## My Towers of Hanoi



Designed and made by Diniar Namdarian, 2016; sleeve made by J. A. Storer. (laser cut plastic, $7+1 / 8^{\prime \prime}$ high by $5+1 / 8^{\prime \prime}$ wide $\mathrm{x} 1 / 2^{\prime \prime}$ thick)
Referring to the figure on the right above, on the left and right are 4 rectangular pieces where in the left the background color is a light blue and on the right it is a darker blue. Both sides have raised red plastic showing 4 size cups, labeled here as $\boldsymbol{1}$ to $\mathbf{4}$ on the left and $\boldsymbol{A}$ to $\boldsymbol{D}$ on the right. In the center is a single piece, labeled here as $\boldsymbol{S}$, that has two sections connected by strip under the pieces. The $S$ piece forms a slider that can go up and down and carry / push pieces with it.

The puzzle name comes from the classic Towers of Hanoi problem, here the basic problem is:
"Problem 5": Exchange the two towers.
The puzzle directions also give four other problems:
1: Invert the left tower.
2: Exchange 2 and $\boldsymbol{B}$, exchange $\mathbf{4}$ and $\boldsymbol{D}$.
3: Invert both towers, exchange $\boldsymbol{1}$ and $\boldsymbol{A}$, exchange $\mathbf{3}$ and $\boldsymbol{C}$.
4: Invert both towers.
The following pages show solutions where all moves are simple straight-line movement:
Problem 1: 99 moves
Problem 2: 95 moves
Problem 3: 93 moves
Problem 4: 106 moves
Problem 5: 64 moves

## Counting $S$ Moves

In all of the solutions presented, a single slide of the $S$ piece counts as one move, which may have the effect of repositioning the one or two pieces between the two portions of the $S$ piece.
If there are two pieces on the slider, then a single finger can push the slider, and they move as the slider moves. However, if there is only one piece on the slider, then exactly what happens depends on the puzzle construction.
Consider the $S$ move depicted in the figure below. If the connection between the two parts of the slider does not contact the underside of the pieces on the slider (it goes under the puzzle or is a strip recessed down a bit and not as wide as a piece) then a single finger can push the $S$ piece down where it bumps into piece 4 and finishes the slide with piece 4 in the correct position.
However, this puzzle is constructed with a strip connecting the two halves of the $S$ piece that is as wide as a piece and level with the bottom of the pieces (and contacts the bottom of pieces on the strip), and so piece 4 moves with $S$ as soon as it is pushed, and has to be stopped with a finger when it is in position, or it will be dragged too far down and then a second move has to push it back up.

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| XXX A A |  | $X X X \quad S S S$ A A A |
| 111 \$444 \$ B B B |  | 111 \$ S S S \$ B B B |
| $1111 \$ 444$ \$ B B B | slide S down | $1111 \$$ S S S \$ B B B |
| $222 \$$ S S S \$ C C C | , | $222 \$ 444$ \$ C C C |
| $\begin{array}{llll}2 & 2 & 2\end{array}$ |  | $222 \$ 444$ \$ C C |
| $\begin{array}{llll}3 & 3 & 3 & \text { S S S D D D }\end{array}$ |  | $\begin{array}{lll}3 & 3 & 3 \\ \text { D D D }\end{array}$ |
| 333 S S S D D D |  | 333 D D D |
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If one wants to count these situations as two moves, then the move counts for the solutions presented here can be adjusted upward accordingly.

## My Towers of Hanoi Problem 1 Solution

Here are moves $11,22,33,44,55,66,77,88,99$ of a 99 rectilinear moves solution:


## My Towers of Hanoi Problem 2 Solution

Here are moves $11,22,33,44,55,66,77,88,95$ of a 95 rectilinear moves solution:


## My Towers of Hanoi Problem 3 Solution

Here are moves $10,20,30,40,50,60,70,80,93$ of a 93 rectilinear moves solution:


## My Towers of Hanoi Problem 4 Solution

Here are moves $12,24,36,48,60,72,84,96,106$ of a 106 rectilinear moves solution:


## My Towers of Hanoi Problem 5 Solution

Here are moves $7,14,21,28,35,42,49,56,64$ of a 64 rectilinear moves solution:

|  |  | Mo | ve | X |  |  |  |  |  | 14 | M | Mov | Ve |  |  |  |  |  |  | \#21 |  |  | e |  |  |  |  |  |  |
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| A | A | A |  | S | S S |  | 2 | 2 | 2 | B B | B |  | S |  | S |  | 2 | 2 | 2 | B B | B |  | S | S | S |  | 3 | 3 | 3 |
| A | A | A |  | S | S S |  | 2 | 2 | 2 | B B | B |  | S |  | S |  | 2 | 2 | 2 | B B | B |  | S | S | S |  | 3 | 3 | 3 |
| B | B | B | \$ | S | S S | \$ | 3 | 3 | 3 | C C | C | \$ | S |  | S | \$ | 3 | 3 | 3 | C C | C | \$ | S | S | S | \$ | 4 | 4 | 4 |
| B | B | B | \$ | S | S S | \$ | 3 | 3 | 3 | C C | C | \$ | S |  | S | \$ | 3 | 3 | 3 | C C | C | \$ | S | S | S | \$ | 4 | 4 | 4 |
| C | C | C | \$ | 1 | 11 | \$ | 4 | 4 | 4 | D D | D | \$ | A |  | A | \$ | 4 | 4 | 4 | D D | D | \$ | 2 | 2 | 2 | \$ | X | X | X |
| C | C | C | \$ | 1 | 11 | \$ | 4 | 4 | 4 | D D | D | \$ | A |  | A | \$ | 4 | 4 | 4 | D D | D | \$ | 2 | 2 | 2 | \$ | X | X X | X |
| D | D | D |  |  |  |  | X | X X | X | 11 | 1 |  |  |  |  |  | X | X | X | 11 | 1 |  |  |  |  |  | A | A $A$ | A |
| D | D | D |  |  |  |  | X | X X | X | 1 | 1 |  |  |  |  |  | X | X | X | 11 | 1 |  |  |  |  |  | A | A | A |
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| C | C | C |  | S | S S |  | 3 | 3 | 3 | C C | C |  | S |  | S |  | 4 | 4 | 4 | D D | D |  | S | S | S |  | 4 | 4 | 4 |
| D | D | D | \$ | S S | S S | \$ | 4 | 4 | 4 | D | D | \$ | S |  | S | \$ | X | X | x | 11 | 1 | \$ | S | S | S | \$ | X | X | X |
| D | D | D | \$ | S | S S | \$ | 4 | 4 | 4 | D D | D | \$ | S | S S | S | \$ | X | X | X | 11 | 1 | \$ | S | S | S | \$ | X | X | X |
| 1 | 1 | 1 | \$ | B | B B | \$ | X | X X | X | 11 | 1 | \$ | 3 | 3 | 3 | \$ | A | A | A | 22 | 2 | \$ | C | C | C | \$ | A | A $A$ | A |
| 1 | 1 | 1 | \$ | B | B B | \$ | X | X | X | 11 | 1 | \$ | 3 | 3 | 3 | \$ | A | A | A | 22 | 2 | \$ | C | C | C | \$ | A | A | A |
| 2 | 2 | 2 |  |  |  |  | A | A | A | 22 | 2 |  |  |  |  |  | B | B | B | 33 | 3 |  |  |  |  |  | B | B | B |
| 2 | 2 | 2 |  |  |  |  | A | A | A | 22 | 2 |  |  |  |  |  | B | B | B | 33 | 3 |  |  |  |  |  | B | B | B |
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| D | D | D |  | S | S S |  | X | X X | X | 11 | 1 |  | S | S | S |  | X | X | X | 11 | 1 |  | S | S | S |  | A | A $A$ | A |
| 1 | 1 | 1 | \$ | S | S S | \$ | A |  | A | 22 | 2 | \$ | S | S S | S | \$ | A | A | A | 22 | 2 | \$ |  |  |  | \$ | B | B B | B |
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| 2 | 2 | 2 | \$ | 4 | 44 | \$ | B | B B | B | 33 | 3 | \$ | D | D D | D | \$ | B | B | B | 33 | 3 | \$ | X | X | X | \$ | C | C | C |
| 2 | 2 | 2 | \$ | 4 | 44 | \$ | B | B | B | 33 | 3 | \$ | D | D D | D | \$ | B | B | B | 33 | 3 | \$ |  |  |  | \$ | C | C | C |
| 3 | 3 | 3 |  |  |  |  | C | C | C | 44 | 4 |  |  |  |  |  | C | C | C | 44 | 4 |  | S | S | S |  | D | D | D |
| 3 | 3 | 3 |  |  |  |  | C | C | C | 44 | 4 |  |  |  |  |  | C | C | C | 44 | 4 |  | S | S | S |  | D | D D | D |
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