Math Games

sliding-block Puzzles

Ed Pegg Jr., December 13, 2004

The December issue of *The Economist* contains an **article** with a prominent question. **Has an inventor found the hardest possible simple sliding-block puzzle?** It goes on to describe the ***Quzzle* puzzle**, by Jim Lewis. The article finishes with "Mr Lewis claims that *Quzzle*, as he dubs his invention, is 'the world's hardest simple sliding-block puzzle.' Within the terms of his particular definition of 'simple,' he would seem to have succeeded."

The claim is wrong. Quzzle is *not* the world's hardest simple sliding-block puzzle, no matter how you define "hard."

Sometimes, hard means *lot and lots of moves*. Junk Kato has succinctly described a record setting series for the most moves required with *n* pieces. The basic underlying principle is **Edouard Lucas**' **Tower of Hanoi** puzzle. Move the red piece to the bottom slot.

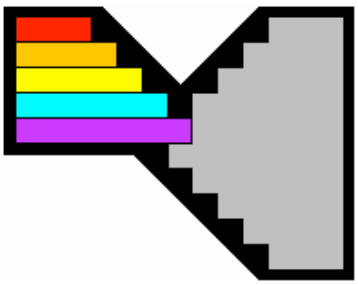

Figure 1. Junk's Hanoi by Junk Kato. From **Modern sliding-block Puzzles**.

Often, "hard" maximizes complexity with a sparse number of pieces. Usually, the fewer the pieces, the better the puzzle. Two people at the forefront of making really hard sliding-block puzzles with just a few pieces are Oskar van Deventer and James Stephens. James started with **Sliding-Block Puzzles**. The two of them collaborated to create the **ConSlide Puzzle** and the **Bulbous Blob Puzzle**. Oskar even went so far as to make a prototype of the excellent **Simplicity** puzzle. If any sliding-block puzzle deserves the label "hardest simple puzzle," it might be this one.

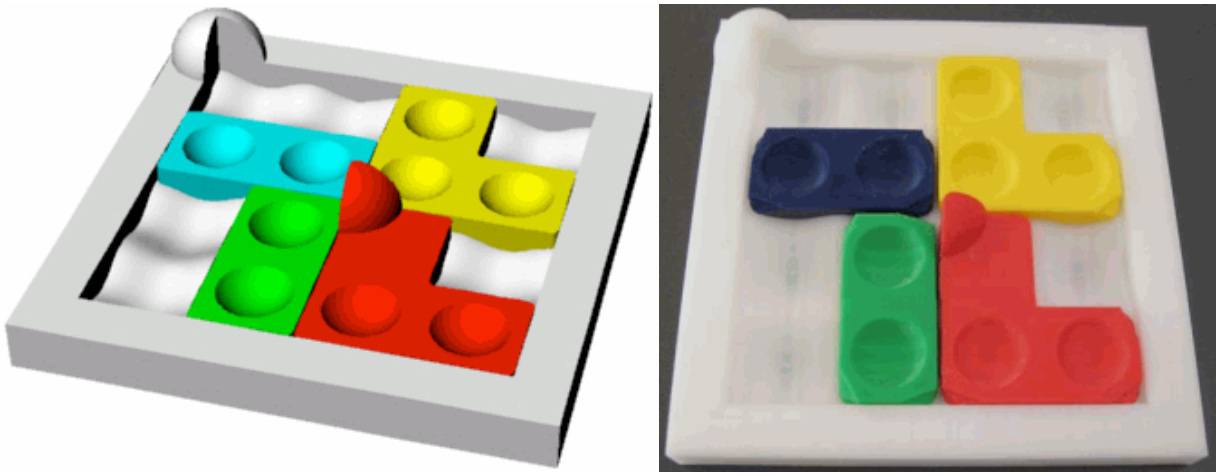

Figure 2. **Simplicity** by James Stephens, prototyped by Oskar. Move the red piece to the upper corner.

Oskar's prototype for **Bulbous Blob** is a thing of beauty, both aesthetically and puzzlistically. I hope it gets sold in stores eventually. These pieces are the diominos and triominos from the set of **rounded polyominos**.

Figure 3. The **Bulbous Blob** puzzle by James Stephens and Oskar van Deventer.

For puzzles with a small number of pieces, a high number of moves, and difficult solutions, my own puzzles used to be record setters. **F-penta** and **Centrifuge** are very difficult 4 and 5 piece sliding-block puzzles, requiring 41 and 63 moves. While writing this column, I tried a tiny modification to Simplicity (which I will call Simplicity 2), and found it requires 68 moves. (Hopefully, someone will topple this new record for 4 piece puzzles.)

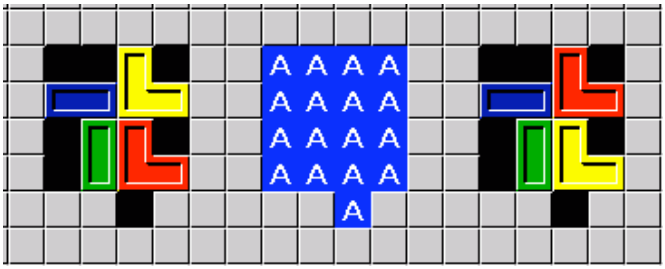

Figure 4. Simplicity 2, "solution found" display for the **Taniguchi solver**.

I created **my puzzles** with **Taniguchi's sliding-block Puzzle Solver**. It's an excellent free program that I regularly mention it on **mathpuzzle.com**, in the hopes someone will create a fantastic new series of puzzles. Is Simplicity 2 the hardest 4 piece puzzle? **Let me know**.

An old-style sliding-block puzzle fits into a 4×5 rectangle, has all pieces 1×1, 1×2, or 2×2, and has goal "move the large piece to position B." The most famous puzzle of this type is **Dad's puzzle**. For comparison purposes, Nick Baxter has put together **a page of 4×5 puzzles**. In ***Winning Ways***, John Conway published the Century puzzle, which has long been the "hardest" old-style puzzle. Here's the solution:

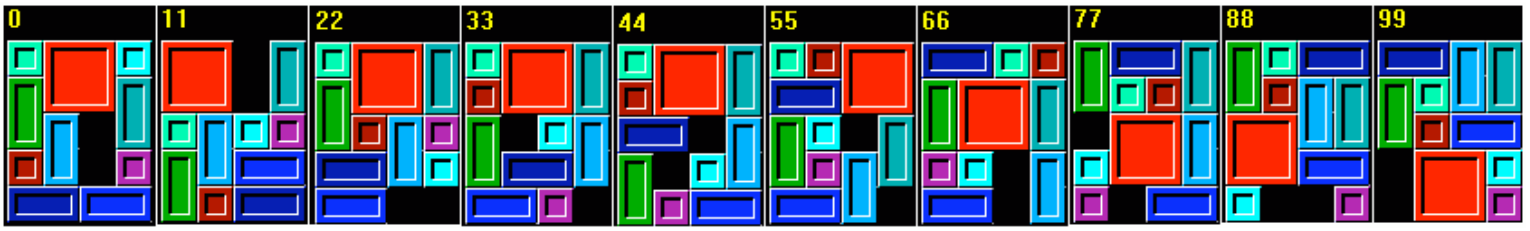

Figure 5. Steps in the solution of John Conway's Century puzzle.

He was later bested by Junk Kato, with the 123-move puzzle **Supercompo**. Cut to last week. Gil Dogon, a software Engineer and part time puzzler/mathematician/hacker, was a "bit irked" by the claims in *The Economist* that Jim Lewis had created the "Hardest 4×5 sliding puzzle problem." He proceeded to find a 138 move puzzle which he calls **Super-Century**. Quzzle requires 84 moves.

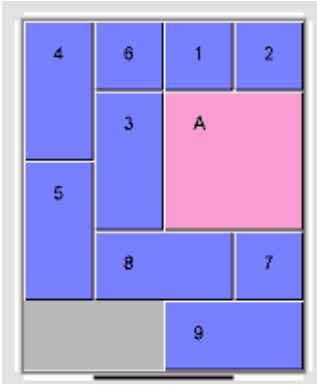

Figure 6. **Super-Century** by Gil Dogon. Move piece A to the lower middle of the board.

Another old-style puzzle has goal "go from board position A to board position B." Conway's **Century and a Half** requires flipping the board position, and needs 150 moves. Junk Kato's **HiFi** requires swapping 2 pieces, and needs 200 moves. Gil Dogon's position above requires 229 moves to flip the board position.

If L-trominoes or other polyforms are tossed into the mix, greater complexities appear. Michael McKee has made very, *very* difficult puzzles with the pentominoes and a 9×9 board-- see **Pentomino Mosaics**. So -- just how difficult can sliding-block puzzles be?

In February 1964, Martin Gardner devoted his *Mathematical Games* column to sliding-block puzzles. He said: "These puzzles are very much in want of a theory. Short of trial and error, no-one knows how to determine if a given state is obtainable from another given state, and if it is obtainable, no-one knows how to find the minimum chain of moves for achieving the desired state."

Recently, Erik Demaine and Robert Hearn proved that sliding-block puzzles are PSPACE-complete. **In their paper**, you can see the AND-gates, OR-gates and other wiring necessary to build a sliding-block puzzle that emulates a computer. In fact, a puzzle made just with dominoes is PSPACE-complete. Since sliding-block puzzles are PSPACE-complete, "hardest" is meaningless. For any question a computer can answer with YES or NO, an equivalent sliding-block puzzle can be set up whose solvability depends on the answer.

What is the hardest sliding-block puzzle with *n* pieces, fitting within a board containing *k* squares? We don't know. For a related problem, Sokoban, Erich Friedman has **compiled some results**.

What is the most elegant sliding-block puzzle? There are two that I particularly like. Serhiy Grabarchuk's **Sliding Digits** is a twist on the Sam Loyd **15 puzzle**. Just swap the positions of the 7 and 8. In Rectangular Jam, the pieces may be rotated around, if enough room exists. The size of the pieces is very delicately calculated, to produce a puzzle with much more complexity than is apparent at first glance. **Rightangular Jam** and **Triangular Jam** are similarly elegant.

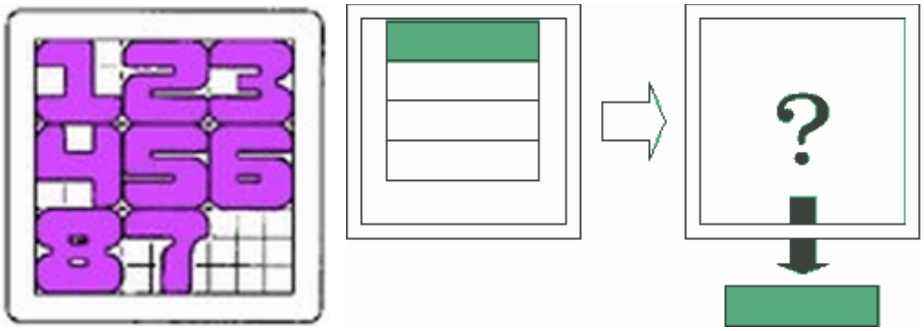

Figure 7. Sliding Digits by **Serhiy Grabarchuk**, and Rectangular Jam by **Hirokazu Iwasawa**.

Many good sliding-block puzzles were discovered *this year*. Worse, some of the nice puzzles were discovered by *me*, just bumbling around. Both of those are good signs that there are lots of elegant, undiscovered sliding-block puzzles, waiting for someone to careen into them.

Incidentally, if you have a good mechanical puzzle, you might want to enter the **Nob Yoshigahara Puzzle Design Competition for 2005**. Any puzzle designer, anywhere in the world, is welcome to enter. Quzzle was an entry in the **2004 competition**, along with many other wonderful puzzles.

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Math Games archives.

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