" and "B" is substituted for "11". With this in mind, the two configurations would generate the following numbers:

```
Surrounding L (left): B33333333333 (sum = 44, base-10)
Surrounding I (right): B33333333322 (sum = 42)
```

Questions #2 and #4 above are actually somewhat trivial. By stringing the 12 pieces together in a long configuration, you can generate a minimal number of 222222222211 (sum = 22); any lesser number or sum can only be generated by disconnected figures.

But questions #1 and #3 don't seem to be quite as simple.

Anyone want to give them a shot?

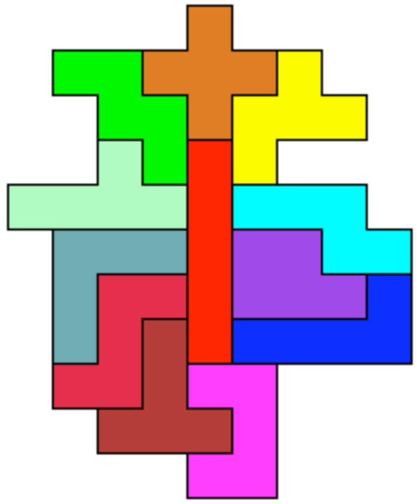
- Eric Harshbarger, 1 February 2005

Update (1 Feb 2005): Within a few hours of writing this article, Aad van de Wetering of Holland sent me two text files. One of them lists all 2339 generated 12 digit strings and their sums. The second prints out all of the configurations in text format (a handy thing to have for this and other 6x10 questions).

From the first file one learns that the greatest 12 digit number generated from 6x10 retangles is 864433333333 (and there is only one solution that generates that number).

It's sum is only 46, however; less than the maximum sum found therein. That maximum is 50, which is achieved two dozen times (the least sum found is 38; occurring only 3 times).

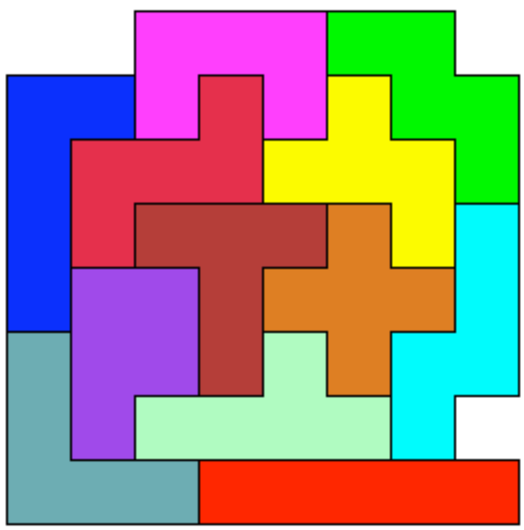
Update (early Feb 2005): while I was in Europe, Aad van de Wetering and I traded several more emails about the topics on this page. At first he believed B33333333333 == 44 was probably the greatest sum which included a 'B' (a pieces with 11 neighbors). I replied, that, no, at least a sum of 46 could be found... I had sketched an example on graph paper, but did not have the time to update these pages right then. Here answered the next day with affirmation: he had found this example, B44333333333:



There are other examples summing to 46, but I suspect that this is now maximized.

Then I mentioned that I had found various configurations whose neighborly sums had reached 52 (though, obviously, no single piece in the formations touched all the other pieces). I suspected 54 might be possible, but had no answer (I mentioned that it would likely involve a configuration that was an 8x8 square less four unit squares).

Soon, he responded with an answer which was, "far from easy to find."



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Longest Narrow Path

This is a pentomino problem that I did not think of myself, but which kept me busy nonetheless. I'm not sure who originally thought of the version I was working on, but I'll describe how I came to know it.

In late February 2005 I stumbled across the pentomino contest described on another webpage. Actually the problem involves using both pentominoes and tetrominoes (the four shapes each formed with 4 squares). The puzzle is to use the pieces to enclose a "narrow" (single unit width) path of longest length. The path must be closed at both ends, cannot be a loop, have no branches, and must be properly enclosed (no pathway corners touching).

Sometime during the next week I was playing boardgames with some friends. Someone brought out a set of pentominoes and that reminded me on the contest. I could not remember the exact path length given as an example of the contest page, but I knew "it was somewhere in the forties."

What I had completely forgotten was the tetrominoes were used as well. So, while playing Russian Rails over the next few hours, my friends and I tried to enclose the longest path possible. I was stumped by the fact that we were coming nowhere near 40. My friend Robert kicked things off by getting a length of 31.

About an hour later I squeaked out a score of 32, and with half an hour of that I upped my answer to 33.

But I *knew* the example I had seen was in the 40s.

The 33 was the highest achieved that session, and when I got home I immediately searched for the original webpage again to see what was going on.

Oh... that example used tetrominoes too...

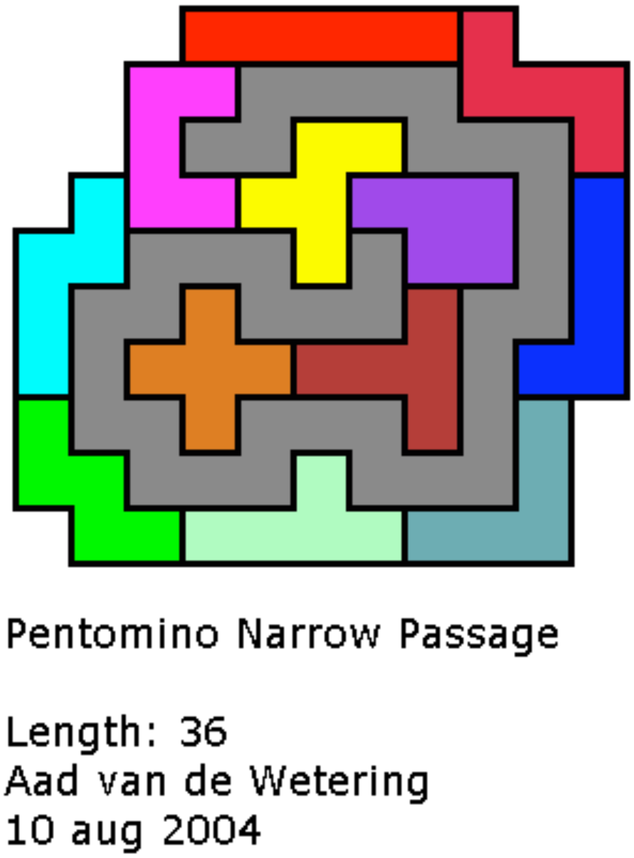
That explained that.

But what about using just pentominoes?

The answer of 33 seems pretty good.

Surely someone has looked into this... and maybe better answers found.

I emailed some friends-in-pentominoes about the question, and by the next morning Aad de Wetering of the Netherlands told me that the best solution he had ever seen was one he had created. It is pictured below and has a path length of 36.



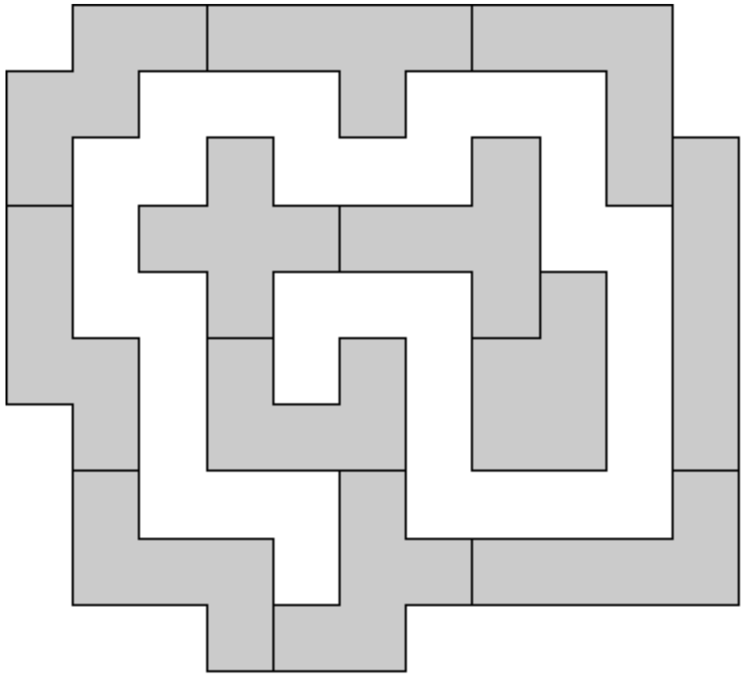
He was pretty sure that no answer above 36 was possible.

36. Wow. Granted, I had only played with the problem a couple of hours the night before, but an improvement of 3 over my best answer certainly impressed me.

That next day I decided to tackle the problem again. Even if I could not get 37 (which had not been formally ruled out yet), I still wanted to see if I could achieve my own configuration for 36 (though it was not even clear that there could be multiple 36 length solutions).

I spent about 7 hours attacking the problem. At my house on the couch. At the local coffeeshop. Back at home at my desk in fromt of my computer. By 10:00 at night I was about ready to concede defeat, when, "eureka!"

I found my own answer for path length of 36:



I still don't know if 37 is possible, but I, too, am now highly skeptical.

If anyone finds a better answer, please let me know (or, if you find your own 36 solution, I'll be happy to present that as well).

- Eric Harshbarger, 5 March 2005

Update (12 Mar 2006): Dan Edlefsen contacted me to say that he had found his own 36-length narrow path with pentominoes. He also doubts that a path of length 37 is possible. However, he did pose an additional question. Using *two* complete sets of pentominoes, what is the longest narrow path that can be form? He has managed a length of 79 but speculates that up to 82 my be possible.

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